Binding energies of proton-rich nuclei in the vicinity of ^{100}Sn

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Shell-model calculations suggest that binding energies of several proton-rich nuclei in the vicinity of ^{100}Sn differ appreciably from the estimates given in the most recent Mass Tables. We give improved estimates, which should be of considerable interest to researchers at GSI and elsewhere who are studying β-decay and β-delayed proton emission from these nuclei.

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In recent years, much of the experimental effort in nuclear spectroscopy has been devoted to studies of nuclei close to the limits of particle stability. Of special interest has been the region close to ^{100}Sn, since this provides the most massive $N = Z$ nuclei which are stable to proton emission. The region is also of interest because of the prediction of a $N = Z = 50$ closed shell, and the strong Gamow-Teller β^+ decays arising from $g_{9/2} \rightarrow g_{7/2}$ transitions. Research groups at GSI have been producing nuclei in the vicinity of ^{100}Sn using fusion-evaporation reactions and on-line mass separation, recent examples of their work being investigations of the β decay of ^{100}In and ^{102}In [1], and a study of β-delayed proton emission following decay of ^{101}Sn [2]. Analysis of the data has been hampered by the fact that no accurate binding energies are known for many of the nuclei, with only "estimates from systematic trends" being given in the latest atomic mass compilation [3]. Hence β-decay Q values, and proton separation energies, are very poorly known.

In an attempt to provide more accuracy mass data we have carried out a number of shell-model calculations for nuclei with neutron number 50, 51, and 52. For $N = 50$, Gloekner and Serduke [4] obtained proton interactions with a $(g_{9/2}p_{1/2})^n$ model space and ^{88}Sr core. Their "full energy" interaction is referred to below as GS. We have repeated their least-squares fit of nine two-particle matrix elements and two single-particle energies, using a much larger set of experimental data (65 levels) than was available to them. The resulting interaction, referred to below as JS1, gives χ^2 equal to 62 keV, to be compared with 72 keV for GS. We have also carried out a least-squares fit of the two-particle matrix elements and three single-particle energies in a $(g_{9/2}p_{1/2}p_{3/2})^n$ model space with ^{84}Se core. This interaction, referred to below as JS2, gives χ^2 of 60 keV to 100 levels between ^{85}Br and ^{96}Pd. Binding energies of nuclei between ^{90}Zr and ^{100}Sn, calculated using JS1 and JS2, are compared in Table I with values given in the 1993 Atomic Mass Tables [3].

Calculation of even-parity levels in $N = 51$ nuclei have been reported in an earlier paper [5]. These used the GS interaction for protons, and carried out a least-squares fit to determine a neutron-proton interaction with the 51st neutron allowed to occupy the $d_{5/2}$, $s_{1/2}$, $d_{3/2}$, and $g_{7/2}$ orbitals. Free parameters were the $g_{9/2}$ $d_{5/2}$ matrix elements, the neutron single-particle energy gaps, and the $T = 0$ and $T = 1$ strengths of a volume Δ force used for the remainder of the np interaction. The energy centroid for interaction of a $p_{1/2}$ proton with the neutron in any

TABLE I. Binding energies of $N = 50$ nuclei calculated with interactions JS1 and JS2, and energies from the 1993 mass tables [3]. The symbol # denotes an estimate from systematic trends.

	JS1	JS2	Mass tables
^{90}Zr	783.82	783.87	783.894(2)
^{91}Nb	789.10	789.10	789.053(3)
^{92}Mo	796.43	796.47	796.509(4)
^{93}Tc	800.63	800.64	800.596(4)
^{94}Ru	806.83	806.81	806.843(13)
^{95}Rh	809.96	809.93	809.900(150)
^{96}Pd	815.06	814.99	815.030(150)
^{97}Ag	817.12	817.13	816.940(400)#
^{98}Cd	821.15	821.14	820.890(210)#
^{99}In	822.12	822.38	821.640(500)#
^{100}Sn	825.08	825.39	824.480(450)#

TABLE II. Binding energies of $N = 51$ nuclei calculated with interactions JS3 and JS4, and energies from the 1993 mass tables [3]. The symbol # denotes an estimate from systematic trends.

	JS3	JS4	Mass tables
^{91}Zr	791.03	791.00	791.089(2)
^{92}Nb	796.97	796.97	796.936(3)
^{93}Mo	804.55	804.53	804.579(4)
^{94}Tc	809.26	809.25	809.219(4)
^{95}Ru	815.77	815.75	815.797(12)
^{96}Rh	819.37	819.35	819.261(13)
^{97}Pd	824.72	824.67	824.720(300)
^{98}Ag	827.25	827.19	827.090(150)
^{99}Cd	831.17	831.09	831.360(210)#
^{100}In	833.06	832.94	832.530(400)#
^{101}Sn	836.19	836.03	835.650(500)#

TABLE III. Binding energies of $N = 52$ nuclei calculated with interactions JS5 and JS6, and energies from the 1993 mass tables [3]. The symbol # denotes an estimate from systematic trends.

	JS5	JS6	mass tables
^{92}Zr	799.72	799.72	799.724(2)
^{93}Nb	805.91	805.94	805.766(2)
^{94}Mo	814.30	814.33	814.257(2)
^{95}Tc	819.29	819.32	819.152(5)
^{96}Ru	826.48	826.49	826.490(8)
^{97}Rh	830.31	830.31	830.300(40)
^{98}Pd	836.27	836.24	836.296(22)
^{99}Ag	839.01	838.96	839.010(150)
^{100}Cd	843.87	843.79	843.840(110)
^{101}In	845.59	845.42	845.280(300) #
^{102}Sn	849.44	849.24	848.910(400) #

orbital was set equal to -374 keV, the value for $d_{5/2}$ deduced from ^{90}Y. We have also derived another interaction in the same model space, making somewhat different assumptions. The proton interaction was the improved version JS1 described above, and the neutron single-particle gaps were held fixed at the values suggested by neutron stripping on ^{88}Sr—i.e., the $s_{1/2}$, $d_{3/2}$, and $g_{7/2}$ levels were placed 1.03, 2.01, and 2.67 MeV above the $d_{5/2}$. The $p_{1/2}d_{5/2}$ centroid was again set equal to -374 keV, but the other centroids were allowed to be free parameters together with the $g_{9/2}d_{5/2}$ matrix elements and the Δ-force strengths. Binding energies calculated with these two interactions, referred to as JS3 and JS4, are compared to mass table values in Table II.

Calculations for $N = 52$ nuclei have been carried out in the $(g_{9/2}p_{1/2})^n \times (d_{5/2}s_{1/2}d_{3/2}g_{7/2})^2$ model space using JS3 and JS4 for the proton-proton and proton-neutron interactions. The most important part of the neutron-neutron interaction, the $d^2_{5/2}$ matrix elements, was chosen to reproduce the lowest 0^+, 2^+, and 4^+ levels of ^{92}Zr, while the remainder of the interaction was taken to be a volume Δ force with strength chosen to optimize well-known binding energies. Results with these interactions, referred to as JS5 and JS6, are given in Table III.

In Tables I–III, discrepancies between theory and mass table values are small for those nuclei whose binding energies have been determined from experiment, but become very large for several of those estimated from systematic trends. Estimated values are consistently lower. Our results for proton-rich nuclei are summarized in Table IV, which gives averages of values from JS1 and JS2 for $N = 50$, JS3 and JS4 for $N = 51$, and JS5 and JS6 for $n = 52$. The tentative error estimates are based on the variation between different interactions, and on the magnitude of the discrepancies between theory and experiment for nuclei with well-defined experimental values.

TABLE IV. Binding energies predicted by the calculations, with estimated possible errors.

^{97}Ag	817.12(5)
^{98}Cd	821.15(5)
^{99}Cd	831.13(10)
^{99}In	822.25(15)
^{100}In	833.00(10)
^{100}Sn	825.23(20)
^{101}In	845.50(15)
^{101}Sn	836.11(10)
^{102}Sn	849.34(15)

In view of the good agreement for the lighter nuclei it is tempting to propose that the calculated energies of the proton-rich nuclei close to ^{100}Sn are also accurate. However, the results of Ji and Wildenthal [6] offer a warning that such extrapolations can be dangerous. These authors carried out an approximate least-squares fit of interaction parameters within a $(f_{5/2}p_{3/2}p_{1/2}g_{9/2})^n$ model space for $N = 50$ nuclei between ^{82}Ge and ^{96}Pd. Although the binding energies of lighter nuclei are well fitted, the calculated value for ^{100}Sn is over 2 MeV greater than given by JS1 or JS2 (and 3.25 MeV greater than the systematic trends estimate). This clearly shows the danger of extrapolating results from the lower part of a model space to the upper part where some of the two-body matrix elements appear with much higher weighting. For example, the main component of the energy of ^{94}Ru (a nucleus quite well fitted by Ji and Wildenthal) involves a weighting of only 6 for the important $g^2_{9/2}$ matrix elements, whereas for ^{100}Sn these have a weighting of 45. Moreover, it may be that effective parameters close to the drip line differ from those applicable to lighter nuclei due, for example, to a change in the radial wave functions.

The Ji and Wildenthal binding energies for ^{95}Rh and ^{96}Pd, at the upper end of their fitted nuclei, are too large by 350 and 590 keV, whereas in our calculations binding energies remain close to experimental values for all nuclei with known energies. There is therefore no evidence that our energies diverge as do those of Ji and Wildenthal, but it is clear that the errors given in Table IV must remain open to question until new experimental data becomes available.

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