Simple formula for conditional fission barriers of rotating nuclei

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(Received 16 May 1994)

A simple formula for conditional fission barriers of rotating nuclei is proposed based on an extension of Swiatecki's dimensionless expressions for conditional barrier energies. The calculated barrier heights are compared with the measured values. The utility of the present formula in the analysis of experimental results in a typical heavy ion fusion reaction is discussed.

PACS number(s): 25.70.Gh, 25.70.Jj, 25.85.Ge, 21.60.Ev

All macroscopic models of fusion reactions in medium energy heavy ion collisions suggest a long dynamical path between the capture of the projectile by the target and the full equilibration of the fused composite system leading to the formation of a "compound nucleus." Possible signatures of this phase of the fusion reaction process have also been identified in several experiments such as anomalous fission fragment angular distributions and giant dipole resonance γ-ray measurements [1,2]. Based on a dynamical model of nuclear shape evolutions describing fusion, Swiatecki [3] has shown that the details of the dynamics preceding the compound nucleus formation sensitively depend on the conditional fission barriers. While Swiatecki gave simple analytical expressions for the dimensionless energies of the conditional fission barriers, these could only be used at a qualitative level and not in a quantitative analysis of experimental data because of the relative simplicity of the model and the consequent systematic overprediction of all barrier heights. In particular, finite range effects are known to be important in the calculation of fission barrier heights but are not included in Swiatecki's model. We propose here a simple extension of Swiatecki's expressions for the conditional barrier heights, to include the effects of the finite range of the nuclear force and angular momentum. The calculated conditional barrier heights are compared with the measured values, wherever possible. The utility of the present formula in the analysis of experimental data on heavy ion fusion reactions is also discussed.

Swiatecki's model of fusion is based on a dynamical evolution of the system in a three-dimensional space of nuclear shapes. The three shape degrees of freedom are (1) distance variable $\rho = \frac{r}{R_1+R_2}$, (2) window-opening variable $\alpha = (\frac{\sin\theta}{\sin\theta_{\max}})^2$, and (3) asymmetry variable $\Delta = \frac{R_1-R_2}{R_1+R_2}$. The quantities R_1 and R_2 are the radii of the two nuclei, r is the seperation between the nuclei and θ is the window-opening angle (see Fig. 1). Swiatecki has also shown that the potential energy of a configuration taken with respect to the energy of tangent spheres and written in units of $8\pi\overline{R}^2\gamma$ is well approximated as

$$\eta = \frac{PE - PE_{tg}}{8\pi\overline{R}^2\gamma} = \nu\sigma - \nu^2 + \nu^3 - x\sigma, \tag{1}$$

where the variables ν and σ are defined in terms of the three shape degrees of freedom; $\nu = \sqrt{\alpha}$ and $\sigma = \frac{\rho^2-1}{1-\Delta^2}$ and x is the effective fissility parameter; $\overline{R}[=R_1R_2/(R_1+R_2)]$ is the reduced radius of the system and γ is the surface energy coefficient. The effective fissility parameter x is dependent on the mass asymmetry:

$$x = x_0\frac{(1-D)^2}{1+3D},$$

where $x_0 = \frac{3Z^2e^2}{40\pi\gamma R^3}$, $D = \Delta^2$, and $\gamma = 19.008(1 - 2.84I^2)/4\pi r_0{}^2$ with $I = (N-Z)/A$ and the radius parameter $r_0 = 1.2$. The quantities N, Z, A, and R refer to the neutron, charge, and mass numbers and radius of the combined system. The conditional saddle-point $(\overline{\nu}, \overline{\sigma})$ can be located by differentiating η with respect to ν and σ. The saddle-point energy thus calculated is

$$\eta(\overline{\nu}, \overline{\sigma}) = -x^2 + x^3. \tag{2}$$

The barrier heights are to be calculated with respect to the energy of a sphere, which is $x-1$ in the above units:

$$\eta_{\rm bar} = 1 - x - x^2 + x^3. \tag{3}$$

Swiatecki's expression for barrier height implies an important scaling law— "The dimensionless conditional saddle-point properties (such as the relative degree of window opening and energy deviation from tangent spheres expressed in units of $8\pi\overline{R}^2\gamma$) are functions of the

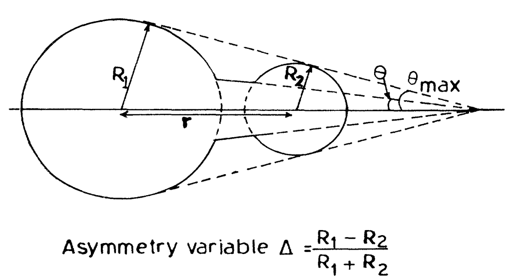

FIG. 1. The nuclear configuration defining the three shape variables ρ, α, and Δ.

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effective fissility parameter x alone otherwise independent of asymmetry."

In spite of the simplicity and elegance of the Swiatecki model, it could not be used for direct analysis of the experimental results because of the well-known overestimation of the fission barrier heights by the simple liquid drop model. For example, Fig. 2 shows the calculated fission barrier heights using the present formula compared with Swiatecki's expression and with the predictions of the finite range liquid drop model of Sierk [4]. The discrepencies are obvious and arise due to a neglect of the finite range of the nuclear force in the Swiatecki model. It is known that, for nearly spherical nuclei, finite range effects can easily be taken into account by a simple readjustment of the surface tension. On the other hand, for highly necked-in shapes, finite range effects exhibit themselves as the interaction energy between the two halves [5]. For the overlapping spheres, Blocki *et al.* have given analytical expressions for the interaction energy. However, the conditional saddle shapes deviate from two overlapping spheres, with the addition of a neck. No simple analytical finite range correction could be found for the conditional saddle-point shapes. Based on numerical estimates we propose an expression for interaction energy of the form

$$V_{\mathrm{int}} = -7.36\pi\gamma\overline{R}(1-x)^{1.2}. \tag{4}$$

The resulting expression for the dimensionless fission barrier height is

$$\eta_{\mathrm{bar}} = 1 - x - x^2 + x^3 - \frac{0.92}{\overline{R}}(1-x)^{1.2}. \tag{5}$$

For $x = 0$, the saddle-point configuration is one of touching spheres. The finite range correction is the proximity energy itself [5]. For $x = 1$, the saddle-point shape is spherical and therefore the finite range correction gets absorbed in the surface tension constant itself. Equation (4) reflects these two extreme cases. Figure 2 also shows the calculated barrier heights with the inclusion of the finite range correction. Now we use Swiatecki's scaling law for conditional fission barrier heights. Experimental measurements of conditional barrier heights are available for ^{75}Br, $^{110-111}$In, and ^{244}Cm. Figures 3 and 4 show the calculated barrier heights for the systems ^{75}Br and ^{110}In as function of fragment charge. Figure 5 shows a comparison of the experimental yields and the yields calculated [6] using the present formula for barrier heights for ^{244}Cm. The yield Y for a given complex fragment is calculated from the expression

$$Y(Z) \propto T_Z \exp(-B_Z/T_Z), \tag{6}$$

where B_Z is the conditional barrier and T_Z is the saddle temperature. In all cases, the present simple formula yields a satisfactory agreement with the experimental data.

One of the distinguishing features of heavy ion reactions is that they involve large angular momenta. It is known that the centrifugal repulsion in the case of a rotating nucleus contributes to lowering of the fission barrier heights in a manner very similar to Coulomb repulsion. One therefore expects a scaling law for rotating nuclei similar to Swiatecki's scaling law for asymmetric fission. We therefore explored a scaling law of the form $x = x_0 + x_R$ for symmetric fission barriers where x_R is a rotational part of the fissility parameter.

To obtain the functional form of x_R, the saddle-point energy η is calculated as a function of l^2 for symmetric fission of various nuclei using Sierk's barriers [4]. From Eq. (4), x versus l^2 is found to be almost linear for each nucleus, though the slopes are different. These slopes when plotted against the mass number A, give the mass dependence of x_R. Based on the above analysis, we deduce

$$x_R \approx 0.44\frac{l^2}{A^{1.843}}. \tag{7}$$

One could now combine the two scaling laws. The effec-

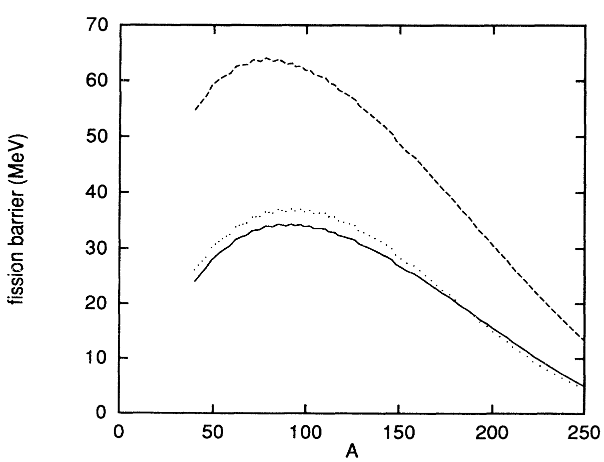

FIG. 2. Fission barrier vs mass of the CN for β stable nuclei. Swiateckti's calculation is shown with dashes, Sierk's calculation with dots, and the present formula with a solid line.

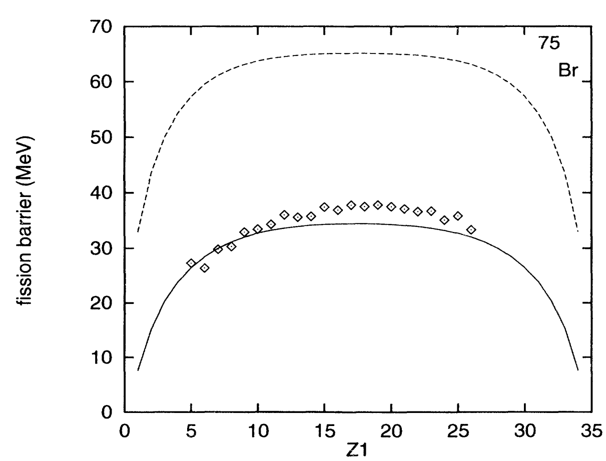

FIG. 3. Fission barrier vs the charge of the emitted fragment for ^{75}Br. Swiatecki's calculation is shown with dashes and the present calculation with a solid line. Experimental points are taken from Ref. [6].

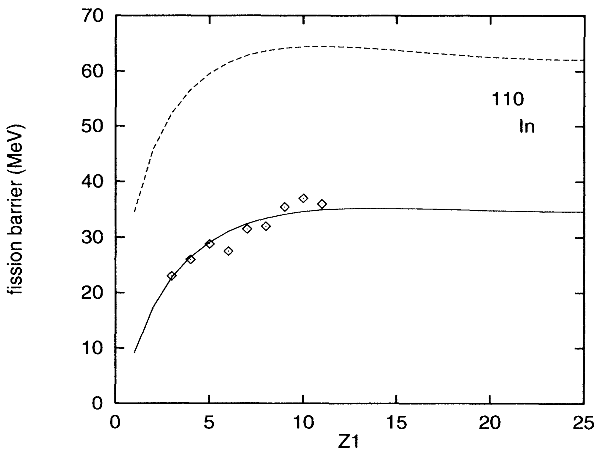

FIG. 4. Fission barrier vs the charge of the emitted fragment for ^{110}In. Swiatecki's calculation is shown with dashes and the present calculation with a solid line. Experimental points are taken from Ref. [7].

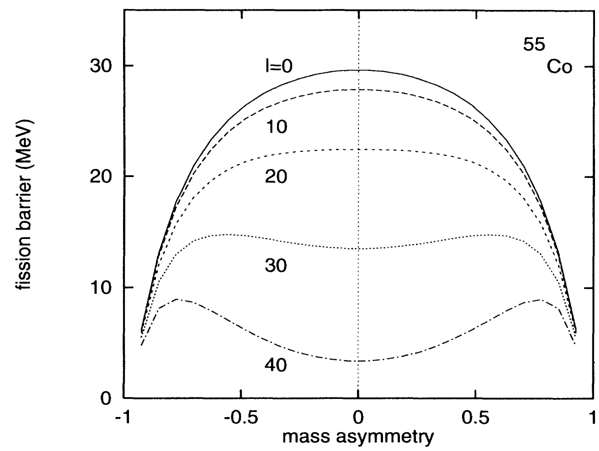

FIG. 6. Fission barrier vs mass asymmetry for various values of l for the system ^{55}Co.

tive fissility parameter x for a calculation of the conditional barrier heights of rotating nuclei can now be written as

$$x = \left(x_0 + 0.44 \frac{l^2}{A^{1.843}} \right) \frac{(1-D)^2}{1+3D} \tag{8}$$

and used in Eq. (5).

Figure 6 shows the calculated conditional barriers for ^{55}Co for various values of angular momentum l. We now describe a simple use of the proposed formula in the analysis of a typical heavy ion fusion experiment.

One of the long standing puzzles in heavy ion fusion reaction studies is the deviations noticed between the measured particle yields and energy spectra and the predictions of the standard statistical theory of deexcitation of excited nuclei. While some groups have gone to the extent of questioning the validity of the statistical theory for nuclei [9], with large excitation energies and spins others have attempted a simple modification of the transmission coefficients, level densities of the emitted nuclei, etc. to fit the measured quantities [10]. It is, however, known that in heavy ion induced fusion reactions, the formation phase of the compound nucleus (CN) plays an important role and may leave characteristic signatures in the fusion observables, particularly, the multiplicity and energy spectra of the emitted particles [11]. The effect is expected to be important when the temperature of the composite system is comparable to the potential barriers encountered and when the angular momentum carried is large. For example, the ^{28}Si+^{27}Al reaction studied by Agnihotri *et al.* [10], is expected to populate the compound nucleus ^{55}Co at an excitation energy of 84 MeV and l values up to $42\hbar$. The corresponding temperature of the CN is about 3.5 MeV. We examine here the ^{28}Si+^{27}Al reaction at 140 MeV in detail to find out whether the deviations observed by Agnihotri *et al.*, might be a consequence of the entrance channel dynamics and not entirely to a modification of the parameters of the statistical model of the compound nucleus. Fig-

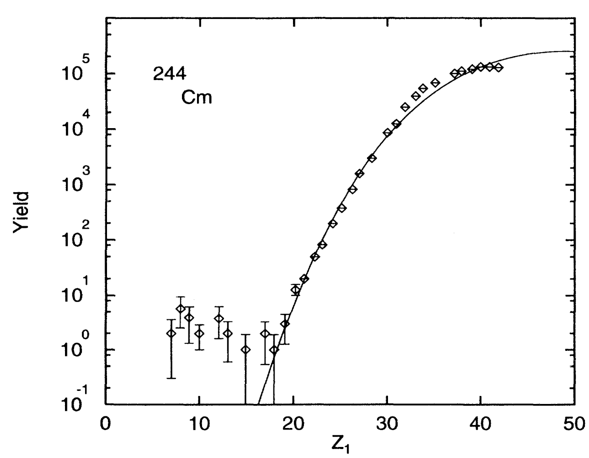

FIG. 5. Yield vs the charge of the emitted fragment for ^{244}Cm. Calculated yields are shown by a solid line. Experimental points are taken from Ref. [8].

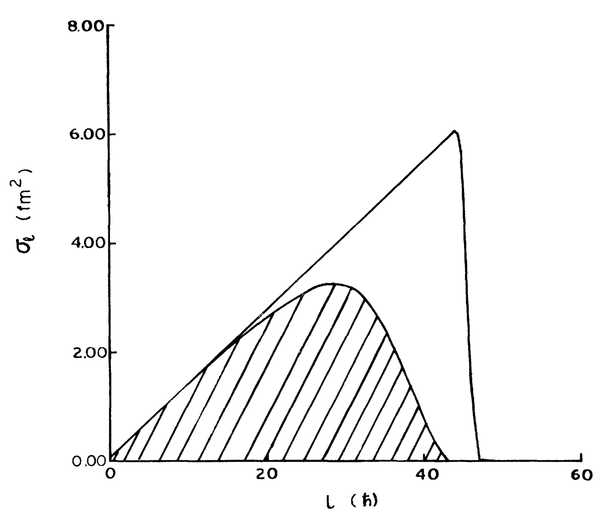

FIG. 7. Partial cross-section σ_l vs l for the system ^{55}Co. Cross section for CN formation is shown by the shaded region.

ure 6 shows the conditional barriers as a function of the mass asymmetry $(\frac{A_2-A_1}{A_2+A_1})$ for different values of l. Figure 7 shows the calculated partial cross sections σ_l for fusion using the one-dimensional barrier penetration model. As discussed in [11], it is the conditional barrier which keeps the system together until it equilibrates in all degrees of freedom resulting in the formation of the compound nucleus. It is known that the shape relaxation is quite slow and takes more than about 10^{-20} sec to reach a nearly spherical CN. We have therefore calculated the probability of forming a CN without premature seperation using the conditional barrier heights and the temperature [12]. The calculated differential cross section for CN formation is also shown in Fig. 7 as a shaded area. Out of the total cross section of 1450.5 mb, only about 50% of the events result in the formation of a fully equilibrated CN which undergoes statistical decay. The remaining fraction of the events seperate out before CN formation resulting in fissionlike fragments and the collision trajectory always remains elongated. Such events involve emission of α's from highly deformed shapes involving lower emission temperatures and modified transmission coefficients. Therefore the results of Ref. [10] might suggest not a deformed CN but the presence of collision events not leading to compound nucleus formation. Any quantitative analysis of the data should therefore involve the precompound nucleus dynamics and the emission of particles during this phase of the reaction. The utility of the proposed pocket formula in such calculations is obvious.

In summary, we propose here a simple formula for conditional fission barriers of rotating nuclei based on the extension of Swiatecki's expressions for the conditional barrier heights to include the effects of finite range of the nuclear force and angular momentum. The utility of the present formula in the analysis of experimental data on heavy ion fusion reactions is also discussed.

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