Color transparency assumptions

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An exactly solvable model is used to investigate the assumptions behind color transparency.

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I. INTRODUCTION

Color transparency (CT) is the anomalously high transparency of the nucleus to nucleons in quasielastic high-momentum transfer nuclear processes, measured with resolution good enough to ensure that no "extra" pions are produced. This means that the absorptive nuclear optical potential representing initial and final state interactions plays no role in such reactions. Color transparency is under active investigation with experiments performed at BNL [1], SLAC [2], and ongoing work at BNL [3] and experiments proposed at CEBAF. Further references can be found in the review Ref. [4].

The existence of color transparency depends upon three assumptions [5,6]:

(i) A small wave packet is formed in a high-momentum transfer reaction. This wave packet is sometimes dubbed a pointlike configuration (PLC).

(ii) The interactions between a small color neutral wave packet and the nucleus are suppressed.

(iii) The wave packet escapes the nucleus before expanding. The expansion time τ is typically stated as $\tau \sim \tau_0 P/M$ where τ_0 is a time in the rest frame (expected to be about 1 fm) needed for a small system to expand and P/M is a time dilation factor. For large enough momentum, P, τ is large.

Each of these assumptions can be questioned. Indeed, the question of whether or not a small wave packet is formed depends crucially on the properties of the ground state wave function, hence on poorly understood features of nonperturbative quantum chromodynamics QCD [7,8]. The reduction of the final state interaction is often explained as due to the cancellation of gluon emission amplitudes that occurs when a color singlet system consists of closely separated quarks and gluons. The legality of adding amplitudes (before squaring) requires a coherent reaction, so that the cancellation is limited to a select class of reactions. An additional issue is the value of the factor M in the time dilation; why couldn't it be as large as P? Another way to phrase this question is that the expansion of the PLC can be slow only if highly excited intermediate states are not important [9,10].

The purpose of this paper is to use a simple model with realistic features to investigate the meaning and limitations of each of the three basic assumptions. The model is defined and described in Sec. II. Each "nucleon" is the ground state of an electrically neutral systems of two quarks interacting via the Coulomb potential. The wave functions for this potential cannot be computed using perturbation theory, but can be computed exactly. In this sense the above force provides an example of a nonperturbative, yet solvable interaction. The two quarks have different masses, with the ratio of the heavier to the lighter being 2. The nucleus is a collection of such nucleons. This model is relevant to color transparency physics because of the analogy between the electric neutrality of quantum electrodynamics QED and the color neutrality of QCD. Within this model, it is straightforward to construct (Sec. III) the wavepacket formed in a high-momentum transfer reaction. We take the hard interaction as simply adding momentum to the heavy quark. This wave packet evolves as it moves through the surrounding nucleus. This evolution is governed by the internal Coulomb force between the heavy and light quarks and also the final state Coulomb interaction with the surrounding nucleons. This interaction is computed, and the dependence on the transverse separation between the heavy and light quarks is presented in Sec. IV. The process we consider in this paper is meant to be analogous to the $(e,e'p)$ scattering, one of the candidate reactions for the observation of CT. In the $(e,e'p)$ reaction hard and soft interactions are of different nature — the first one is electromagnetic; the second one is strong. The smallness of the soft interaction is a consequence of the color neutrality of the proton, which also has electric charge. In our model the soft interaction is of electromagnetic nature, and our nucleon is electrically neutral. The high-momentum transfer interaction has a distinct charge associated with it (only one of the quarks has this charge). The numerical results of the calculations are given in Sec. V. We find that a transparency phenomenon is obtained for the model under consideration. A small size object is initially formed in a high momentum transfer process, and the expansion rate of this object is inversely proportional to the momentum transfer Q. The origin of this slow expansion rate is investigated in Sec. VI. A brief summary is presented in Sec. VII.

II. THE MODEL

We consider a nonrelativistic, electrically neutral system of two quarks interacting with the Coulomb potential. The strength of the the electromagnetic interaction

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is given by the fine structure constant α; α determines the ratio of the binding energy of the system to the mass of its constituents. In Nature $\alpha = 1/137$, but within the present model α can be varied and need not be small. Thus the interaction we use is intended to represent the strong color force.

The form of the bound state and continuum wave functions are well known, and are not presented here. It is worthwhile to discuss the system of units that we use. Coulomb units with $\hbar = 1$, $e = 1$, and the mass of the light quark $m_q = 1.5$ (to set the reduced mass $m_{\rm red}$ to 1) are the natural choice for this problem. With this choice the size of the system in the ground state, the Bohr radius r_B, is the unit of length, and twice the ground state energy is the unit of energy. Note that in the Coulomb units it is the speed of light $c = 1/\alpha$ that sets the energy scale. In this paper $1/\alpha$ will be used instead of c. A relativistic dispersion relation is used for the energy E_X of the state $|X\rangle$ with momentum P_X

$$E_X^2 = M_X^2/\alpha^4 + P_X^2/\alpha^2. \tag{1}$$

The rest mass M_X includes the binding energy ε_X as well as the quark masses

$$M_X = 4.5 + \varepsilon_X \alpha^2. \tag{2}$$

The energy eigenvalues are

$$\varepsilon_n = -1/2n^2 \tag{3}$$

for the discrete part of the spectrum and

$$\varepsilon_k = k^2/2 \tag{4}$$

for the continuous part of the spectrum.

III. EXPANDING WAVE PACKET

A wave packet, possibly a pointlike configuration, is created when a photon of momentum Q is absorbed by the heavy quark of the nucleon in the ground state $|1\rangle$. Thus we write

$$|PLC\rangle \equiv e^{i\vec{q}\cdot\frac{1}{3}\vec{r}}|1\rangle, \tag{5}$$

where spin is ignored, $\frac{1}{3}\vec{r}$ is the position operator for the heavy quark relative to the center of mass. Thus Eq. (5) defines our model of the hard high-momentum transfer reaction. The transferred momentum is defined as $\vec{q} = Q\hat{z}$, which specifies the direction of the z axis.

The use of completeness allows us to express this wave packet in terms of the elastic $F(Q^2)$ and inelastic $f_X(Q^2)$ form factors

$$|PLC\rangle = F(Q^2)|1\rangle + \sum_X \left(\int dX\right) f_X(Q^2)|X\rangle \tag{6}$$

and

$$F(Q^2) = \int d^3r\psi_1^*(r)\exp\left(i\vec{q}\cdot\frac{1}{3}\vec{r}\right)\psi_1(r), \tag{7}$$

$$f_X(Q^2) = \int d^3r\psi_1^*(r)\exp\left(i\vec{q}\cdot\frac{1}{3}\vec{r}\right)\psi_X(r), \tag{8}$$

where $\psi_X(r)$ are the Coulomb eigenfunctions (subscript 1 for the ground state). The summation (integration) is over the complete set of the Coulomb eigenstates, $X \equiv nl$ for the discrete states and $X \equiv kl$ for the continuum states.

If there were no final state interaction (FSI) of the hadronic wave packet with the nucleus, the amplitude of detecting the nucleon in the ground state would be equal to the form factor $F(Q^2)$. Color transparency is concerned with situations in which experimental kinematics constrain the final state interactions to be soft, of low-momentum transfer. We shall model these soft interactions by treating the nuclear medium here as a set of neutral nucleons, (Sec. IV). Then the soft interactions are approximately proportional to the product of the wave-packet-nucleon forward scattering amplitude, with the density of nucleons. In the impulse approximation, the interaction is expressed in terms of a matrix element of an operator $\hat{\chi}(b)$. The main feature of this operator is its dependence on b, the transverse separation of the quarks ($\vec{b}\cdot\hat{z} \equiv 0$). In particular, $\hat{\chi}(b=0) = 0$. We shall discuss a specific model for $\hat{\chi}$ in Sec. IV below.

With this notation, the scattering amplitude $\mathcal{M}_1$ is given by

$$\mathcal{M}_1(Z) = \langle 1|\hat{\chi}\hat{G}(Z)|PLC\rangle, \tag{9}$$

to first order in $\hat{\chi}$ where $\hat{G}(Z)$ is the Green's propagator of a PLC a distance Z (along the $\hat{q}$ direction) through the medium. Color transparency occurs if this term is small compared to the Born amplitude $F(Q^2)$. Note that we use lower case letters (b, z) to denote transverse and longitudinal quark-antiquark separations and upper case letters (B, Z) to denote the displacement of the center of the wave packet from the center of the nucleus. An evaluation of the matrix element of Eq. (9) requires an integration over d^3r, but not over the coordinates (B, Z). (The B dependence of $\mathcal{M}_1$ is suppressed for simplicity.)

In the standard Glauber treatment of final state interactions [11] an optical potential approximation is often used. The potential is proportional to the forward scattering amplitude, hence to the total nucleon-nucleon cross section σ. This cross section determines the rate of the exponential decay of the scattering nucleon wave function. In the present case the b dependence of $|PLC\rangle$ varies with Z, because the Green's function $\hat{G}$ includes the effects of the heavy quark light quark Hamiltonian. Thus the PLC forward scattering amplitude varies with Z. In particular, if the initial state of Eq. (6) corresponds to one of very small transverse size, the PLC expands as Z increases. We shall assume that the eikonal approximation is valid. In that case, each of the states from the complete set of states defined above in (6), acquires a phase $\exp iP_X Z$ as it propagates a distance Z.

It is useful to define an effective cross section, $\sigma_{\rm eff}(Z)$, with

$$\sigma_{\rm eff}(Z) \equiv \mathcal{M}_1/F(Q^2). \tag{10}$$

The term effective cross section replaces the more appropriate terminology—imaginary part of the forward scattering amplitude—to correspond with notations of previous papers. The real part of the forward scattering amplitude plays little role in color transparency physics. The quantity $\sigma_{\text{eff}}(Z)$ depends on the overlap of $\hat{\chi}(b)$ with the quark-antiquark wave function, and is therefore a measure of how the size of the wave packet varies with Z [10]. Some standard manipulations then lead to the result:

$$\sigma_{\text{eff}}(Z) = \sigma + \sum_{l=0,2}\sum_{n=2}^{\infty} \frac{\chi_{nl} f_{nl}(Q^2)}{F(Q^2)} \exp i(P_n - P)Z + \sum_{l=0,2}\int_0^{\infty} dk \frac{\chi_{kl} f_{kl}(Q^2)}{F(Q^2)} \exp i(P_k - P)Z. \quad (11)$$

This result is similar to the one of Jennings and Miller [12]. In that work the matrix elements χ_X and the inelastic form factors f_X were taken from available data, and the color transparency condition $\sigma_{\text{eff}}(Z=0)=0$ was imposed. In the present work we use a specific model to evaluate those terms and can determine whether or not the color transparency condition is satisfied.

To proceed, we further specify our notation. The first term of Eq. (11) is the total cross section for the nucleon ground state to interact with a target nucleon. This is

$$\sigma = \langle 1|\hat{\chi}|1\rangle. \quad (12)$$

The matrix elements χ_X are

$$\chi_X = \langle 1|\hat{\chi}|X\rangle. \quad (13)$$

The orbital angular momentum of the states X are limited to even values by the requirements of parity conservation. We restrict the sum to values of $l = 0, 2$ to anticipate a specific form: $\hat{\chi}(b) \propto b^2$.

The momentum P_X of the excited state $|X\rangle$ is given by the energy conservation relation imposed by the wave equation:

$$P_X^2 + M_X^2/\alpha^2 = P^2 + M_1^2/\alpha^2, P \simeq Q. \quad (14)$$

An important quantity is $\sigma_{\text{eff}}(Z=0)$, which measures the size of the initially formed PLC. This must be small for $\sigma_{\text{eff}}(Z)$ to be small. Note that if $\sigma_{\text{eff}}(Z=0)$ is to be small, cancellations in Eq. (11) have to render $\sigma_{\text{eff}}(0)$ fall off rapidly with Q for $Q \gg 1$. We shall work with a simple form of the soft interaction $\hat{\chi}(b) = \sigma\hat{b}^2/\langle 1|\hat{b}^2|1\rangle$; see the next section. This form allows the evaluation of $\sigma_{\text{eff}}(Z=0)$ with the result

$$\sigma_{\text{eff}}(0) = \frac{2}{Q^2 r_B^2/4 + 1}. \quad (15)$$

This is significant; it says that the effective size $b^2 \sim 1/Q^2$ for large Q^2. This is a property of the ground state Coulomb wave function [7,8]. PLC formation is allowed in this model.

For nonzero values of Z the phase factors $\exp i(P_X - P)Z$ spoil the cancellation of different terms in the sum (11). As a result σ_{eff} grows as Z increases from 0. This indicates that an expanding PLC generally experiences final state interactions. If the PLC leaves the nucleus without significant expansion then the final state interactions are suppressed. How fast a particular term goes out of phase and upsets the cancellation depends on its momentum P_X

$$P_X = \sqrt{P^2 - \Delta M_X^2/\alpha^2}\ , \quad (16)$$

with

$$\Delta M_n^2/\alpha^2 = \frac{n^2-1}{n^2}\left(4.5 - \alpha^2\frac{n^2+1}{4n^2}\right) \quad (17)$$

for the discrete spectrum and

$$\Delta M_k^2/\alpha^2 = (k^2+1)\left[4.5 + \alpha^2/4(k^2-1)\right] \quad (18)$$

for the continuous spectrum.

Suppose that a limit $P \to \infty$ can be taken, such that all relevant $P_X \to P$. In that case, the Z dependence of σ_{eff} disappears, and the PLC does not expand. For the discrete states $P_n \gg \Delta M_n/\alpha$ is true for P as low as $\sim 3/r_B$. The situation is different for the continuous states, since their energy is not bound from above. The value of P large enough to ensure a slow rate of the PLC expansion depends on the structure of the matrix elements χ_{kl} and form factors f_{kl}. This goes back to the idea of Ref. [9], that CT can be observed only for momenta transfer greater than the energy of all important intermediate excited states. We shall use specific calculations within our model to investigate these issues. The use of equations like Eq. (11) in eikonal expressions for PLC wave functions is discussed in Ref. [13].

There is another concern about the contribution of the higher excited states. The eikonal approximation used to derive the phase factors $\exp i(P_X - P)Z$ breaks down if $P_X \to 0$. Therefore any evidence, that the contribution into the sum (11) of the states with $P_X \ll 1$ is important, invalidates the approach developed above.

We shall also investigate the dependence on α. The physical range of α is from 0 to 3. The value $\alpha = 0$ corresponds to the nonrelativistic limit in which the speed of light ($1/\alpha$) is infinite with

$$E_X = \varepsilon_X + \frac{P_X^2}{2m_{\text{red}}}. \quad (19)$$

The upper limit $\alpha = 3$ corresponds to $M_1 = 0$; recall Eq. (2). A further increase in α would yield a negative rest mass of ground state nucleon.

IV. THE WAVE PACKET NUCLEON INTERACTION

We are concerned here with deriving the interaction $\hat{\chi}$, which has been defined above as the forward scattering amplitude between two quarks (with a mass ratio of two) of transverse separation b and a nucleon target. We are using a nonrelativistic Coulomb bound state for the ground state dynamics of the heavy and light quarks, so we also take the nucleon targets to be the ground state of the same system.

The expression for the projectile-nuclear forward scattering amplitude is [11]

$$\hat{f}(\theta=0,b)=\frac{ik}{2\pi}\int d^2B\left[1-e^{-i\chi(B,b,\hat{B}\cdot\hat{b})}\right], \quad (20)$$

where the integration is over the area of the nuclear target and k is the momentum of the wave packet. The phase shift function $\chi(B,b,\hat{B}\cdot\hat{b})$ is given by the integral over dZ of the sum of the light and heavy quark Coulomb potentials. One usually sees expressions in which the integral over b times appropriate wave functions is performed. Here we are dealing with a wave packet that is a coherent sum of physical states, so it is convenient to study $\hat{f}$. Straightforward manipulations lead to the result

$$\chi(B,b,\hat{B}\cdot\hat{b})=\frac{-i\alpha 4\pi}{v}\int\frac{d^2q_\perp}{(2\pi)^2}\frac{\tilde{\rho}(q_\perp)}{q_\perp^2}\,e^{i\vec{q}_\perp\cdot\vec{B}} \times\left(e^{i\frac{2}{3}\vec{q}_\perp\cdot\vec{b}}-e^{-i\frac{1}{3}\vec{q}_\perp\cdot\vec{b}}\right), \quad (21)$$

where v is the speed of the wave packet and $\tilde{\rho}(q_\perp)$ is the Fourier transform of the nucleonic charge density, $\rho(r)$:

$$\rho(r)=\psi_1^2\left(\frac{2}{3}r\right)-\psi_1^2\left(\frac{1}{3}r\right). \quad (22)$$

In particular

$$\tilde{\rho}(q_\perp)=\frac{1}{\left(1+\frac{q_\perp^2 r_B^2}{9}\right)^2}-\frac{1}{\left(1+\frac{q_\perp^2 r_B^2}{36}\right)^2}, \quad (23)$$

where the first (second) term the Fourier transform of the charge density of the light (heavy) quark. The neutrality condition is

$$\tilde{\rho}(q_\perp=0)=0, \quad (24)$$

which is vital in obtaining the result that $\chi(B,b,\hat{B}\cdot\hat{b})$ vanishes at $b=0$. This is because the integral over $d^2q_\perp$ is convergent only because of Eq. (24). Note also that $\tilde{\rho}(q_\perp)/q_\perp^2$ is proportional to the Fourier transform of the Coulomb potential.

We stress that expression (21) is obtained by summing coherently the Coulomb interactions of both the heavy and light quarks with target nucleons. This coherence, and the consequent "small interactions at small separations" is lost if one is computing the cross section for an inclusive process in which one sums over all final states of the ejected wave packet.

We first assess Eq. (20) by expanding the exponent in powers of χ. If the usual fine structure constant is used, the leading term will dominate. There is no term of 0th order in $\chi(B,b,\hat{B}\cdot\hat{b})$. The first-order term $\int d^2B\,\chi(B,b,\hat{B}\cdot\hat{b})$ vanishes for all values of b because the integration over d^2B sets $q_\perp$ to 0. However, $\int d^2B\,\chi^2(B,b,\hat{B}\cdot\hat{b})$ does not vanish. Keeping this term leads to the result

$$\hat{f}(\theta=0,b)=\frac{ik}{2\pi}\left(\frac{4\pi\alpha}{v}\right)^2\int\frac{d^2q_\perp}{2\pi}\frac{\tilde{\rho}^2(q_\perp)}{q_\perp^4}2\left[1-J_0(q_\perp b)\right]. \quad (25)$$

The zeroth-order cylindrical Bessel function has small argument limit

$$\lim_{x\to 0}J_0(x)=1-x^2/4,$$

so that one immediately finds

$$\hat{f}(\theta=0,b)\sim ib^2. \quad (26)$$

This term is purely imaginary, so that its influence is to damp exponentially scattering wave functions.

We next turn to a complete evaluation of Eq. (20). The integration can be simplified by replacing $\vec{B}$ by a shifted value $\vec{B}-\frac{1}{6}\vec{b}$ so that Eq. (21) becomes

$$\chi(B,b,\hat{B}\cdot\hat{b})=\frac{-i\alpha 4\pi}{v}\int\frac{d^2q_\perp}{(2\pi)^2}\frac{\tilde{\rho}(q_\perp)}{q_\perp^2}\,e^{i\vec{q}_\perp\cdot\vec{B}} \times\left(e^{i\frac{1}{2}\vec{q}_\perp\cdot\vec{b}}-e^{-i\frac{1}{2}\vec{q}_\perp\cdot\vec{b}}\right). \quad (27)$$

This expression can be obtained for any ratio of quark masses by using different shifts of $\vec{B}$. An examination of Eq. (27) shows that $\chi(B,b,\hat{B}\cdot\hat{b})$ is an odd function of $\hat{B}\cdot\hat{b}\equiv\cos(\phi)$. The real part of $\hat{f}(\theta=0,b)$ is proportional to the integral of $\sin[\chi(B,b,\hat{B}\cdot\hat{b})]$ over ϕ between 0 and 2π and therefore vanishes. Thus, we have a general theorem that the forward scattering amplitude for the scattering of two neutral systems (made of two particles) that interact via Coulomb forces is purely imaginary.

We now evaluate the imaginary part of $\hat{f}(\theta=0,b)$. A closed form expression for the function $\chi(B,b,\hat{B}\cdot\hat{b})$ can be obtained by performing the integral over $d^2q_\perp$ in Eq. (27). The result is

$$\chi(B,b,\hat{B}\cdot\hat{b})=2\alpha/v\{K_0(3x_2)-K_0(6x_2)+K_0(6x_1)-K_0(3x_1) +\tfrac{1}{2}[3x_2K_1(3x_2)-6x_2K_1(6x_2)+6x_1K_1(6x_1)-3x_1K_1(3x_1)]\}, \quad (28)$$

where $x_1=\mid\vec{B}+\frac{1}{2}\vec{b}\mid/r_B$, $x_2=\mid\vec{B}-\frac{1}{2}\vec{b}\mid/r_B$ and K_i are modified Bessel functions. This expression can be used to evaluate the imaginary part of $\hat{f}$. The results are shown in Fig. 1.

We see that one finds a b^2 behavior for small values of b, and in that regime the amplitude is proportional to α^2. This indicates that perturbation theory is valid at small b, even though the coupling constant α can be large.

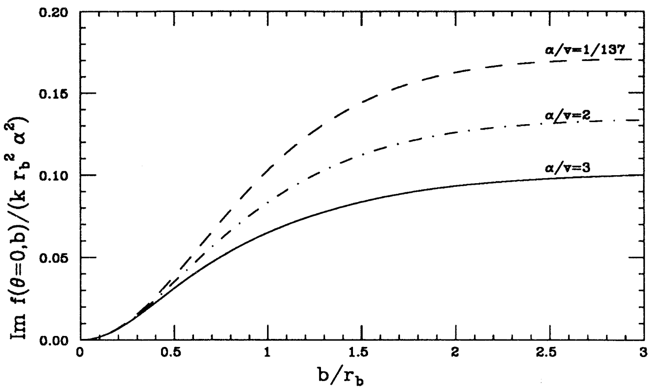

FIG. 1. Imaginary part of the forward scattering amplitude $\hat{f}$ for three different values of α/v.

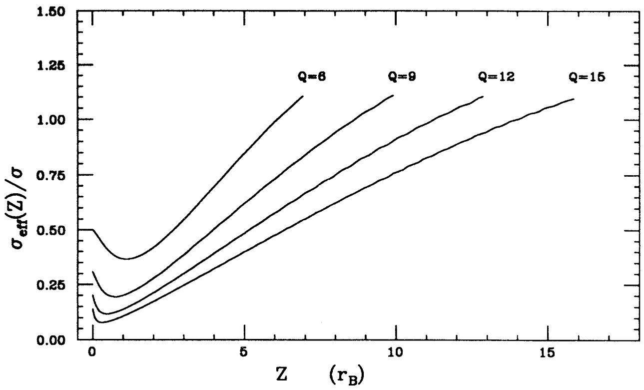

FIG. 2. PLC expansion for four different momenta Q. $\alpha = 2$.

Our purpose in this paper is to focus on the properties of wave packets of small transverse size. In particular, Eq. (15) shows that for large enough Q, the effective value of $b^2 \sim 1/Q^2$, which is small. Thus we use a simplified version of the interaction

$$\hat{\chi}(b) \equiv \hat{f}(\theta = 0) \approx \sigma b^2/\langle b^2\rangle, \tag{29}$$

where $\langle b^2\rangle$ represents the ground state expectation value of the operator b^2.

The calculations of the present section are a Coulomb version of the calculations of Refs. [16–18] which used two gluon exchange. Those references also found a b^2 behavior for projectiles of small transverse size. The origin is color neutrality, which we have modeled here as electrical neutrality. We have made nonperturbative calculations of all orders in α which we have taken as large as three.

V. CALCULATION

Expression (11) for $\sigma_{\text{eff}}(Z)$ is evaluated numerically. The matrix elements $\hat{b}_X^2$ and form factors $f_X(Q^2)$ are calculated for 600 discrete states and a $[0, 10]$ range of the continuous variable k. This is shown to be sufficient to reproduce the analytic result, Eq. (15), for momentum transfer Qr_B of up to 20. These calculations are performed for $P = Q$ which corresponds to the quasielastic kinematics of Ref. [2]. The results for the real part of $\sigma_{\text{eff}}(Z)$ for several values of the momentum transfer Q and $\alpha = 2$ are shown in Fig. 2. The nucleon expands more slowly (is large for a larger value of Z) for higher values of the momentum transfer. There is an initial drop, which is analyzed below and due primarily to the contribution of the states with $l = 2$. After the initial drop for a wide range of Z the effective cross section $\sim Z$, which is consistent with the "linear expansion" model of Refs. [14,15] and also the results of [12].

The Z dependence for a given value of $Qr_B = 6$ and three values of $\alpha = 1/137, 2, 3$, is presented in Fig. 3. The figure shows that the nucleon expansion distance grows with α. This is because P_X moves closer to P as α increases; recall Eqs. (16). This is discussed below in more detail.

To describe quantitatively the expansion let's introduce an expansion distance Z_{exp} defined as follows

$$\sigma_{\text{eff}}(Z_{\text{exp}}) = \sigma. \tag{30}$$

The expansion distance Z_{exp} is shown as a function of the momentum transfer Q in Fig. 4. We see that Z_{exp} is linear with Q; the value of α determines the slope.

We calculate $\sigma_{\text{eff}}(Z)$ for a reduced range of the continuous variable $k \in [0, k_{\max}]$ to investigate further the importance of the higher excited states. If $k_{\max}$ is such that $P(k_{\max}) = k_{\max} \sim \sqrt{Q}$ then the reduced $\sigma_{\text{eff}}(Z)$ is almost identical to the full one. This result is important to support the validity of the eikonal approximation. The $\sigma_{\text{eff}}(Z)$ for $k_{\max} = 1$ is shown in Fig. 5. Even though on the initial stage the expansion picture is different, for most of the range it reproduces the expansion of the "full" PLC well, and the expansion distance is nearly the same as for the full PLC.

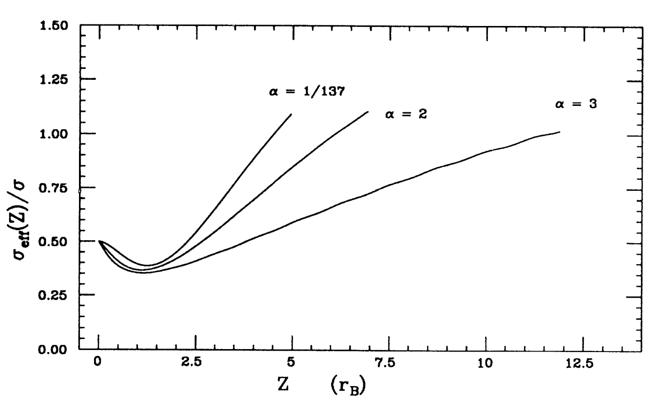

FIG. 3. PLC expansion for three values of α. $Q = 6$.

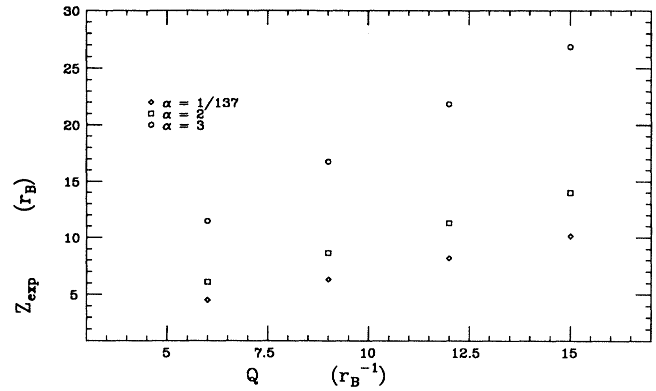

FIG. 4. Expansion distance Z_{exp} for three values of α.

VI. DISCUSSION

A conclusion can be made that a transparency phenomenon is obtained for the model under consideration. A small size object is initially formed in a high-momentum transfer process, and the expansion rate of this object is inversely proportional to the momentum transfer Q. While the first result has been known for this model for some time [7,8], the second result is a new consequence for this model. This QED-based model is therefore in contrast with the works [19] in which the rate of expansion was assumed to be slow. We shall take a closer look at what causes such a decrease (favorable for color transparency) in the expansion rate. As was mentioned above, PLC expansion depends on the momenta of the intermediate states P_X. For $\Delta M_X/\alpha \ll Q$ (16) can be expanded as

$$P_X = Q + \frac{\Delta M_X/\alpha}{2Q} + \cdots . \quad (31)$$

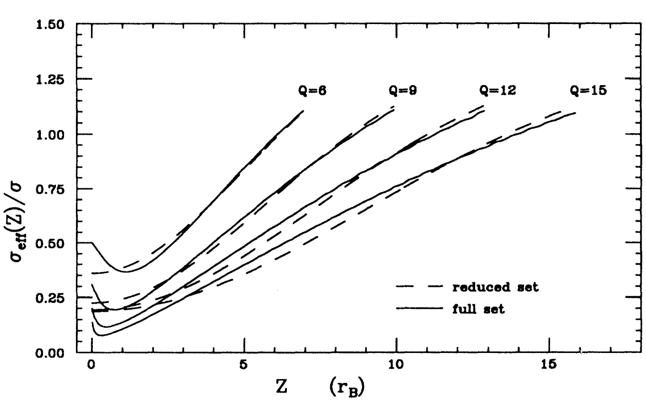

FIG. 5. $\sigma_{\text{eff}}(Z)$ for the "full" PLC and a "reduced" PLC. Full PLC is constructed out of 600 discrete states and [0,10] range of the continuous spectrum. Reduced PLC is constructed out of 600 discrete states and [0,1] range of the continuous spectrum. $\alpha = 2$, $Q = 6$.

Thus the values of ΔM_X, recall Eq. (16), that correspond to the intermediate states X which make important contributions to the sums in Eq. (11), determine the expansion rate. These states have a discrete or continuum nature. But the energies of the exited discrete states are bounded from above and their contributions to $\sigma_{\text{eff}}(Z)$; b_{nl}^2 and $f_{nl}(Q^2)$ decay rapidly and monotonically with the number n. Thus it is more relevant to examine the contributions of the continuum states, which in principle can have very high energies.

The form factors for the continuum $f_{kl}(Q^2)$ display a peaking behavior, which occurs when the momentum transfer Q matches the relative momentum denoted by the quantum number k. See Fig. 6 which shows also that the energy of the states produced in a high-momentum transfer process grows linearly with Q. If these states had appreciable matrix elements b_{kl}^2 for the soft interaction, the expansion would be very rapid. However, b_{kl}^2 decays rapidly with k (Fig. 7). The b_{k2}^2 and f_{k2} matrix elements exhibit similar behavior. As a result, $b_{kl}^2 f_{kl}(Q^2)$ have maxima that experience only a slight increase with Q (Fig. 8). So we see that in this model high excited states are formed for high-momentum transfer, but the soft interaction cuts them off.

The k dependence of $b_{kl}^2 f_{kl}$ determines the expansion of the PLC. We have seen in Figs. 3 and 5 that there is an initial drop for small values of Z. We argue that this drop is caused by the contribution of the states with $l = 2$. To see this we expand Eq. (11) for small Z:

$$\begin{aligned} \text{Re}[\sigma_{\text{eff}}(Z)] = \sigma &+ \sum_{l=0,2} \int_{k_{thr}}^{\infty} dk \frac{\chi_{kl} f_{kl}(Q^2)}{F(Q^2)} \\ &- Z \sum_{l=0,2} \int_{k_{thr}}^{\infty} dk \frac{\chi_{kl} f_{kl}(Q^2)}{F(Q^2)} \text{Im} P_k \\ &- Z^2 \sum_{l=0,2} \int_{0}^{\infty} dk \frac{\chi_{kl} f_{kl}(Q^2)}{F(Q^2)} \text{Re}(P_k - P)^2 . \end{aligned} \quad (32)$$

Here we omit, for simplicity, the contribution of the discrete states; k_{thr} is defined by $P_{k_{thr}} = 0$. The calculations show that $b_{k0}^2 f_{k0} < 0$, whereas $b_{k2}^2 f_{k2} > 0$. In the first integral in (32) the states with $l = 0$ dominate,

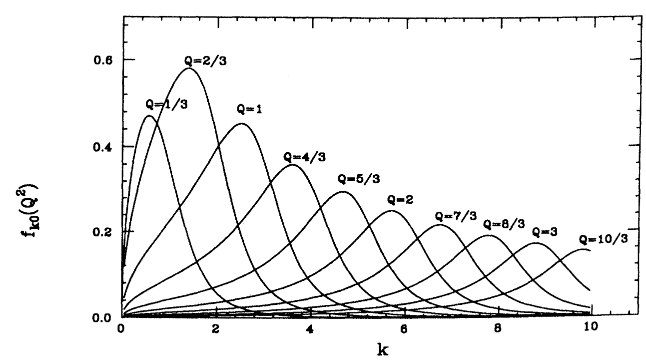

FIG. 6. Inelastic form factors $f_{k0}(Q^2)$ for ten values of Q.

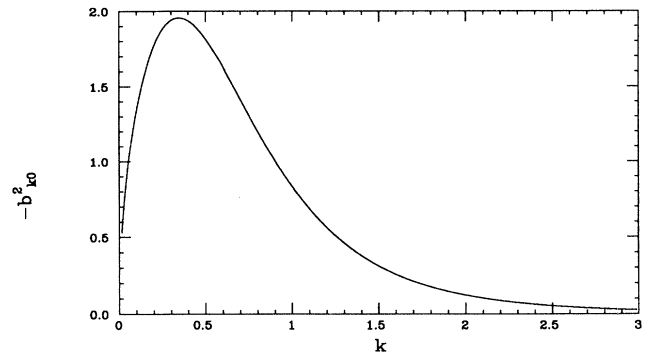

FIG. 7. b^2_{k0} decays rapidly with k.

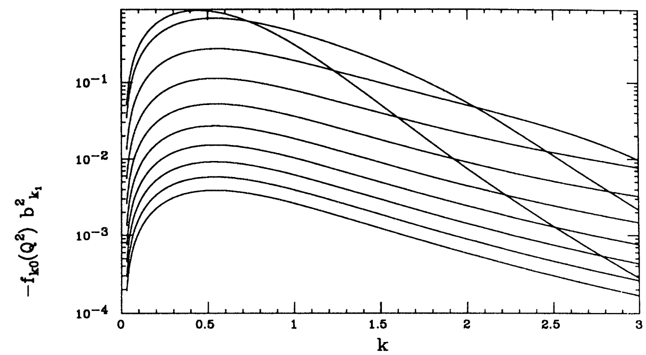

FIG. 8. $b^2_{k0} f_{k0}$ for the same ten values of Q as in Fig. 6.

which results in the small $\sigma_{\rm eff}(0)$. In the second and third integral the situation is reversed, although with the similar result: the dominating $l = 2$ states cause the initial shrinkage of the PLC.

Another piece of information is provided by the α dependence of the expansion distance. The latter decreases with the ΔM_X of the important states $|X\rangle$. For all the discrete states and for the continuum states with $k < 1$ ΔM_X decrease with α. Since $Z_{\rm exp}$ grows with α, it can be seen that only the states with $k < 1$ are important for the PLC expansion.

This analysis leads to the conclusion that the maximum energy of the states which are relevant for the PLC expansion grows much slower than the momentum transfer Q. This result is important in two ways. First, it illustrates that this indeed is the condition necessary for CT to occur. Second, our approach was based on the eikonal approximation, which assumed the momenta of the propagating states to be much greater than the size of the system. This approximation breaks down for the states with energy close to the momentum transfer Q. But since these states are not relevant and their contribution is negligibly small, the validity of the eikonal approximation is proven by the above conclusion.

VII. CONCLUSION

A nucleon model of a hadron has been investigated. In this model hadron consists of two quarks bound by the Coulomb potential with the variable strength. A small transverse size object is formed when nucleon absorbs a high-momentum photon. Such a system expands with the rate inversely proportional to the momentum transfer Q as in [14]. This slow expansion is a consequence of the fact that the states with energy much greater than the ground state energy are not very important.

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