Effects of T- and P-odd weak nucleon interaction in nuclei: Renormalizations due to residual strong interaction, matrix elements between compound states and their correlations with P-violating matrix elements

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Manifestations of T- and P-odd weak interaction between nucleons in a nucleus are considered. Renormalization of this interaction due to residual strong interaction is studied both analytically and numerically. Expression for the effective two-body T- and P- violating interaction between nucleons in a nucleus, which incorporates the effects of the renormalization due to the residual strong interaction, is derived. Mean square matrix elements of the T- and P-odd weak interaction between nuclear compound states are calculated. Correlators between the T- and P-odd and the T-even and P-odd weak interaction matrix elements in the compound states are considered and estimates for these quantities are obtained.

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I. INTRODUCTION

Spatial parity nonconserving weak interaction of nucleons is now a subject for extensive experimental and theoretical investigations. In these studies, quantitative comparison of experimental results and theoretical predictions is possible. Developments in experimental techniques and interpretations of the results obtained allow one to raise questions far beyond the scope of the theory of weak interactions [1–11].

Much less is known, both in experimental and theoretical aspects, about the component of nucleon weak interaction that violates both spatial parity (P) and time reversal invariance (T) (T- and P-odd weak interaction) [12–14]. The problem of possible T violation has been of interest for a long time [15–23].

In the context of nuclear physics, the T- and P-odd interaction, if it exists, induces T- and P-odd nuclear moments [24–30] (electric dipole, magnetic quadrupole moments, "Schiff" moment, etc.). Experimental data exist only for the upper limits of these quantities. At the same time, theoretical values of the constants of this interaction are not unambiguously known, varying by several orders of magnitude from one model to another (see, e.g., [26,30]). In most cases, the scale of T- and P-odd interaction is predicted to be very small.

In this situation, possible sources for enhancement of the effects caused by this interaction, which allow experimental investigation, are crucial for further studies of T- and P-odd interactions. Apparently, the compound nuclear resonances which provide a large statistical enhancement of small perturbations are very convenient in this case.

Here, we consider the weak T- and P-odd nucleon interaction in nuclei beginning at the single-particle level. The effects of the residual strong interaction on the T- and P-odd potential was considered in Ref. [9], where simple analytical expressions for the renormalized constants of the weak potential were obtained. In the present work, we focus attention on renormalization of the two-body T- and P-odd interaction due to the residual strong interaction, which is important for the description of the T- and P-odd effects in nuclear states at excitation energies near or lower than that of the neutron separation threshold B_n. We obtain the expression for the effective two-body T- and P-violating interaction between nucleons in a nucleus that results from the T- and P-odd weak interaction, the residual strong interaction, and nuclear structure effects. We have calculated the mean square T- and P-violating matrix elements between compound states and have considered possible correlations of these matrix elements with the matrix elements of P-odd and T-even weak interaction [8].

The structure of the paper is the following. In Sec. II we consider a T- and P-odd potential, acting on a nucleon that arises in the mean field approximation for the initial two-body weak interaction. We calculate the single-particle matrix elements of this potential and discuss their properties in comparison with the single-particle matrix elements of P-odd and T-even weak interaction. In Sec. III we consider renormalization of the T- and P-odd weak interaction by the residual strong nucleon interaction. In Sec. IV the equations of this renormalization are solved for the Landau-Migdal parametrization of the residual strong interaction and explicit analytical results for the effective two-body T- and P-odd weak interaction between the nucleons in heavy nuclei are derived. Numerical results are obtained for the matrix elements expressed through the constants of the initial weak interaction. It is shown that, contrary to the case of the P-odd and T-even weak interaction, renormalization due to strong interaction does not result in enhancement of the matrix elements, though this renormalization is important for quantitative results.

In Sec. V we calculate the mean square matrix elements of the T- and P-odd interaction between nuclear compound states of opposite parity within the statistical model. Section VI is devoted to discussion of correlations

of T- and P-odd and P-odd and T-even matrix elements between compound states. Calculation of the correlator in the statistical model yields the value about 10%.

The results are summarized in Sec. VII.

II. T- AND P-ODD WEAK NUCLEON INTERACTION. T- AND P-ODD POTENTIAL

The nuclear Hamiltonian H with accounts for the T- and P-odd weak interaction can be written in the form

$$H = H_0 + V_S + \mathcal{W}^{T,P} + F. \quad (1)$$

Here, the first term $H_0 = \vec{p}^2/2m + U_S(r, \vec{\sigma})$ is the single-particle Hamiltonian of the nucleons moving in the strong mean field $U_S(r, \vec{\sigma})$ including the spin-orbit interaction, V_S stands for the residual two-body strong interaction (it will be considered in Sec. IV), $\mathcal{W}^{T,P}$ describes T- and P-odd weak interaction between nucleons, and F denotes other possible interactions, e.g., coupling to electromagnetic field. The two-body weak T- and P-odd interaction $\mathcal{W}^{T,P}$ can be written as follows (see, e.g., [25,26]):

$$\hat{\mathcal{W}}^{T,P}(1,2) = \frac{G}{\sqrt{2}}\frac{1}{2m}\Big((\eta_{12}\vec{\sigma}_1 - \eta_{21}\vec{\sigma}_2)\cdot\vec{\nabla}_1\delta(\vec{r}_1 - \vec{r}_2) + \eta'_{12}[\vec{\sigma}_1 \times \vec{\sigma}_2]\cdot\{\vec{p}_1 - \vec{p}_2, \delta(\vec{r}_1 - \vec{r}_2)\}\Big), \quad (2)$$

where $G = 10^{-5}$ m^{-2} is the Fermi constant, m is the nucleon mass, and p and $\vec{\sigma}$ are the nucleon momentum and doubled spin, respectively. Hereafter, $\vec{a} \times \vec{b}$ means the exterior vector product, and $\{a, b\}$ denotes an anti-commutator. The dimensionless constants $\eta_{1,2}$ and $\eta'_{1,2}$ which determine the scale of the T- and P-odd effects are predicted to be very small, e.g., within the Kobayashi-Maskawa model (see e.g., [25,26,29]).

The analysis of the T- and P-odd effects in nuclei is similar to that in the case of the P-odd weak interaction (see, e.g., [8,10]). It is convenient to introduce the T- and P-odd "weak potential" $w^{T,P}(1)$ acting on a valence nucleon 1, which arises from the summation $\mathcal{W}^{T,P}(1,2)$ over the states of the nucleon 2 (see, e.g., Ref. [26]):

$$w^{T,P}(1) = \frac{G}{2\sqrt{2}m}\eta(\vec{\sigma}\cdot\vec{\nabla})\rho(r), \quad (3)$$

where ρ is the nucleon density and the dimensionless constants η_p and η_n characterize the strength of the T- and P-odd potential for protons (neutrons); they are connected to the parameters of the initial two-body interaction $\mathcal{W}^{T,P}(1,2)$ by the relations

$$\eta_p = \frac{Z}{A}\eta_{pp} + \frac{N}{A}\eta_{pn}, \quad \eta_n = \frac{N}{A}\eta_{nn} + \frac{Z}{A}\eta_{np}, \quad (4)$$

where Z, N, and A are the nuclear charge, a neutron number, and its mass number, respectively. The limits on these constants (η_p, η_n) were obtained from the atomic [27] and molecular [28] electric dipole moment measurements. The T- and P-odd weak potential $w^{T,P}$ is a single-particle operator; it obeys the same selection rules as the P-odd and T-even weak potential w^P:

$$\Delta l = \pm 1, \quad \Delta j = 0. \quad (5)$$

The values of the matrix elements of the T- and P-odd

TABLE I. The single-particle matrix elements of T- and P-odd weak interaction (3) for the protons given in units of eV (the fifth column). The latin indices a,b denote the quantum number sets, $a \equiv \{n_a l_a j_a\}$ where n, l, and j are the radial quantum number, the orbital angular momentum, and the total angular momentum, respectively. The states with energies closest to the Fermi energy are marked by asterisks. The energy separations are given by the third column, in units of MeV. The fourth column gives the single-particle elements of the P-odd and T-even weak potential.

a	b	$\varepsilon_a - \varepsilon_b$ (MeV)	w^P_{ab} (eV)	w^{TP}_{ab} (eV)	$\tilde{w}^{TP}_{ab}$ (eV)
$2p_{3/2}$	$2d^*_{3/2}$	−8.554	$0.513g_{pp} + 0.748g_{pn}$	$0.080\eta_{pp} + 0.098\eta_{pn}$	$0.053\eta_{pp} + 0.065\eta_{pn}$ $+0.003\eta_{np} + 0.003\eta_{nn}$
$1g_{9/2}$	$1h^*_{9/2}$	−11.054	$0.599g_{pp} + 0.842g_{pn}$	$0.112\eta_{pp} + 0.129\eta_{pn}$	$0.074\eta_{pp} + 0.086\eta_{pn}$
$2p_{1/2}$	$3s^*_{1/2}$	−8.443	$-0.500g_{pp} - 0.722g_{pn}$	$-0.066\eta_{pp} - 0.078\eta_{pn}$	$-0.044\eta_{pp} - 0.052\eta_{pn}$
$2p_{1/2}$	$3s^*_{1/2}$	−8.443	$-0.500g_{pp} - 0.722g_{pn}$	$-0.066\eta_{pp} - 0.078\eta_{pn}$	$-0.044\eta_{pp} - 0.052\eta_{pn}$
$1g_{7/2}$	$2f_{7/2}$	−9.745	$-0.517g_{pp} - 0.720g_{pn}$	$-0.068\eta_{pp} - 0.071\eta_{pn}$	$-0.045\eta_{pp} - 0.047\eta_{pn}$
$2d_{5/2}$	$2f_{5/2}$	−10.084	$0.553g_{pp} + 0.812g_{pn}$	$0.078\eta_{pp} + 0.107\eta_{pn}$	$0.052\eta_{pp} + 0.071\eta_{pn}$
$2d^*_{3/2}$	$2p_{3/2}$	8.554	$-0.513g_{pp} + -0.748g_{pn}$	$0.080\eta_{pp} + 0.098\eta_{pn}$	$0.053\eta_{pp} + 0.065\eta_{pn}$ $+0.003\eta_{np} + 0.003\eta_{nn}$
$2d^*_{3/2}$	$3p_{3/2}$	-8.732	$-0.558g_{pp} - 0.803g_{pn}$	$-0.050\eta_{pp} - 0.073\eta_{pn}$	$-0.033\eta_{pp} - 0.048\eta_{pn}$
$3s^*_{1/2}$	$3p_{1/2}$	−9.186	$0.549g_{pp} + 0.806g_{pn}$	$0.055\eta_{pp} + 0.091\eta_{pn}$	$0.037\eta_{pp} + 0.060\eta_{pn}$ $+0.002\eta_{np} + 0.003\eta_{nn}$
$1h^*_{9/2}$	$1g_{9/2}$	11.054	$-0.599g_{pp} - 0.842g_{pn}$	$0.112\eta_{pp} + 0.129\eta_{pn}$	$0.074\eta_{pp} + 0.086\eta_{pn}$
$1h^*_{9/2}$	$2g_{9/2}$	−9.417	$-0.575g_{pp} - 0.789g_{pn}$	$-0.055\eta_{pp} - 0.064\eta_{pn}$	$-0.037\eta_{pp} - 0.042\eta_{pn}$
$2f_{7/2}$	$1g_{7/2}$	9.745	$0.517g_{pp} + 0.720g_{pn}$	$-0.068\eta_{pp} - 0.071\eta_{pn}$	$-0.045\eta_{pp} - 0.047\eta_{pn}$
$2f_{5/2}$	$1d_{5/2}$	26.505	$0.096g_{pp} + 0.134g_{pn}$	$-0.067\eta_{pp} - 0.066\eta_{pn}$	$-0.044\eta_{pp} - 0.044\eta_{pn}$
$2f_{5/2}$	$2d_{5/2}$	10.084	$-0.553g_{pp} - 0.812g_{pn}$	$0.078\eta_{pp} + 0.107\eta_{pn}$	$0.052\eta_{pp} + 0.071\eta_{pn}$
$3p_{3/2}$	$1d_{3/2}$	25.840	$-0.054g_{pp} - 0.063g_{pn}$	$0.036\eta_{pp} + 0.036\eta_{pn}$	$0.024\eta_{pp} + 0.024\eta_{pn}$
$3p_{3/2}$	$2d^*_{3/2}$	8.732	$0.558g_{pp} + 0.803g_{pn}$	$-0.050\eta_{pp} - 0.073\eta_{pn}$	$-0.033\eta_{pp} - 0.048\eta_{pn}$

TABLE II. The same as in TableI, but for neutrons.

a	b	$\varepsilon_a - \varepsilon_b$ (MeV)	w_{ab}^P (eV)	w_{ab}^{TP} (eV)	$\tilde{w}_{ab}^{TP}$ (eV)
$3p_{3/2}^*$	$2d_{3/2}$	7.784	$0.541g_{nn}+ 0.778g_{np}$	$-0.071\eta_{nn}$ -$0.048\eta_{np}$	$-0.040\eta_{nn} - 0.027\eta_{np}$
$3p_{3/2}^*$	$3d_{3/2}$	−8.733	$0.446g_{nn}+ 0.661g_{np}$	$0.060\eta_{nn}+ 0.026\eta_{np}$	$0.033\eta_{nn} + 0.015\eta_{np}$
$2f_{5/2}^*$	$2d_{5/2}$	10.055	$-0.536g_{nn} -0.790g_{np}$	$0.108\eta_{nn}+ 0.078\eta_{np}$	$0.060\eta_{nn} + 0.044\eta_{np}$
$2f_{5/2}^*$	$3d_{5/2}$	−6.633	$-0.539g_{nn} -0.773g_{np}$	$-0.044\eta_{nn} -0.017\eta_{np}$	$-0.024\eta_{nn} - 0.009\eta_{np}$
$2f_{5/2}^*$	$2d_{5/2}$	10.055	$-0.536g_{nn} -0.790g_{np}$	$0.108\eta_{nn}+ 0.078\eta_{np}$	$0.060\eta_{nn} + 0.044\eta_{np}$
$2f_{5/2}^*$	$3d_{5/2}$	−6.633	$-0.539g_{nn} -0.773g_{np}$	$-0.044\eta_{nn} -0.017\eta_{np}$	$-0.024\eta_{nn} - 0.009\eta_{np}$
$3p_{1/2}^*$	$1s_{1/2}$	34.531	$0.004g_{nn} -0.003g_{np}$	$0.023\eta_{nn}+ 0.022\eta_{np}$	$0.013\eta_{nn} + 0.012\eta_{np}$ $0.001\eta_{pn} + 0.001\eta_{pp}$
$3p_{1/2}^*$	$2s_{1/2}$	23.964	$0.037g_{nn}+ 0.050g_{np}$	$-0.044\eta_{nn} -0.042\eta_{np}$	$-0.025\eta_{nn} - 0.023\eta_{np}$ $-0.001\eta_{pn} - 0.001\eta_{pp}$
$3p_{1/2}^*$	$3s_{1/2}$	8.811	$-0.528g_{nn} -0.775g_{np}$	$0.090\eta_{nn}+ 0.054\eta_{np}$	$0.050\eta_{nn} + 0.030\eta_{np}$ $0.002\eta_{pn} + 0.001\eta_{pp}$
$3p_{1/2}^*$	$4s_{1/2}$	−6.645	$-0.452g_{nn} -0.660g_{np}$	$-0.035\eta_{nn} -0.012\eta_{np}$	$-0.020\eta_{nn} - 0.007\eta_{np}$
$2g_{9/2}$	$1h_{9/2}$	7.590	$0.561g_{nn}+ 0.770g_{np}$	$-0.058\eta_{nn} -0.048\eta_{np}$	$-0.033\eta_{nn}$-$0.027\eta_{np}$
$3d_{5/2}$	$1f_{5/2}$	24.931	$0.023g_{nn}+ 0.039g_{np}$	$0.012\eta_{nn}+ 0.004\eta_{np}$	$0.006\eta_{nn} + 0.002\eta_{np}$
$3d_{5/2}$	$2f_{5/2}^*$	6.633	$0.539g_{nn}+ 0.773g_{np}$	$-0.044\eta_{nn} -0.017\eta_{np}$	$-0.024\eta_{nn} - 0.003\eta_{np}$
$4s_{1/2}$	$1p_{1/2}$	37.016	$-0.038g_{nn} -0.040g_{np}$	$0.001\eta_{nn}+ 0.006\eta_{np}$	$0.000\eta_{nn} + 0.003\eta_{np}$
$4s_{1/2}$	$2p_{1/2}$	23.364	$0.039g_{nn}+ 0.053g_{np}$	$0.009\eta_{nn} -0.004\eta_{np}$	$0.005\eta_{nn}$-$0.002\eta_{np}$
$4s_{1/2}$	$3p_{1/2}^*$	6.645	$0.452g_{nn}+ 0.660g_{np}$	$-0.035\eta_{nn} -0.012\eta_{np}$	$-0.020\eta_{nn} - 0.007\eta_{np}$
$2g_{7/2}$	$1f_{7/2}$	28.563	$0.034g_{nn}+ 0.068g_{np}$	$-0.066\eta_{nn} -0.061\eta_{np}$	$-0.037\eta_{nn} - 0.034\eta_{np}$
$2g_{7/2}$	$2f_{7/2}$	11.326	$-0.518g_{nn} -0.770g_{np}$	$0.101\eta_{nn}+ 0.061\eta_{np}$	$0.057\eta_{nn} + 0.034\eta_{np}$
$3d_{3/2}$	$2p_{3/2}$	25.118	$-0.021g_{nn} -0.020g_{np}$	$-0.028\eta_{nn} -0.015\eta_{np}$	$-0.016\eta_{nn} - 0.008\eta_{np}$ $-0.001\eta_{pn} - 0.001\eta_{pp}$
$3d_{3/2}$	$3p_{3/2}^*$	8.733	$-0.446g_{nn} -0.661g_{np}$	$0.060\eta_{nn}+ 0.026\eta_{np}$	$0.033\eta_{nn} + 0.015\eta_{np}$

weak potential (3) between single-particle nuclear states calculated for ^{209}Pb are presented in Tables I and II, expressed through the dimensionless weak constants η_{12}. The numerical calculations have been performed with the use of the single-particle basis of states obtained by numerical solution of the eigenvalue problem in the Woods-Saxon potential with spin-orbital interaction in the form

$$U_S(r,\boldsymbol{\sigma}) = -U_0 f(r) + U_{ls}(\vec{\sigma}\cdot\vec{l})[\hbar/(m_\pi c)]^2 \frac{1}{r}\frac{df}{dr} + U_c \quad (6)$$

with $f(r) = \{1+\exp[(r-R)/a]\}^{-1}$. Here, $\vec{l}$ is the orbital angular momentum and U_c is the Coulomb correction for protons; $U_c = 3Ze^2/(2R)[1-r^2/(3R^2)], r \le R$, and $U_c = Ze^2/r, r > R$, for R, a, and r the nuclear radius, diffusity parameter, and radial variable, respectively. The parameter values were used in accordance with Bohr-Mottelson formulas (see Ref. [31]) for the case of ^{233}Th: they are close to those established for heavy nuclei like lead (Ref. [32]) to reproduce the single-particle properties.

The coefficients before the weak constants η_{12} in the expressions for the matrix elements characterize the strength of the matrix elements and incorporate the effects of nuclear structure. As it is seen from Tables I and II, these quantities for the single-particle T- and P-odd matrix elements (the fifth column) are numerically suppressed (by about an order of magnitude) as compared to the corresponding quantities for the matrix elements of the P-odd and T-even potential $\hat{w}^P(1) = \langle \hat{W}^P(1,2)\rangle = \frac{Gg}{2\sqrt{2}m}\{(\boldsymbol{\sigma}\cdot\mathbf{p})\rho + \rho(\boldsymbol{\sigma}\cdot\mathbf{p})\}$ (the fourth column) that arises from the corresponding two-body P-violating interaction [24]

$$\hat{W}^P(1,2) = \frac{G}{\sqrt{2}}\frac{1}{2m}\Big((g_{12}\vec{\sigma}_1 - g_{21}\vec{\sigma}_2)\cdot\{(\vec{p}_1-\vec{p}_2),\delta(\vec{r}_1-\vec{r}_2)\} + g'_{12}[\vec{\sigma}_1\times\vec{\sigma}_2]\cdot\vec{\nabla}_1\delta(\vec{r}_1-\vec{r}_2)\Big) \quad (7)$$

in the same way as the T- and P-odd potential. The difference between these two cases is due to the surface character of the potential (3) which is proportional to the nuclear density derivative and peaked at the nuclear surface.

On average, the mixing of the single-particle states of opposite parity due to potential (3) that gives rise to nuclear T- and P-odd nuclear moments (see [25,26]) is

$$f_{ab} = |w_{ab}^{TP}/(\varepsilon_a - \varepsilon_b)| \sim 10^{-8}\eta, \quad (8)$$

with ε_a and ε_b being the energies of the single-particle shell model states a and b coupled by the T- and P-violating potential (3).

As the selection rules are the same for the T- and P-odd potential and the T-even and P-odd potential [see Eq.(5)], the following considerations are valid for the T- and P-odd potential. It is well known [33,34] that doublets of single-particle states with the same total angular momentum but of opposite parity usually do not appear in the same spherical nuclear shell. Hence the energy separation $|\varepsilon_a - \varepsilon_b|$ between levels in such doublets is about $5,...,8$ MeV, which is the average energy distance be-

tween different shells. Thus the coherent single-particle T- and P-odd contribution (3) does not work effectively in mixing any excited nuclear states (including the compound states) with energies below $B_n \sim 4, ..., 6$ MeV, the neutron separation energy, because the many-particle wave functions in this energy region are dominated by nucleon excitations within the valence shells [31]. Therefore, the main P-odd effects in this energy region are to be determined by the purely two-particle "residue," $:\hat{W}(1,2):$, of the weak interaction $\hat{W}(1,2)$, given by the difference

$$:\mathcal{W}^{T,P}(1,2): \equiv \mathcal{W}^{T,P}(1,2) - \langle \mathcal{W}^{T,P}(1,2)\rangle = \mathcal{W}^{T,P}(1,2) - w^{T,P}(1) , \quad (9)$$

which does not contain coherent summation in contrast to (3) (where such summation results in the nuclear density factor ρ).

First, we consider the case where the strong interaction V_S is "switched off." From the technical viewpoint, it is convenient to include the corrections due to the T and P-odd potential (5) into the single-particle wave functions using a unitary transformation. As is known from Refs. [25], [26], in the simple model with the strong potential $U(r)$ which is proportional to the nuclear density ρ [$\rho(r) = \rho(0)U(r)/U(0)$], it is easy to find the result of the action of the perturbation $\hat{w}(1)$:

$$\tilde{\psi} = \exp(-\hat{\alpha})\psi^0 \simeq (1 + \theta\vec{\sigma}\cdot\vec{\nabla})\psi^0, \qquad \hat{\alpha} = -\theta\vec{\sigma}\cdot\vec{\nabla},$$
$$\theta = \eta\frac{G}{2\sqrt{2}m}\frac{\rho(0)}{U(0)} = -2\times 10^{-8}\eta \text{ fm}, \quad (10)$$

where ψ^0 is the unperturbed wave function, and $\tau_z = -1$ $(+1)$ is the isospin projection for the proton (neutron). To get this solution, one should also neglect spin-orbit interactions. Accordingly, the matrix elements of any operator O, including the Hamiltonian, can be calculated by using the unperturbed wave functions ψ^0 and the transformed operator $\tilde{O}$:

$$\langle\tilde{\psi}_a|O|\tilde{\psi}_b\rangle = \langle\psi^0_a|\tilde{O}|\psi^0_b\rangle = \langle\psi^0_a|e^{\hat{\alpha}}Oe^{-\hat{\alpha}}|\psi^0_b\rangle \simeq \langle\psi^0_a|O + [\hat{\alpha}, O]|\psi^0_b\rangle,$$

where $e^{\hat{\alpha}} \equiv e^{-i\theta(\vec{\sigma}\cdot\vec{\nabla})}$ is the operator of the corresponding unitary transformation with the single-particle anti-Hermitian operator $\hat{\alpha}$. This transformation compensates the single-particle T- and P-odd potential in the Hamiltonian $e^{\hat{\alpha}}He^{-\hat{\alpha}}$. Thereby, the effects caused by this potential are accounted for in the renormalized operators $\tilde{O}$, rather than the wave functions $\tilde{\psi}$.

III. RENORMALIZATION OF THE T- AND P-ODD EFFECTS DUE TO RESIDUAL STRONG INTERACTION

To take the strong interaction V_S into account, let us seek now for an operator $e^{\hat{\mathcal{A}}}$ which should play the same role as $e^{\hat{\alpha}}$ above, but will incorporate the renormalization effects due to the residual strong interaction V_S. Eventually, as we will see below, the operator $\hat{\mathcal{A}}$ differs from $\hat{\alpha}$ mainly due to the renormalization of the weak interaction constant by the residual strong interaction V_S. The transformed Hamiltonian looks like

$$\tilde{H} = e^{\mathcal{A}}He^{-\mathcal{A}} = H_0 + V_S + \hat{F} + [\hat{\mathcal{A}}, F] + w^{T,P} + :\hat{\mathcal{W}}^{T,P}: + [\hat{\mathcal{A}}, H_0] + [\hat{\mathcal{A}}, V_S] , \quad (11)$$

where the P-violating terms are given by the second line, and we have used the decomposition of the T- and P-odd weak interaction $\mathcal{W}^{T,P}$ [Eq.(9)]. We neglected all terms above first order in the weak interaction. To obtain the effective two-particle T- and P-odd interaction acting in the valence shells we should find the operator $\hat{\mathcal{A}}$ in such a way that the single-particle T- and P-odd contribution in $e^{\hat{\mathcal{A}}}He^{-\hat{\mathcal{A}}}$ will be compensated. The last term in (11) is a two-body operator. We employ the same decomposition as in (9): $[\hat{\mathcal{A}}, V_S] \equiv \langle[\hat{\mathcal{A}}, V_S]\rangle + :[\hat{\mathcal{A}}, V_S]:$, where the first single-particle term is the average over the paired nucleons, and the second one, $:[\hat{\mathcal{A}}, V_S]:$, which yields zero under such averaging, is the effective induced two-particle T- and P-odd interaction (ITPNCI) which we are seeking:

$$\mathcal{W}^{TP}_{\text{ITPNCI}} =: [\hat{\mathcal{A}}, V_S] :, \qquad \langle\mathcal{W}^{TP}_{\text{ITPNCI}}\rangle \equiv 0. \quad (12)$$

Now we choose the operator $\hat{\mathcal{A}}$ to match the "compensation equation"

$$\hat{w}^{T,P} + [\hat{\mathcal{A}}, H_0] + \langle[\hat{\mathcal{A}}, V_S]\rangle = 0. \quad (13)$$

After that, the transformed Hamiltonian (11) takes the form

$$\tilde{H} = H_0 + V_S + F + :\hat{\mathcal{W}}^{T,P}: + \mathcal{W}^{T,P}_{\text{ITPNCI}} + [\hat{\mathcal{A}}, F], \quad (14)$$

where T- and P-odd single-particle terms are canceled. The sources of symmetry violations present in Eq. (14) can be classified as follows.

(i) The term $[\hat{\mathcal{A}}, F]$ which gives a direct contribution of the symmetry violating potential $w_1^{T,P}$ to the matrix elements of an external field F ($\langle\psi|F + [\hat{\mathcal{A}}, F]|\psi'\rangle = \langle\tilde{\psi}|F|\tilde{\psi}'\rangle$).

(ii) The two-body residual weak interaction $:\mathcal{W}^{T,P}:$

(iii) $\mathcal{W}^{T,P}_{\text{ITPNCI}}$, which play the same role as $:\mathcal{W}^{T,P}:$. We note that the induced T- and P-odd interaction $\mathcal{W}^{T,P}_{ITPNCI}$ is not enhanced in comparison with the two-particle residual T- and P-odd interaction $:\mathcal{W}^{T,P}:$, contrary to the case of the P-odd and T-even interaction that turns out to be enhanced by $\sim A^{1/3}$ times (see [10]). In fact, the $\mathcal{W}^{T,P}_{\text{ITPNCI}}$ incorporates the effects of the admixture of the distant components of the many-body wave function (e.g., the "small" components of the compound states [8,10]).

The effects of renormalization of the P-odd and T-even interaction were considered in detail in Ref. [10]; below we focus our attention on the T- and P-odd interaction.

IV. EXPLICIT FORM OF THE RESULTING TWO-PARTICLE T- AND P-ODD INTERACTION

To solve Eq. (13) and find an explicit form of the ITPNCI we use the Landau-Migdal interaction [35,36,32]. It

is the most widely used particle-hole interaction of contact type with spin- and isospin-exchange terms which goes backwards to the Landau Fermi liquid theory (Ref. [35]); for the case of a nucleus it was established in the theory of finite Fermi systems [36,32,37] by summation of all graphs irreducible in the particle-hole direction. This interaction can be written explicitly as follows:

$$V(\vec{r}_1,\vec{\sigma}_1,\vec{r}_2,\vec{\sigma}_2) = C\delta(\vec{r}_1-\vec{r}_2)[f+f'\tau_1\tau_2 + g\vec{\sigma}_1\cdot\vec{\sigma}_2 + g'\tau_1\tau_2\vec{\sigma}_1\cdot\vec{\sigma}_2], \quad (15)$$

where $C = \frac{\pi^2}{p_F m} = 300$ MeV fm^3 is the universal Migdal constant [36,32,37] and the strengths f, f', g, and g' are in fact functions of r via the density dependence: $f = f_{\rm in} - (f_{\rm ex} - f_{\rm in})[\rho(r) - \rho(0)]/\rho(0)$ (the same for f', g, and g'). (Quantities subscripted by "in" and "ex" describe interaction strengths in the depth of the nucleus and on its surface, respectively.) With the parameter values listed below, this interaction has been successfully used by many authors (see Refs. [32]) to quantitatively describe many properties of heavy nuclei.

The conventional choice of the constants widely used for heavy nuclei is (see [36,32,37]) $f_{\rm ex} = -1.95$, $f_{\rm in} = -0.075$, $f'_{\rm ex} = 0.05$, $f'_{\rm in} = 0.675$, $g_{\rm in} = g_{\rm ex} = 0.575$, and $g'_{\rm in} = g'_{\rm ex} = 0.725$.

It can be seen that, in the approximation of constant density as used above, the operator $\hat{\mathcal{A}}$ is proportional to $\hat{\alpha}$: $\hat{\mathcal{A}} = -\tilde{\theta}(\vec{\sigma}\vec{\nabla})$. Evaluating the commutator in (12) and (13), we obtain

$$\begin{aligned}[\hat{\mathcal{A}}, V_S] = &-\tilde{\theta}_1 C\vec{\sigma}_1\cdot[\vec{\nabla}_1,\delta(\vec{r}_1-\vec{r}_2)(f+f'\tau_1\tau_2)] - \tilde{\theta}_1 C(g+g'\tau_1\tau_2)\vec{\sigma}_2\cdot[\vec{\nabla}_1,\delta(\vec{r}_1-\vec{r}_2)]\\ &-\tilde{\theta}_2 C\vec{\sigma}_2\cdot[\vec{\nabla}_2,\delta(\vec{r}_1-\vec{r}_2)(f+f'\tau_1\tau_2)] - \tilde{\theta}_2 C(g+g'\tau_1\tau_2)\vec{\sigma}_1\cdot[\vec{\nabla}_2,\delta(\vec{r}_1-\vec{r}_2)]\\ &+iC(g+g'\tau_1\tau_2)\vec{\sigma}_1\times\vec{\sigma}_2\cdot\{\tilde{\theta}_1\vec{\nabla}_1 - \tilde{\theta}_2\vec{\nabla}_2,\delta(\vec{r}_1-\vec{r}_2)\}. \end{aligned} \quad (16)$$

Contrary to the case of P-odd and T-even weak interaction [9,10], averaging over the core nucleons here yields a nonzero result

$$\langle[\hat{\mathcal{A}}, V_S]\rangle \neq 0,$$

and, consequently, gives a nonzero contribution to the "compensation equation" (13). Taking together the terms with the same operator structures, we obtain from (13) and (16), the equation [9]

$$\tilde{\theta}(\vec{\sigma}\cdot\vec{\nabla})U = \theta(\vec{\sigma}\cdot\vec{\nabla})U + \gamma\frac{\rho(0)}{U(0)}(\vec{\sigma}\cdot\vec{\nabla})U, \quad (17)$$

which is equivalent to a system of two linear algebraic equations relating new (renormalized) interaction strengths $\tilde{\eta}_{1,2}$ with their initial values η_{12} (without strong interaction). Here, $\gamma = C[\theta_p\frac{Z}{A}(g\pm g') + \theta_n\frac{N}{A}(g\mp g')]$ for the protons (upper signs) and for the neutrons (lower signs). The solutions for this system of equations for the constants are the following:

$$\tilde{\eta}_p = \frac{1}{D}\left[\left(1+\tilde{C}g_{pp}\frac{N}{A}\right)\left(\frac{Z}{A}\eta_{pp}+\frac{N}{A}\eta_{pn}\right) - \tilde{C}g_{pn}\frac{N}{A}\left(\frac{N}{A}\eta_{nn}+\frac{Z}{A}\eta_{np}\right)\right],$$

$$\tilde{\eta}_n = \frac{1}{D}\left[\left(1+\tilde{C}g_{pp}\frac{Z}{A}\right)\left(\frac{N}{A}\eta_{nn}+\frac{Z}{A}\eta_{np}\right) - \tilde{C}g_{pn}\frac{Z}{A}\left(\frac{Z}{A}\eta_{pp}+\frac{N}{A}\eta_{pn}\right)\right], \quad (18)$$

with $D = 1 + \tilde{C}g_{pp} + 4\tilde{C}^2 g_{pn}^2 ZN/A^2$. Here, $g_{pp} = g_{nn} = g - g'$, $g_{pn} = g - g'$, and $\tilde{C} = C\rho/|U| = \frac{4}{3}\frac{\epsilon_F}{|U|} = \frac{4}{3}(1+\frac{B_n}{\epsilon_F})^{-1} \simeq 1$. We have used the well known relations

$$C = \frac{\pi^2}{p_F m}, \quad \rho = \frac{2p_F^3}{3\pi^2}, \quad \epsilon_F = \frac{p_F^2}{2m}, \quad |U| = \epsilon_F + B_n, \quad (19)$$

where p_F is the Fermi momentum and B_n is the nucleon separation energy. The renormalized matrix elements of the T- and P-odd weak potential for ^{209}Pb are presented in the last column of Tables I and II. It is seen that the strong residual interaction reduces the values of the T- and P-odd potential constants 1.5 to 2 times, on average.

To this end, using the explicit expression for $:\mathcal{W}^{T,P}:$ (9) given by Eq. (7), we obtain from (16) the resulting purely two-body T- and P-odd weak interaction in a nucleus:

$$\begin{aligned}\mathcal{W}^{T,P}_{\rm eff} &= :\mathcal{W}^{T,P}: + \mathcal{W}^{T,P}_{\rm ITPNCI}\\ &= \frac{G}{\sqrt{2}}\frac{1}{2m}\Big[:\Big((\eta_{12}-\tilde{\eta}_2 g_{12}\tilde{C})\vec{\sigma}_1 - (\eta_{21}-\tilde{\eta}_1 g_{12}\tilde{C})\vec{\sigma}_2\Big)\cdot[\vec{\nabla}_1,\delta(\vec{r}_1-\vec{r}_2)]\\ &\quad +\tilde{C}(\tilde{\eta}_1\vec{\sigma}_1 - \tilde{\eta}_2\vec{\sigma}_2)\cdot[\vec{\nabla}_1,\delta(\vec{r}_1-\vec{r}_2)f_{12}(r_1)]:\\ &\quad -i\vec{\sigma}_1\times\vec{\sigma}_2\cdot\{(\tilde{\eta}_1 g_{12}\tilde{C}+\eta'_{12})\vec{\nabla}_1 - (\tilde{\eta}_2 g_{12}\tilde{C}+\eta'_{12})\vec{\nabla}_2,\delta(\vec{r}_1-\vec{r}_2)\}\Big], \end{aligned} \quad (20)$$

where the renormalized weak constants $\tilde{\eta}_1$ and $\tilde{\eta}_2$ are given by Eq. (18). We used here the fact that the spin constant of the strong interaction (16) does not depend on r, while the constants $f_{pp} = f_{nn} = f(r) + f'(r)$ and $f_{pn} = f_{np} = f(r) - f'(r)$ do.

It should be noted that the induced T- and P-odd interaction $\mathcal{W}^{TP}_{\rm ITPNCI}$ has the same operator structure as the initial two-body T- and P-odd interaction $\mathcal{W}^{TP}$. Thus

TABLE III. Reduced matrix elements of T- and P-odd weak interaction $\mathcal{W}^{TP,J}_{abcd}$ for states of valence shells in the nuclei in the U–Th region in terms of the dimensionless weak constants η_{12}. The Latin indices a,b,... denote the full set of quantum numbers, $a \equiv \{p(n)n_a l_a j_a\}$, where p (n) means proton (neutron) states.

J	a	b	c	d	$\mathcal{W}^{TP,J}_{abcd}$ (eV)
2	$p1h_{9/2}$	$p1h_{9/2}$	$n1j_{15/2}$	$n1i_{11/2}$	$0.021\eta'_{pn}$
3	$p1h_{9/2}$	$p1h_{9/2}$	$n1j_{15/2}$	$n1i_{11/2}$	$-0.003\eta_{pn} + 0.014\eta'_{pn}$
4	$p1h_{9/2}$	$p1h_{9/2}$	$n1j_{15/2}$	$n1i_{11/2}$	$0.010\eta'_{pn}$
5	$p1h_{9/2}$	$p1h_{9/2}$	$n1j_{15/2}$	$n1i_{11/2}$	$-0.006\eta_{pn} + 0.009\eta'_{pn}$
6	$p1h_{9/2}$	$p1h_{9/2}$	$n1j_{15/2}$	$n1i_{11/2}$	$0.006\eta'_{pn}$
7	$p1h_{9/2}$	$p1h_{9/2}$	$n1j_{15/2}$	$n1i_{11/2}$	$-0.011\eta_{pn} + 0.006\eta'_{pn}$
8	$p1h_{9/2}$	$p1h_{9/2}$	$n1j_{15/2}$	$n1i_{11/2}$	$0.004\ \eta'_{pn}$
9	$p1h_{9/2}$	$p1h_{9/2}$	$n1j_{15/2}$	$n1i_{11/2}$	$-0.024\eta_{pn} + 0.005\eta'_{pn}$
2	$n1i_{11/2}$	$n1j_{15/2}$	$n2g_{9/2}$	$n2g_{9/2}$	$-0.003\eta_{nn} - 0.009\eta'_{nn}$
3	$n1i_{11/2}$	$n1j_{15/2}$	$n2g_{9/2}$	$n2g_{9/2}$	$0.001\eta_{nn} - 0.006\eta'_{nn}$
4	$n1i_{11/2}$	$n1j_{15/2}$	$n2g_{9/2}$	$n2g_{9/2}$	$-0.002\eta_{nn} - 0.004\eta'_{nn}$
5	$n1i_{11/2}$	$n1j_{15/2}$	$n2g_{9/2}$	$n2g_{9/2}$	$0.002\eta_{nn} - 0.004\eta'_{nn}$
6	$n1i_{11/2}$	$n1j_{15/2}$	$n2g_{9/2}$	$n2g_{9/2}$	$-0.001\eta_{nn} - 0.003\eta'_{nn}$
7	$n1i_{11/2}$	$n1j_{15/2}$	$n2g_{9/2}$	$n2g_{9/2}$	$0.004\eta_{nn} - 0.003\eta'_{nn}$
8	$n1i_{11/2}$	$n1j_{15/2}$	$n2g_{9/2}$	$n2g_{9/2}$	$-0.001\eta_{nn} - 0.002\eta'_{nn}$
9	$n1i_{11/2}$	$n1j_{15/2}$	$n2g_{9/2}$	$n2g_{9/2}$	$0.009\eta_{nn} - 0.002\eta'_{nn}$
2	$n1j_{15/2}$	$n1i_{11/2}$	$n1i_{11/2}$	$n2g_{9/2}$	$-0.003\eta_{nn} - 0.010\eta'_{nn}$
3	$n1j_{15/2}$	$n1i_{11/2}$	$n1i_{11/2}$	$n2g_{9/2}$	$0.002\eta_{nn} + 0.002\eta'_{nn}$
4	$n1j_{15/2}$	$n1i_{11/2}$	$n1i_{11/2}$	$n2g_{9/2}$	$-0.003\eta_{nn} - 0.003\eta'_{nn}$
5	$n1j_{15/2}$	$n1i_{11/2}$	$n1i_{11/2}$	$n2g_{9/2}$	$0.003\eta_{nn} + 0.001\eta'_{nn}$
6	$n1j_{15/2}$	$n1i_{11/2}$	$n1i_{11/2}$	$n2g_{9/2}$	$-0.004\eta_{nn} - 0.001\eta'_{nn}$
7	$n1j_{15/2}$	$n1i_{11/2}$	$n1i_{11/2}$	$n2g_{9/2}$	$0.003\eta'_{nn}$
8	$n1j_{15/2}$	$n1i_{11/2}$	$n1i_{11/2}$	$n2g_{9/2}$	$-0.005\eta_{nn} - 0.001\eta'_{nn}$
9	$n1j_{15/2}$	$n1i_{11/2}$	$n1i_{11/2}$	$n2g_{9/2}$	$0.003\eta'_{nn}$

$\mathcal{W}^{TP}_{\rm ITPNCI}$ differs from $\mathcal{W}^{TP}$ only due to the renormalization of the strength constants which turns out to be weak, because the response of the nucleus to the T- and P-odd potential (3) as a function of the interaction constants has poles ($D = 0$) at $g = \tilde{C}^{-1} \simeq -1$ and $g' \simeq \tilde{C}^{-1} \simeq -1$ (for $N \simeq Z$), while the actual nuclear strong interaction "drives" the solution of the renormalization equations (18) in the direction opposite the poles. As a result, the induced T- and P-odd interaction does not play a special role in the present case and causes renormalization of order 1 to 2. Thus there is an essential difference with the case of the P-odd and T-even weak interaction [10] where the analogous induced P-odd interaction is enhanced by about $A^{1/3}$ times and practically dominates the results.

In practical calculations, it is convenient to treat $\mathcal{W}^{TP}_{\rm eff}$ in the second quantized version using a multipole expansion in the particle-hole channel: $\mathcal{W}^{TP}_{\rm eff} = \frac{1}{2}\sum_J[(a^\dagger b)_J \mathcal{W}^{TP,J}_{{\rm eff}abcd}(c^\dagger d)_J]_0$ where $(\cdots)_J$ means the coupling of nucleon creation $a^\dagger$ and destruction a operators to a given angular momentum J [31]. The numerical results for some reduced matrix elements of $\mathcal{W}^{TP,J}_{\rm eff}$ as compared to those of the initial interaction : $\mathcal{W}^{TP,J}$: between valence shell states for the Th–U region are presented in Table III.

V. T- AND P-ODD MATRIX ELEMENTS BETWEEN COMPOUND STATES

In the work (Ref. [8]) we introduced a method to calculate the mean square matrix elements (MSME's) of operators between compound states and obtained the results for the P-odd and T-even weak interaction. Here, we apply the method to calculation of MSME's of the T- and P-odd interaction. Consider the mean squared value of the matrix element:

$$\overline{\mathcal{W}^{T,P^2}} = \overline{(p|\mathcal{W}^{T,P}|s)(s|\mathcal{W}^{T,P}|p)} = \overline{(p| : \mathcal{W}^{T,P} : +\mathcal{W}^{T,P}_{\rm ITPNCI}|s)(s| : \mathcal{W}^{T,P} : +\mathcal{W}^{T,P}_{\rm ITPNCI}|p)}\ . \tag{21}$$

Now, we can expand the compound states $|C^{J\pi}) = |s), |p)$ in terms of their simple components (multiparticle excitations) $|\alpha^{J\pi}\rangle$ of the same quantum numbers of angular momentum J and parity π,

$$|C) \quad = \quad \sum_\alpha C_\alpha |\alpha\rangle, \tag{22}$$

having for the MSME's the expression

$$\overline{\mathcal{W}^{T,P\,2}} = \sum_{\alpha\beta} \overline{C_\alpha C_\beta (p| : \mathcal{W}^{T,P} : + \mathcal{W}^{T,P}_{\rm ITPNCI}|\alpha\rangle\langle\beta| : \mathcal{W}^{T,P} : + \mathcal{W}^{T,P}_{\rm ITPNCI}|p)} \ . \tag{23}$$

The number of different terms in Eq. (22), $\mathcal{N}$, is very large $\sim 10^5$–10^7. The main contribution in Eq. (22) is dominated by the set of $\bar{N}$ "principal components" $|\bar{\alpha}\rangle$ with shell model energies $E_{\bar{\alpha}}$ close to the energy of a compound state E. We can make use of the statistical independence of the coefficients C_α to take their second moments in the form (Refs. [31,38])

$$\overline{C_\alpha C_\beta} = \overline{C^2_\alpha}\delta_{\alpha\beta} = \delta_{\alpha\beta}\frac{1}{\bar{N}}\Delta(\Gamma_{\rm spr}, E - E_\alpha). \tag{24}$$

The overbar means averaging over a rather broad set of the compound states. Here, the spreading width $\Gamma_{\rm spr}$ is related to the number of principal components $\bar{N}^{-1/2} \simeq \sqrt{\frac{2d}{\pi\Gamma_{\rm spr}}}$ and d is the average energy distance between the resonances. The Breit-Wigner-type factor Δ that describes quenching of the weights of states distanced in energy,

$$\Delta(\Gamma_{\rm spr}, E - E_\alpha) = \frac{\Gamma^2_{\rm spr}/4}{(E - E_\alpha)^2 + \Gamma^2_{\rm spr}/4}, \tag{25}$$

is a widely "spread" δ function. It is normalized so as to be of order unity for $|E - E_\alpha| \le \Gamma_{\rm spr}/2$ and with the conventional limit $\Delta(\Gamma_{\rm spr}, E - E_\alpha) \to \frac{\pi\Gamma_{\rm spr}}{2}\delta(E - E_\alpha)$ for $\Gamma_{\rm spr} \to 0$. For the principal components, $|E_{\bar{\alpha}} - E| \lesssim \Gamma_{\rm spr}$, the expression (25) reflects the "chaotic" nature of a broad mixture of the simple components in the compound state due to the strong interaction. For the small (energy distanced) components it reduces to the perturbation theory result. From (23)–(25), we obtain for the MSME's

$$\overline{\mathcal{W}^{T,P\,2}} = \sum_\alpha \frac{1}{\bar{N}}\Delta(\Gamma_{\rm spr}, E - E_\alpha)\overline{(p| : \mathcal{W}^{T,P} : + \mathcal{W}^{T,P}_{\rm ITPNCI}|\alpha\rangle\langle\alpha| : \mathcal{W}^{T,P} : + \mathcal{W}^{T,P}_{\rm ITPNCI}|p)}. \tag{26}$$

Here the argument of the function Δ is the change of the energy, $E - E_\alpha = \epsilon_a - \epsilon_b + \epsilon_c - \epsilon_d$, and $: \mathcal{W}^{T,P}:$ and $\mathcal{W}^{T,P}_{\rm ITPNCI}$ are given by Eq. (20). Summation over α in (26) is equivalent to summation over different components of the operator $\mathcal{W}^{T,P}_{\rm eff}$ in Eq. (5), i.e., the problem is reduced to the calculation of $(p|\mathcal{W}^{T,P}\mathcal{W}^{T,P}|p)$. The coefficients before the "principal" components $\tilde{C}_\alpha$ in (22) are governed by the microcanonical ensemble rule [31,38]. Then, to calculate the averaging over p-resonance "principal" components $\overline{(p|\cdots|p)}$ in $\overline{(\mathcal{W}^{T,P})^2}$, we use, instead of the present microcanonical ensemble, an equivalent canonical one. The latter can always be introduced for a system with a large number degrees of freedom by introducing the effective nuclear temperature T and chemical potentials λ_n and λ_p. In the second quantization representation, the average expectation value in (26) is reduced to a canonical ensemble average with the standard contractor rules $(p|\overline{a^+ b}|p) = \delta_{ab}\nu^T_a$, for ν^T_a the finite temperature Fermi occupation probabilities, $\nu^T_a = \{\exp[(\epsilon_a - \lambda)/T] + 1\}^{-1}$. The canonical ensemble parameters T and λ_τ (τ means isospin projection) are to be determined from conventional "consistency" equations $E = \sum_a \nu_a\epsilon_a$, $Z = \sum_p \nu_p$ and $N = \sum_n \nu_n$ for the excitation energy E (being equal to the neutron separation energy B_N), nuclear charge Z, and neutron number N, correspondingly.

Using the same considerations, we obtain the following result for $\sqrt{\overline{\mathcal{W}^{T,P\,2}}}$:

$$\sqrt{\overline{\mathcal{W}^{T,P\,2}}} = \sqrt{\frac{2d}{\pi\Gamma_{\rm spr}}}\Big\{\frac{1}{2}\sum_{abcd}\nu^T_a(1-\nu^T_b)\nu^T_c(1-\nu^T_d) \times |\,\mathcal{W}^{T,P}_{{\rm eff}\ ab,cd}\,|^2\,\Delta(\Gamma_{\rm spr}, \epsilon_a - \epsilon_b + \epsilon_c - \epsilon_d)\Big\}^{\frac{1}{2}}. \tag{27}$$

Here, $\Delta(\Gamma_{\rm spr}, \varepsilon_a - \varepsilon_b + \varepsilon_c - \varepsilon_d)$ can be viewed as an approximate energy conservation law with accuracy up to the width of the states.

The numerical calculations for ^{233}Th have been performed with the use of a single-particle basis of states obtained numerically [see Sec. II, Eq. (6) and the discussion below it].

The value of temperature $T = 0.6$ MeV was used in accordance with the consistency condition for the excitation energy. The result for the mean square matrix elements of T- and P-odd interaction between compound states is

$$\sqrt{\overline{\mathcal{W}^{T,P\,2}}} = 0.2\eta_0 \quad \text{meV}.$$

The ratio of the T and P-odd matrix elements to the P-odd ones is $\sqrt{\overline{\mathcal{W}^{T,P\,2}}}/\sqrt{\overline{\mathcal{W}^{P\,2}}} = 0.1\eta/g$. Here, we use equal values of the constants η_{12} in (2), $\eta_{12} = \eta_0$. The corresponding mixing coefficient for compound states $|F_{sp}|$ is

$$|F_{sp}| \simeq \frac{\sqrt{\overline{\mathcal{W}^{T,P\,2}}}}{|E_s - E_p|} \simeq 1 \times 10^{-5}\eta_0,$$

which is about 10^3 times larger than single-particle mixing $f_{12} \simeq 10^{-8}\eta_0$ [Eq. (8)]. We assumed in this estimate that $|E_s - E_p| = D_s$ where $D_s \simeq 20$ eV is the average energy interval between compound resonances in an s-wave.

VI. CORRELATIONS BETWEEN T- AND P-ODD AND P-ODD AND T-EVEN MATRIX IN COMPOUND STATES

The question of possible correlations between matrix elements of T- and P-odd weak interaction and those of

P-odd and T-even weak interaction is very interesting. If the correlator

$$C(W^P,\mathcal{W}^{T,P})=\frac{\overline{(p|W^P|s)(p|\mathcal{W}^{T,P}|s)}}{\sqrt{\overline{\mathcal{W}^{T,P^2}}}\sqrt{\overline{W^{P^2}}}} \tag{28}$$

$(0<|C(W^P,\mathcal{W}^{T,P})|<1)$ can be calculated, one can make predictive estimates for the values and the signs of the T- and P-odd effects in compound states based on the information about the corresponding quantities for P-odd effects (the latter are much easier to measure) in the case when the quantity $C(W^P,\mathcal{W}^{T,P})$ differs considerably from zero. $C(W^P,\mathcal{W}^{T,P})$ can be calculated, in principle, by the same technique [8] as the mean square matrix element [39]. To calculate the numerator in the right-hand side of Eq. (28), $c(W^P,\mathcal{W}^{T,P})=\overline{(p|W^P|s)(p|\mathcal{W}^{T,P}|s)}$, one can employ the same method of reducing to the average over the ensemble as used above to calculate the MSME's:

$$\begin{aligned}c(W^P,\mathcal{W}^{T,P})&=\overline{(p|W^P|s)(s|\mathcal{W}^{T,P}|p)}\\&=\overline{(p|W^P_{\mathrm{IPNCI}}|s)(s|:\mathcal{W}^{T,P}:+\mathcal{W}^{T,P}_{\mathrm{ITPNCI}}|p)}.\end{aligned} \tag{29}$$

However, in this a case more careful treatment is needed. In fact, the present thermodynamical approach does not distinguish the cases when "external" averaging (canonical) goes over p-states or s-states. This approximation is reasonable for the case of the mean square matrix element because the latter is not considerably affected by the change $\overline{(p|W^P|s)(s|W^P|p)}\to\overline{(s|W^P|p)(p|W^P|s)}$. The latter is not the case for the quantity $c(W^P,\mathcal{W}^{T,P})$. The reason is that the matrix elements of the P-odd weak operator W^P are imaginary and change sign when substituting final states instead of initial states. In contrast, the matrix elements of the T- and P-odd weak operator $\mathcal{W}^{T,P}$ are real and symmetric under such a substitution. Two-body matrix elements of W^P and $\mathcal{W}^{T,P}$ obey the following symmetry rules, respectively:

$$\begin{aligned}W^P_{ab,cd}&=-W^P_{ba,dc}=-W^P_{dc,ba}=W^P_{cd,ab},\\\mathcal{W}^{T,P}_{ab,cd}&=\mathcal{W}^{T,P}_{ba,dc}=\mathcal{W}^{T,P}_{dc,ba}=\mathcal{W}^{T,P}_{cd,ab}.\end{aligned} \tag{30}$$

As a result, we have some cancellations of the different terms in the sum of the products $\mathcal{W}^{T,P}W^P$. Thus an appropriate symmetrization should be done when the quantity $\bar{C}(W^P,\mathcal{W}^{T,P})$ is calculated by the present method of reduction to the ensemble averaging:

$$\begin{aligned}c(W^P,\mathcal{W}^{T,P})&=\frac{1}{2}\left[\overline{(p|\mathcal{W}^{T,P}_{\mathrm{eff}}|s)(s|W^P_{\mathrm{IPNCI}}|p)}+\overline{(s|W^P_{\mathrm{IPNCI}}|p)(p|\mathcal{W}^{T,P}_{\mathrm{eff}}|s)}\right]\\&=\frac{1}{2}\left[\overline{(p|W^P_{\mathrm{IPNCI}}|s)(s|\mathcal{W}^{T,P}_{\mathrm{eff}}|p)}-\overline{(s|W^P_{\mathrm{IPNCI}}|p)(p|\mathcal{W}^{T,P}_{\mathrm{eff}}|s)}\right].\end{aligned} \tag{31}$$

As a result of the symmetrization and due to the negative sign before the second term in the last line, some cancellations of similar terms in the large sum of the same type as in Eq. (23) are possible. It is seen from the last equation that we cannot pretend to obtain the correct sign of the correlator within the present statistical method, because the compound states of positive and negative parity are treated on the same footing. Without having additional information about occupancies of particular single-particle levels with a given total angular momentum and parity, only absolute values of the correlator can be estimated.

After the same calculations as in the previous section and thermal averaging, we obtain the following expression for the numerator in (28):

$$\begin{aligned}|c(W^P,\mathcal{W}^{T,P})|=\frac{d}{\pi\Gamma_{\mathrm{spr}}}\Bigg|&\sum_{abcd}\nu^T_a(1-\nu^T_b)\nu^T_c(1-\nu^T_d)\\&\times W^P_{ab,cd}\mathcal{W}^{T,P}_{\mathrm{eff}\ dc,ba}\Bigg|\\&\times\Delta(\Gamma_{\mathrm{spr}},\epsilon_a-\epsilon_b+\epsilon_c-\epsilon_d).\end{aligned} \tag{32}$$

Using this result and Eqs. (23)–(26) we obtain, for the same value of temperature and the same single-particle basis as in the calculations of mean square matrix elements, the following absolute value of the correlator (28) for the ^{233}Th:

$$|\,C(W^P,\mathcal{W}^{T,P})\,|\ \simeq\ 0.1.$$

This result means that correlations in the matrix elements are weak. Of course, the present statistical calculation of the correlator is rather approximate, and a more refined technique is needed to obtain a precise result for the correlator.

VII. SUMMARY

We have considered the T- and P-odd nucleon interaction in heavy nuclei. Effects of the renormalization of this interaction are considered. An effective two-body T and P-odd interaction acting near the Fermi surface is obtained and the corresponding matrix elements are calculated. This interaction accumulates the effects of the distant state admixtures. We obtained the results for mean squared values of the T- and P-violating interaction between compound states of opposite parity. As well as in the case of P-odd and T-even weak interaction, statistical enhancement of T- and P-odd effects in neutron resonances take place. As a result, the mixing between the compound states of opposite parity is enhanced by 10^3 times, in comparison to the single-particle T- and P-odd mixing. The ratio of the T- and P-odd matrix elements to the P-odd ones is $\sqrt{\overline{\mathcal{W}^{T,P^2}}}/\sqrt{\overline{\mathcal{W}^{P^2}}}=0.1\eta/g$. So the simplest estimate ($\sim\eta/g$) of the sensitivity of the T- and P experiments planned in Los Alamos, St. Petersbourg, and KEK should be reduced by 10 times.

Correlations between matrix elements of T- and P-odd and P-odd and T-even interactions in compound states are found to be weak within the statistical model.

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