Simulation of the exclusion principle in the neutron-^{16}O interaction through a repulsive term and application to a three-body calculation of the ^{16}O$(d,p)^{17}$O reaction

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A two-term s-wave separable potential which generates only the $2s_{1/2}$ single-particle state in ^{17}O and reproduces the low energy phase shift is constructed. The two-term potential, together with a d-wave separable potential which reproduces the $1d_{5/2}$ single-particle state in ^{17}O, is used in a three-body Faddeev calculation of the ^{16}O$(d,p)^{17}$O reaction.

Despite the success of the distorted-wave Born approximation (DWBA) as a tool to analyze deuteron stripping, a more complete description which takes into account the three-body dynamics became the subject of many theoretical works. Most of these assume that the neutron and the proton belonging to the incident deuteron interact with the target nucleus through standard optical potentials. In order to solve the three-body Schrödinger equation, one expands the wave function in a basis which consists of the eigenstates of the neutron-proton relative motion. The resulting coupled equations for the center-of-mass motion are treated through the adiabatic approximation [1] or solved more accurately using the CDCC (continuum-discretized coupled-channels) method [2]. An alternative expansion was recently considered by Laid *et al.* [3]. In these approaches the stripping amplitude cannot be obtained from the asymptotic model wave function and one has to use the integral form of the transition amplitude, which requires the three-body wave function only in the vicinity of the target nucleus.

The simultaneous inclusion of the rearrangement channels in the CDCC calculation is difficult. An approximate calculation [4] finds that the coupling to the proton channel has a substantial effect at low energies.

A natural approach to deuteron reactions in the three-body model is provided by the Faddeev formalism which allows a simultaneous treatment of both the elastic, the stripping, and the breakup channels. For local potentials, the Faddeev equations are difficult to solve numerically and most of the applications have been done using nonlocal separable potentials. This does not represent a great drawback as low-energy scattering amplitudes generated by short-range potentials may also be obtained by a superposition of separable potentials. So far, only the $d+\alpha$ system has been successfully treated in the Faddeev approach [5–7].

Faddeev calculations have been attempted also to describe the stripping reaction ^{16}O$(d,p)^{17}$O [8,9]. One assumes a three-body model consisting of an inert core of ^{16}O plus two nucleons (the neutron and the proton in the incident deuteron). In the process, the neutron cannot be captured in the single-particle states already occupied in the target nucleus. This constraint has been approximated [9,10] by using for the neutron-^{16}O interaction, a separable potential which generates only the $1d_{5/2}$ and the $2s_{1/2}$ single-particle states, which are allowed by the exclusion principle. The s-wave interaction which generates the $2s_{1/2}$ state without producing the forbidden $1s_{1/2}$ state may be taken of the form

$$\langle \mathbf{Q}|V_{s_{1/2}}|\mathbf{Q}'\rangle = -\frac{\Lambda_{s_{1/2}}}{2m}v_{s_{1/2}}(Q)v_{s_{1/2}}(Q')$$
$$\times \sum_{\mu}\langle\hat{\mathbf{Q}}|\mathcal{Y}_{0\frac{1}{2}\mu}\rangle\langle\mathcal{Y}_{0\frac{1}{2}\mu}|\hat{\mathbf{Q}}'\rangle \ , \tag{1}$$

where $\mathbf{Q}$ and m are the momentum and the mass of the neutron and $\Lambda_{s_{1/2}}$ is the magnitude of the interaction. For the form factor, we may choose

$$v_{s_{1/2}}(Q) = (Q^2 + \alpha_{s_{1/2}}^2)\left(\frac{3}{2} - \beta_{s_{1/2}}^2 Q^2\right)e^{-(1/2)\beta_{s_{1/2}}^2 Q^2} \ . \tag{2}$$

Taking

$$\frac{1}{\Lambda_{s_{1/2}}} = \int_0^\infty dQ \, Q^2 \frac{[v_{s_{1/2}}(Q)]^2}{Q^2 + \alpha_{s_{1/2}}^2} \ , \tag{3}$$

we get a bound state with energy $\varepsilon_{s_{1/2}} = -\alpha_{s_{1/2}}^2/(2m)$ and wave function

$$\phi_{0\frac{1}{2}\mu}(\mathbf{Q}) = N_{s_{1/2}}(\tfrac{3}{2} - \beta_{s_{1/2}}^2 Q^2)$$
$$\times e^{-(1/2)\beta_{s_{1/2}}^2 Q^2}\langle\hat{\mathbf{Q}}|\mathcal{Y}_{0\frac{1}{2}\mu}\rangle \ , \tag{4}$$

where $N_{s_{1/2}}$ is the normalization factor. This function is identical to the eigenfunction describing the $2s_{1/2}$ state of the harmonic oscillator. The value of the parameter $\alpha_{s_{1/2}}$ (0.3974 fm^{-1}) is fixed by using the experimental value $\varepsilon_{s_{1/2}} = -3.275$ MeV. The value of $\beta_{s_{1/2}}$ (2.176 fm) is chosen to give the correct mean-square radius (4.07 fm) of the $2s_{1/2}$ neutron orbit in ^{17}O [11]. The parameter $\Lambda_{s_{1/2}}$ is fixed by Eq. (3) and its value turns out to be 17.280 fm^5. We mention here that, for simplicity in the
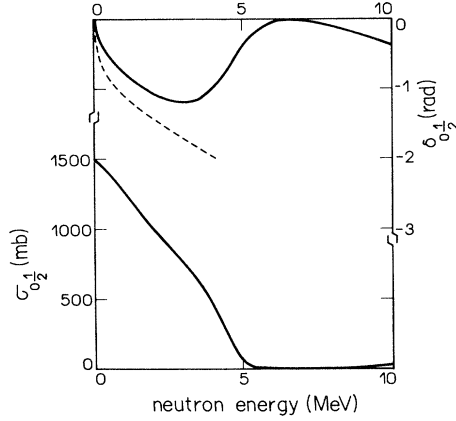
FIG. 1. Phase shift and total cross section predicted by the one-term s-wave separable potential [Eq. (1)]. The values of the parameters are $\alpha_{s_{1/2}} = 0.3974$ fm^{-1}, $\beta_{s_{1/2}} = 2.176$ fm, and $\Lambda_{s_{1/2}} = 17.280$ fm^5. The dashed line represents the nonresonant component of the measured phase shift.

three-body kinematics, the mass of the core is assumed to be infinite.

Figure 1 shows the phase shift and the cross section corresponding to the one-term potential defined by Eqs. (1)–(3). The phase shift exhibits clearly an anomalous behavior, namely, it does not decrease monotonically with the energy, as it happens for the nonresonant component of the phase shift obtained from experiment [12]. In fact, the phase shift bends back toward the value zero and this oscillation generates a slight hump in the cross section.

The one-term potential given by Eq. (1) is not satisfactory also because it generates scattering wave functions which are not orthogonal to the forbidden $1s_{1/2}$ state and thus the exclusion principle is not completely satisfied.

The purpose of this work is to construct an improved s-wave neutron-^{16}O potential and then to apply it in a model three-body Faddeev calculation of the ^{16}O$(d,p)^{17}$O reaction. Since the Pauli principle is properly taken into account by the new potential, we consider the model a refinement of those used in other three-body Faddeev calculations of the same reaction [9,10]. As in [9], we also use a separable potential which acts in the $d_{5/2}$ wave producing the $1d_{5/2}$ bound state.

In the case of the neutron-alpha interaction, many authors [5] use a repulsive separable potential in the s wave in order to simulate the effect of the Pauli principle which forbids the occurrence of a $1s_{1/2}$ bound state in the neutron-alpha system. This suggests that we add to the original potential [Eq. (1)] a repulsive term, so that

$$\langle \mathbf{Q}|V_{s_{1/2}}|\mathbf{Q}'\rangle = \left[-\frac{\Lambda_{s_{1/2}}}{2m}v_{s_{1/2}}(Q)v_{s_{1/2}}(Q') \right.$$

$$\left. +\frac{\lambda_{s_{1/2}}}{2m}g_{s_{1/2}}(Q)g_{s_{1/2}}(Q') \right]$$

$$\times \sum_\mu \langle \hat{\mathbf{Q}}|\mathcal{Y}_{0\frac{1}{2}\mu}\rangle\langle \mathcal{Y}_{0\frac{1}{2}\mu}|\hat{\mathbf{Q}}'\rangle . \qquad (5)$$
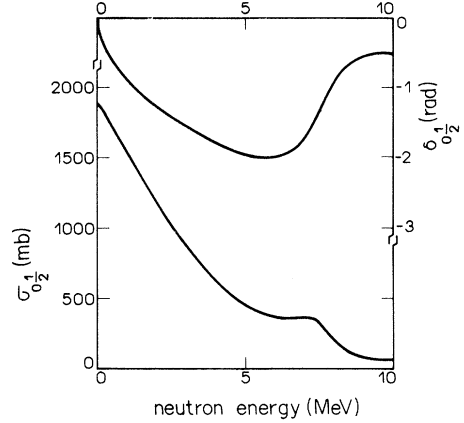

FIG. 2. Same as Fig. 1 for the two-term separable potential [Eq. (5)] with $\lambda_{s_{1/2}} = 3$ fm.

The form factor $g_{s_{1/2}}(Q)$ is chosen as

$$g_{s_{1/2}}(Q) = e^{-(1/2)\beta_{s_{1/2}}^2 Q^2} , \qquad (6)$$

the parameter $\beta_{s_{1/2}}$ being the same as in Eq. (2). As a consequence of the orthogonality relation

$$\int_0^\infty dQ \, Q^2 e^{-(1/2)\beta_{s_{1/2}}^2 Q^2}\left(\tfrac{3}{2} - \beta_{s_{1/2}}^2 Q^2\right)e^{-(1/2)\beta_{s_{1/2}}^2 Q^2}$$

$$= 0 , \qquad (7)$$

the bound state corresponding to the original potential remains unchanged. However, the scattering states are affected by the new term. It can be shown that, for $\lambda_{s_{1/2}}$ very large, the corresponding scattering wave functions Ψ become orthogonal to $g_{s_{1/2}}$. Since $g_{s_{1/2}}$ was chosen identical to a $1s_{1/2}$ harmonic-oscillator wave function appropriate to describe the forbidden $1s_{1/2}$ state, we may say that the exclusion principle is satisfied in the limit of large values of $\lambda_{s_{1/2}}$. Figure 2 refers to a value $\lambda_{s_{1/2}} = 3$ fm. Although the hump in the cross section is more pronounced than in Fig. 1, we notice that the anomaly has moved to a higher energy and, for energies below the anomaly, the phase shift is improved, becoming closer to experiment. In fact, performing calculations with increasing values of
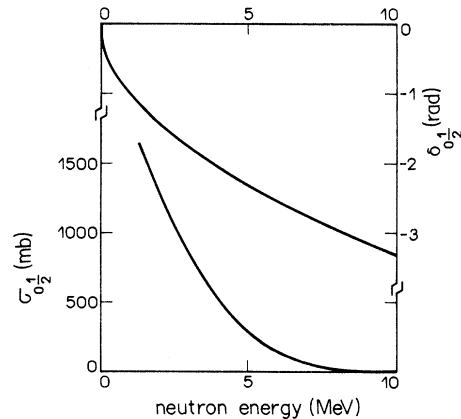

FIG. 3. Same as Fig. 2 with $\lambda_{s_{1/2}} = 50$ fm.

$\lambda_{s_{1/2}}$, we can remove the anomaly to energies well above the energy region relevant to processes involving a low-energy neutron. At the same time, in the low-energy region, the phase shift improves and the magnitude of the overlap integral $\langle g_{s_{1/2}}|\Psi\rangle$ decreases. For $\lambda_{s_{1/2}} = 50$ fm, the calculated phase shift is shown in Fig. 3 and is in good agreement with experiment (dashed line in Fig. 1). The anomaly is pushed to a region around 50 MeV and, for scattering energies below 10 MeV, $|\langle g_{s_{1/2}}|\Psi\rangle|^2$ decreases about 2 orders of magnitude when compared to its values for $\lambda_{s_{1/2}} = 0$. By increasing $\lambda_{s_{1/2}}$ even more, we observe that the phase shift changes only very slightly in the energy region between 0 and 10 MeV. This means that at low energies the effect of the repulsion almost saturates when $\lambda_{s_{1/2}} = 50$ fm. For this reason, we shall consider the two-term potential with $\lambda_{s_{1/2}} = 50$ fm as an appropriate effective potential to describe the low-energy s-wave neutron-^{16}O interaction. One should also use repulsive interactions in the $p_{3/2}$ and $p_{1/2}$ partial waves to project out the $1p_{3/2}$ and $1p_{1/2}$ single-particle states which are occupied in the core. However, the repulsion in these partial waves is expected to be less effective than in the s wave because of the centrifugal barrier. In fact, the low-energy $p_{3/2}$ and $p_{1/2}$ phase shifts are small once the compound elastic resonances are removed [12]. For this reason, the p-wave interactions are neglected. Alternative ways of introducing projection operators to eliminate Pauli forbidden components are given in Refs. [7,13–15].

For the $d_{5/2}$ interaction, we use the potential

$$\langle\mathbf{Q}|V_{d_{5/2}}|\mathbf{Q}'\rangle = -\frac{\Lambda_{d_{5/2}}}{2m}v_{d_{5/2}}(Q)v_{d_{5/2}}(Q')$$
$$\times \sum_{\mu}\langle\hat{\mathbf{Q}}|\mathcal{Y}_{2\frac{5}{2}\mu}\rangle\langle\mathcal{Y}_{2\frac{5}{2}\mu}|\hat{\mathbf{Q}}'\rangle ,\qquad(8)$$

where the form factor is taken to be [9]

$$v_{d_{5/2}}(Q) = (Q^2 + \alpha^2_{d_{5/2}})Q^2 e^{(1/2)\beta^2_{d_{5/2}}Q^2} .\qquad(9)$$

If the parameter $\Lambda_{d_{5/2}}$ is constrained by a relation analogous to Eq. (3), the binding energy is given by $\varepsilon_{d_{5/2}} = -\alpha^2_{d_{5/2}}/(2m)$ and the bound-state wave function is exactly the $1d_{5/2}$ oscillator wave function,

$$\phi_{2\frac{5}{2}\mu}(\mathbf{Q}) = N_{d_{5/2}}Q^2 e^{-(1/2)\beta^2_{d_{5/2}}Q^2}\langle\hat{\mathbf{Q}}|\mathcal{Y}_{2\frac{5}{2}\mu}\rangle ,\qquad(10)$$

$N_{d_{5/2}}$ being the normalization factor. The experimental values [11] $\varepsilon_{d_{5/2}} = -4.146$ MeV and $(\langle r^2\rangle_{d_{5/2}})^{1/2} = 3.44$ fm require the following values of the parameters: $\Lambda_{d_{5/2}} = 34.664$ fm^9, $\alpha_{d_{5/2}} = 0.4472$ fm^{-1}, and $\beta_{d_{5/2}} = 1.839$ fm.

Concerning the proton-core interaction, we do not take into account the Coulomb repulsion since there is no reliable way to treat numerically this interaction in the Faddeev approach to the three-body problem. For deuteron energies well above the Coulomb barrier ($\cong 3.5$ MeV), we expect that it is a reasonable first approximation for the treatment of the (d,p) stripping channels, since the Coulomb force will not deflect appreciably both the incoming deuteron and the outgoing proton. For the nu-
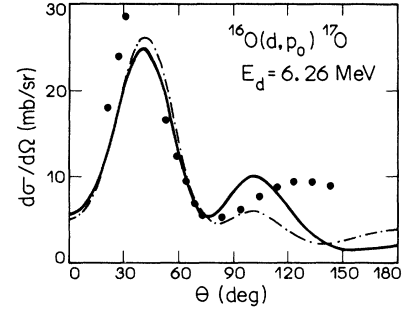

FIG. 4. Angular distribution for the stripping to the ground state ($1d_{5/2}$ sp level) of ^{17}O. The solid (dot-dashed) line corresponds to using two terms (one term) in the s-wave nucleon-core interaction. The experimental values (represented by dots) are from [18].

clear part of the proton-core interaction, we use the same potential as for the neutron, that is, our model treats the neutron and the proton in a symmetrical way (symmetric model).

For the neutron-proton triplet s-wave interaction, we use the separable potential

$$\langle\mathbf{p}|V_{12}|\mathbf{p}'\rangle = -\frac{\Lambda}{m}v(p)v(p')\sum_{M_S}|1,M_S\rangle\langle1,M_S| ,\qquad(11)$$

where $\mathbf{p}$ is the relative momentum and $|1,M_S\rangle$ is the spin-wave function of the triplet state. The form factor is chosen to be of the Yamaguchi form, $v(p) = 1/(p^2+\alpha^2)$. The experimental values $a = 5.42$ fm and $r_0 = 1.76$ fm for
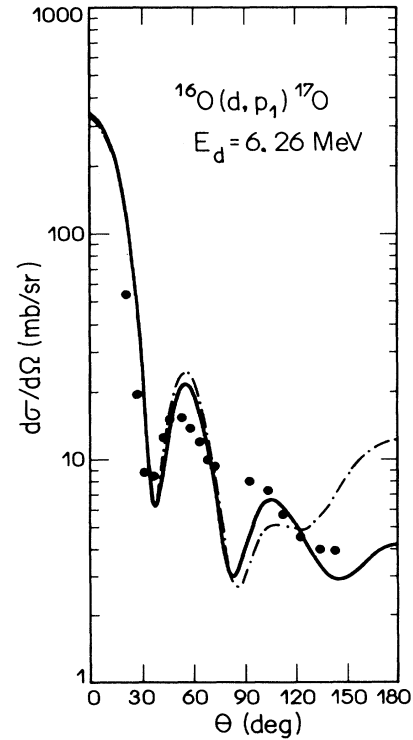

FIG. 5. Angular distribution for the stripping to the excited state ($2s_{1/2}$ sp level) of ^{17}O. The drawing convention is the same as in Fig. 4.

the scattering length and the effective range of the triplet s-wave neutron-proton scattering are reproduced if one takes $\alpha = 1.406$ fm^{-1} and $\Lambda = 0.382$ fm^{-3}. With this choice, we get 2.223 MeV for the binding energy of the deuteron. Since the model is symmetric and the process is initiated by the deuteron which has isobaric spin T = 0, the singlet s-wave interaction will not come into play.

We worked out the Alt, Grassberger, and Sandhas (AGS) [16] equations for the interactions described previously. The derivation of the final form of the equations is contained in Ref. [9], except for modifications due to the fact that we use a two-term separable potential for the s-wave nucleon-core interaction. The coupled integral equations were solved numerically by the method of contour rotation [17]. Using the Gauss quadrature method to approximate the integrals by finite sums, we end up with a system of algebraic equations.

The calculation was performed for a deuteron energy $E_d = 6.26$ MeV. At this energy, there are available the (energy averaged) experimental data of Cords et al. [18]. It was necessary to include 13 partial waves $(J^\pi = 0^-, 1^+, 1^-, \cdots, 6^+, 6^-)$ in order to reach convergence. We did not consider a higher energy (one really well above the Coulomb barrier) as this would need the computation of a much larger number of partial waves.

The results (solid line) are shown in Figs. 4–6. One sees that a reasonable agreement with experiment is obtained despite the simplicity of the model. In fact, our stripping cross sections are close to the ones obtained by Cords et al. using the DWBA.

In order to see the importance of the repulsive term in the $s_{1/2}$ potential, we performed a calculation omitting this term. The results are the curves drawn with a dot-dashed line in Figs. 4–6. One sees that the repulsive term has a small effect, but in the case of the (d, p_1) and (d, d) reactions (Figs. 5 and 6) the results are slightly improved when it is included. The smallness of the effect may be due to the fact that the additional term modifies the nucleon-core interaction in only one partial wave while
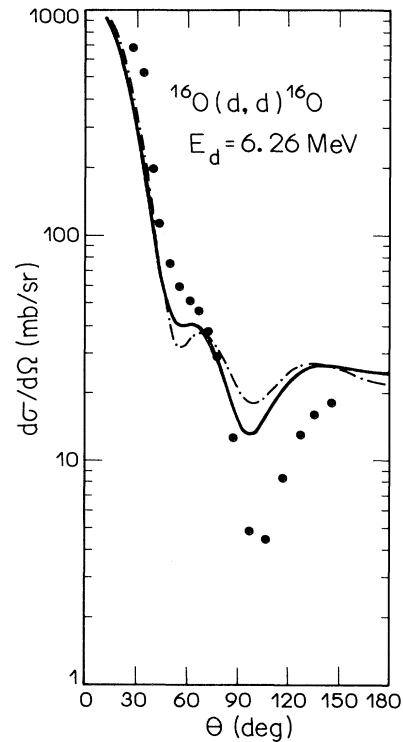

FIG. 6. Angular distribution for the elastic scattering of the deuteron.

many partial waves of the nucleon-core subsystem are involved in the calculation.

For lower deuteron energies $(E_d \cong 2$ MeV) we found that the contribution of the repulsive term of the s-wave interaction becomes more important. However, this conclusion is not definite since, at these energies, it is essential to take into account the Coulomb force in order to obtain meaningful results.

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