

## $\Xi^-$ -hypernuclear states in heavy nuclei

S. Tadokoro, H. Kobayashi, and Y. Akaishi

*Institute for Nuclear Study, University of Tokyo, Tanashi, Tokyo 188, Japan*

(Received 6 September 1994)

The possibility of observing heavy  $\Xi$ -hypernuclei by  $(K^-, K^+)$  reactions is investigated. Several narrow peaks are clearly seen in the  $K^+$  spectrum of  $^{208}\text{Pb}(K^-, K^+)_{\Xi^-}\text{Hg}$  reaction calculated by the Green's function method. They correspond to  $\Xi^-$  bound states with spin-stretched high angular momentum. The excitation of the  $\Xi^-$  bound states amounts to about  $0.4 \mu\text{b/sr MeV}$  for the  $^{208}\text{Pb}$  target in the case of 2 MeV detector resolution, which is considerably larger than that for the  $^{12}\text{C}$  target. It is discussed whether or not the mass-number dependence of nucleus- $\Xi$  potential depths is observed between  $A = 12$  and 208.

PACS number(s): 25.80.Nv, 21.80.+a, 21.60.-n

### I. INTRODUCTION

Strangeness  $S = -2$  hypernuclei would give important information concerning the  $\Xi N$  and  $\Lambda\Lambda$  interactions. Evidence for the formation of double- $\Lambda$  hypernuclei is obtained from old emulsion experiments [1] and a recent experiment at KEK [2]. As for  $\Xi$  hypernuclei seven candidates from emulsion data are reviewed in a pioneering theoretical work by Dover and Gal [3]. Recently, there appear two events in the KEK emulsion experiment which may be attributed to the formation of  $\Xi$ -atoms and/or nuclei [4]. However, the supply of data on double-strangeness ( $S = -2$ ) hypernuclei is still limited. At BNL, 1–2 GeV/c highly-intense and well-separated  $K^-$  beam becomes available and the  $S = -2$  spectroscopy is now feasible by means of  $(K^-, K^+)$  reactions. We can expect to get experimental information on the nucleus- $\Xi$  interaction by populating  $\Xi$ -hypernuclear states. Thus, a key problem at present would be the possible existence of bound  $\Xi$ -hypernuclei.

We investigate  $\Xi^-$ -hypernuclear spectra from  $^{208}\text{Pb}(K^-, K^+)$  as well as  $^{12}\text{C}(K^-, K^+)$  by using the Green's function method [5]. In the case of  $^{12}\text{C}$  Ikeda *et al.* discussed in detail the formation of  $\Xi^-$  states and calculated the transition rate from  $\Xi^-$  states to possible double- $\Lambda$  states [6]. A remarkable feature they obtained is the narrowness of  $\Xi$ -state widths, which encourages the search for  $\Xi$  hypernuclei. Yamamoto has suggested that the depth of the nucleus- $\Xi$  potential derived from the Nijmegen potential [7] has no saturation property but changes from about  $-12$  MeV for  $A = 12$  to about  $-24$  MeV for  $A = 208$  [8]. This is due to weak exchange force of the meson-theoretical  $\Xi N$  potential. Thus, the mass-number dependence of nucleus- $\Xi$  potential depth is an interesting problem in relation to the nature of  $\Xi N$  interaction. In order to know whether the mass-number dependence really exists or not, the  $(K^-, K^+)$  experiment should be performed not only on light-nucleus target but also on heavy-nucleus target. Because core-excited states densely distribute in the heavy-target case, the reaction is required to have a selective population mechanism. We will show that the  $^{208}\text{Pb}(K^-, K^+)$  reaction with an inci-

dent momentum 1.65 GeV/c excites selectively  $\Xi^-$  bound states with spin-stretched high angular momentum. We propose an experimental search for some prominent  $\Xi^-$  peaks in  $^{208}\text{Pb}$ .

In Sec. II we explain the Green's function method. The nucleus- $\Xi$  potentials employed are discussed in Sec. III. The  $\Xi^-$ -hypernuclear spectrum from  $^{208}\text{Pb}(K^-, K^+)$  is given in Sec. IV and compared with that from  $^{12}\text{C}(K^-, K^+)$ . Summary and conclusions are given in Sec. V.

### II. GREEN'S FUNCTION METHOD

The double-differential cross section for the  $(K^-, K^+)$  reaction is given within the distorted-wave impulse approximation (DWIA) as

$$\frac{d^2\sigma}{dE_{K^+}d\Omega_{K^+}} = \beta \left[ \frac{d\sigma}{d\Omega_{K^+}} \right]^{(\text{el})} S(E), \quad (1)$$

where  $\beta$  is a kinematical factor,  $[d\sigma/d\Omega_{K^+}]^{(\text{el})}$  is the Fermi-averaged differential cross section for the elementary process  $K^- + p \rightarrow K^+ + \Xi^-$  and  $S(E)$  is the strength function of a hypernuclear system. The kinematical factor  $\beta$  is defined by

$$\beta = \left\{ 1 + \frac{E_{K^+}^{(2)}}{E_{\Xi}^{(2)}} \frac{p_{K^+}^{(2)} - p_{K^-} \cos \theta_{K^+}}{p_{K^+}^{(2)}} \right\} \frac{p_{K^+} E_{K^+}}{p_{K^+}^{(2)} E_{K^+}^{(2)}}, \quad (2)$$

where  $E_{K^+}^{(2)}$  ( $p_{K^+}^{(2)}$ ) is total energy (momentum) of  $K^+$  in the laboratory system determined by the free  $K^- N$  two-body kinematics at the incident momentum  $p_{K^-}$ . It is noted that  $\beta$  is not constant but proportional to  $p_{K^+} E_{K^+}$ . The strength function  $S(E)$  is obtained by the Green's function method [5],

$$S(E) = -\frac{1}{\pi} \text{Im} \sum_{\alpha\alpha'} \int d\mathbf{r} d\mathbf{r}' f_{\alpha}(\mathbf{r}) G_{\alpha\alpha'}(E; \mathbf{r}, \mathbf{r}') f_{\alpha}(\mathbf{r}'), \quad (3)$$

$$f_{\alpha}(\mathbf{r}) = \chi^{(-)*}(\mathbf{p}_{K^+}, M_C/M_{\text{HYR}}) \chi^{(+)}(\mathbf{p}_{K^-}, M_C/M_{\text{HYR}}) \times \langle \alpha | \psi_N(\mathbf{r}) | i \rangle, \quad (4)$$

where kets  $|i\rangle$  and  $|\alpha\rangle$  are states of the target and of a core nucleus, respectively and  $\psi_{N(Y)}$  is the annihilation operator of  $N(Y)$ . Recoil effects are taken into account through the factor  $M_C/M_{HY}$  in Eq. (4) [9]. Here  $G_{\alpha\alpha'}(E; \mathbf{r}, \mathbf{r}')$  is Green's function for the hyperon interacting with the core nucleus.

$$G_{\alpha\alpha'}(E; \mathbf{r}, \mathbf{r}') = \langle \alpha | \psi_Y(\mathbf{r}) \frac{1}{E - H + i\eta/2} \psi_Y^\dagger(\mathbf{r}') | \alpha' \rangle, \quad (5)$$

with

$$H = T_{\text{nucl}-\Xi} + U_{\text{nucl}-\Xi}. \quad (6)$$

If  $U_{\text{nucl}-\Xi}$  is taken to be a single-particle optical potential, Green's function becomes diagonal with respect to core-nucleus states  $|\alpha\rangle$ . We can take into account spreading widths of proton-hole states (nuclear core states) and the energy resolution of a detector by putting a finite value into  $\eta$  in Eq. (5).

The distortion of meson waves is calculated in the eikonal approximation as

$$\chi^{(+)}(\mathbf{p}_{K^-}, \mathbf{r}) = \exp \left[ i\mathbf{p}_{K^-} \cdot \mathbf{r} - iv_{K^-}^{-1} \int_{-\infty}^{z^-} U_{K^-}(\mathbf{b}_-, z') dz' \right], \quad (7)$$

$$\chi^{(-)*}(\mathbf{p}_{K^+}, \mathbf{r}) = \exp \left[ -i\mathbf{p}_{K^+} \cdot \mathbf{r} - iv_{K^+}^{-1} \int_{z_+}^{\infty} U_{K^+}(\mathbf{b}_+, z') dz' \right], \quad (8)$$

$$z_- = \hat{\mathbf{p}}_{K^-} \cdot \mathbf{r}, \quad z_+ = \hat{\mathbf{p}}_{K^+} \cdot \mathbf{r}, \quad (9)$$

where  $v_{K^\pm}$  is the velocity of the meson and  $\mathbf{b}_-$  ( $\mathbf{b}_+$ ) is the component of  $\mathbf{r}$  perpendicular to  $\mathbf{p}_{K^-}$  ( $\mathbf{p}_{K^+}$ ). The meson-nucleus optical potential  $U_m$  ( $m$  stands for  $K^-$  or  $K^+$ ) is given by

$$U_m(\mathbf{r}) = -i \frac{v_m}{2} \bar{\sigma}_{mN} \rho(\mathbf{r}), \quad (10)$$

$$\bar{\sigma}_{mN} = \sigma_{mN}^{\text{tot}} (1 - i\bar{\alpha}_{mN}), \quad \bar{\alpha}_{mN} = \text{Re}\bar{f}(0)/\text{Im}\bar{f}(0), \quad (11)$$

where  $\sigma_{mN}^{\text{tot}}$  is the isospin-averaged total meson-nucleon cross section and  $\bar{f}(0)$  is the isospin-averaged forward elementary amplitude. We neglect the effect of  $\bar{\alpha}_{mN}$  because the differential cross section is rather insensitive to it [10], and use the following meson-nucleon total cross sections [11]:

$$\begin{aligned} \sigma_{K^-p}^{\text{tot}} &= 32.5 \text{ mb}, & \sigma_{K^-n}^{\text{tot}} &= 25.5 \text{ mb}, \\ \sigma_{K^+p}^{\text{tot}} &= 19.6 \text{ mb}, & \sigma_{K^+n}^{\text{tot}} &= 20.1 \text{ mb}, \end{aligned} \quad (12)$$

which correspond to the case of an incident momentum  $p_{K^-} = 1.65 \text{ GeV}/c$ .

Straightforward angular momentum algebra gives partial-wave decomposition of  $S(E)$ :

$$\begin{aligned} S(E) &= \sum_{JM} \sum_{l_Y, j_Y} \sum_{n_N, l_N, j_N} \sum_{l'_Y, j'_Y} \sum_{n'_N, l'_N, j'_N} \sqrt{(2j_N + 1)(2j'_N + 1)} \\ &\quad \times (j_N \frac{1}{2} J 0 | j_Y \frac{1}{2} \frac{1}{2}) \frac{1}{2} (1 + (-1)^{l_N + l_Y + J}) (j'_N \frac{1}{2} J 0 | j'_Y \frac{1}{2} \frac{1}{2}) \frac{1}{2} (1 + (-1)^{l'_N + l'_Y + J}) \\ &\quad \times S_{(l_Y j_Y, n_N l_N j_N), (l'_Y j'_Y, n'_N l'_N j'_N)}^{JM}(E), \end{aligned} \quad (13)$$

$$\begin{aligned} S_{(l_Y j_Y, n_N l_N j_N), (l'_Y j'_Y, n'_N l'_N j'_N)}^{JM}(E) &= -\frac{1}{\pi} \text{Im} \int dr \int dr' r^2 r'^2 \tilde{j}_{JM}(p_{K^-}, p_{K^+}, \theta_{K^+}; r) \phi_{n_N l_N j_N}(r) \\ &\quad \times G_{(l_Y j_Y, n_N l_N j_N), (l'_Y j'_Y, n'_N l'_N j'_N)}^J(E; r, r') \tilde{j}_{JM}(p_{K^-}, p_{K^+}, \theta_{K^+}; r') \phi_{n'_N l'_N j'_N}(r'), \end{aligned} \quad (14)$$

where the function  $\tilde{j}_{JM}(p_{K^-}, p_{K^+}, \theta_{K^+}; r)$  is defined by

$$\chi^{(-)*}(\mathbf{p}_{K^+}, M_C/M_{HY}\mathbf{r}) \chi^{(+)}(\mathbf{p}_{K^-}, M_C/M_{HY}\mathbf{r}) = \sum_{J=0}^{\infty} \sum_{M=-J}^{M=J} \tilde{j}_{JM}(p_{K^-}, p_{K^+}, \theta_{K^+}; r) Y_J^M(\hat{\mathbf{r}}), \quad (15)$$

$\phi_{n_N l_N j_N}(r)$  is a radial-part wave function of the proton which absorbs the incident meson and  $J$  is the total spin of the hypernucleus. In the case of forward  $K^+$  production  $\theta_{K^+} = 0^\circ$ , Eq. (13) and Eq. (14) are further reduced to

$$S(E) = \sum_J \sum_{l_Y, j_Y} \sum_{n_N, l_N, j_N} \sum_{l'_Y, j'_Y} \sum_{n'_N, l'_N, j'_N} [W] S_{(l_Y j_Y, n_N l_N j_N), (l'_Y j'_Y, n'_N l'_N j'_N)}^J(E), \quad (16)$$

$$[W] = (2J + 1) \sqrt{(2j_N + 1)(2j'_N + 1)} (j_N \frac{1}{2} J 0 | j_Y \frac{1}{2} \frac{1}{2}) \frac{1}{2} (1 + (-1)^{l_N + l_Y + J}) (j'_N \frac{1}{2} J 0 | j'_Y \frac{1}{2} \frac{1}{2}) \frac{1}{2} (1 + (-1)^{l'_N + l'_Y + J}), \quad (17)$$

$$\begin{aligned} S_{(l_Y j_Y, n_N l_N j_N), (l'_Y j'_Y, n'_N l'_N j'_N)}^J(E) &= -\frac{1}{\pi} \text{Im} \int dr \int dr' r^2 r'^2 \tilde{j}_J(p_{K^-}, p_{K^+}; r) \phi_{n_N l_N j_N}(r) \\ &\quad \times G_{(l_Y j_Y, n_N l_N j_N), (l'_Y j'_Y, n'_N l'_N j'_N)}^J(E; r, r') \tilde{j}_J(p_{K^-}, p_{K^+}; r') \phi_{n'_N l'_N j'_N}(r'), \end{aligned} \quad (18)$$

where we use the relation

$$\tilde{j}_{JM}(p_{K^-}, p_{K^+}, \theta_{K^+} = 0^\circ; r) = i^J \sqrt{4\pi(2J + 1)} \tilde{j}_J(p_{K^-}, p_{K^+}; r) \delta_{M0}. \quad (19)$$

We neglect couplings between  $(n_N l_N j_N)$  and  $(n'_N l'_N j'_N) [\neq (n_N l_N j_N)]$ , and between  $(l_Y j_Y)$  and  $(l'_Y j'_Y) [\neq (l_Y j_Y)]$  by adopting single-particle potentials.

The differential cross section to a definite hypernuclear bound state is obtained by integrating Eq. (1) on small energy interval suitably chosen, and is expressed as

$$\frac{d\sigma}{d\Omega_{K^+}} = \alpha \left[ \frac{d\sigma}{d\Omega_{K^+}} \right]^{(el)} Z^{\text{eff}}, \quad (20)$$

$$\alpha = \left( \frac{p_{K^+}}{p_{K^+}^{(2)}} \right)^2 \frac{p_{K^+}^{(2)}/E_{K^+}^{(2)} + (p_{K^+} - p_{K^-} \cos \theta_{K^+})/E_{\Xi}^{(2)}}{p_{K^+}/E_{K^+} + (p_{K^+} - p_{K^-} \cos \theta_{K^+})/E_{\text{HY}}}, \quad (21)$$

where  $Z^{\text{eff}}$  is the effective proton number of the exclusive reaction. One should avoid confusions between  $\alpha$  and  $\beta$ , and between  $Z^{\text{eff}}$  and  $S(E)$ .

### III. NUCLEUS- $\Xi$ POTENTIAL

Dover and Gal [3] made theoretical analyses of old emulsion data attributed to  $(K^-, K^+)$   $\Xi$ -hypernuclear productions [12] and extracted nucleus- $\Xi$  potentials of Woods-Saxon type;

$$U_{\text{nucl}-\Xi} = \frac{V_0 + iW_0}{1 + \exp\{(r - R)/a\}} \quad (22)$$

with  $V_0 = -24 \pm 4$  MeV (real-part strength),  $R = 1.1A^{1/3}$  fm and  $a = 0.65$  fm.

We assume the nucleus- $\Xi^-$  potential of Woods-Saxon form with the same values of range and diffuseness parameters as the above. The imaginary-part strength is taken to be  $W_0 = -1$  MeV. This imaginary part, together with the real part  $V_0 = -24$  MeV, gives level widths of 1.2 MeV for  $1s$  state and of 0.5 MeV for  $1p$  state in the  $\Xi^-$ - $^{11}\text{B}$  system. These widths are in good agreement with the  $\Xi^- p \rightarrow \Lambda\Lambda$  conversion widths obtained from the Nijmegen model- $D$  potential [7] by Ikeda *et al.* [6]. We examine also a stronger conversion case of  $W_0 = -3$  MeV to know how results are sensitive to the imaginary potential.

The real-part strength  $V_0$  is assumed to be  $-24$  MeV for  $^{208}\text{Pb}$ . We, however, employ  $V_0 = -16$  MeV in addition to  $-24$  MeV for  $^{12}\text{C}$ . Reasons why we investigate the shallow potential case are the followings. Yamamoto has suggested that the depth of the nucleus- $\Xi$  potential derived from the Nijmegen interaction has no saturation property but depends on the mass number of the core nucleus [8]. This property is attributed to the fact that no single meson carries double strangeness and therefore the exchange force is weak between  $\Xi$  and N. Aoki *et al.* observed the twin  $\Lambda$ -hyperfragment production via  $\Xi^-$  capture by the emulsion-counter hybrid experiment at KEK [4]. The energy of the stopped  $\Xi^-$  on  $^{12}\text{C}$  is estimated to be  $-0.54 \pm 0.20$  MeV, which deviates from the finite-size Coulomb levels,  $-0.97$  MeV for  $1s$ ,  $-0.28$  MeV for  $2p$  and  $-0.26$  MeV for  $2s$ . The  $V_0 = -16$  MeV po-

tential, when it is added to the Coulomb potential, shifts down the  $2p$  state to  $-0.56$  MeV, giving a plausible (but not unique) explanation for the stopped  $\Xi^-$  energy.

### IV. $\Xi^-$ -HYPERNUCLEAR SPECTRA FROM $(K^-, K^+)$ REACTIONS

Spectra for  $^{12}\text{C}(K^-, K^+)$  and  $^{208}\text{Pb}(K^-, K^+)$  reactions are calculated at forward-angle  $\theta_{K^+} = 0^\circ$  with an incident momentum  $p_{K^-} = 1.65$  GeV/ $c$ . We adopt  $[d\sigma/d\Omega_{K^+}]^{(el)} = 35$   $\mu\text{b}/\text{sr}$  [13]. In the calculation two proton-hole states  $1p_{3/2}^{-1}$  and  $1s_{1/2}^{-1}$  are included for the  $^{12}\text{C}$  case and six proton-hole states  $3s_{1/2}^{-1}$ ,  $2d_{3/2}^{-1}$ ,  $1h_{11/2}^{-1}$ ,  $2d_{5/2}^{-1}$ ,  $1g_{7/2}^{-1}$  and  $1g_{9/2}^{-1}$  for the  $^{208}\text{Pb}$  case. Proton-hole wave functions are calculated by using the Woods-Saxon potential with geometrical parameters of  $r_0 = 1.27$  fm and  $a = 0.67$  fm. We employ strength parameters of  $V_0 = -61$  MeV,  $V_{LS} = 22$  MeV. Hypernuclear spectra do not strongly depend on the detail of proton-hole wave functions, especially in the  $\Xi^-$  bound state region. Spreading widths of the proton-hole states are taken from  $^{12}\text{C}(p, 2p)$  [14] and  $^{208}\text{Pb}(e, e'p)$  [15] experiments. We assume  $\Delta E = 2$  MeV as the energy resolution of a spectrometer. Results are shown in Figs. 1 and 2. The quasifree production of  $\Xi^-$  gets much stronger than the bound-state population, because of large momentum transfer  $0.5$  GeV/ $c$  which is about two times as large as the Fermi momentum in a nucleus. It is noted that all the deep-hole states sizably contribute to the continuum spectrum.

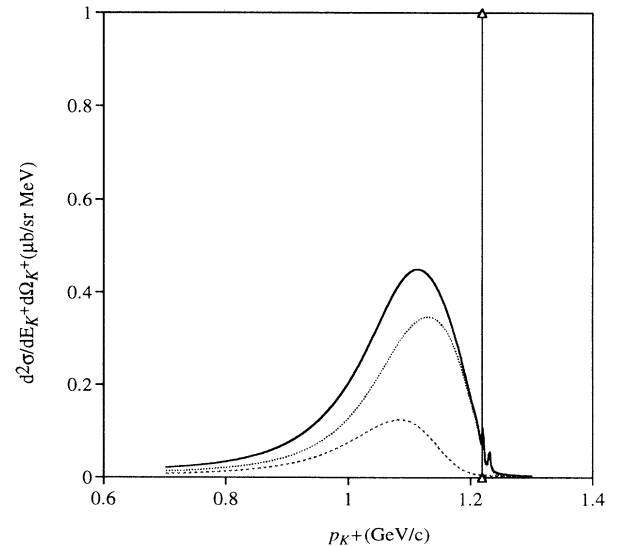


FIG. 1. The  $\Xi^-$ -hypernuclear spectrum from  $^{12}\text{C}(K^-, K^+)$  for  $p_{K^-} = 1.65$  GeV/ $c$  and  $\theta_{K^+} = 0^\circ$ . The threshold of  $\Xi^-$  and the core nucleus (ground state) is denoted by the vertical line. The full curve represents the total double-differential cross section. The dotted and dashed curves are for the contributions from proton hole states  $1p_{3/2}^{-1}$  and  $1s_{1/2}^{-1}$ , respectively.

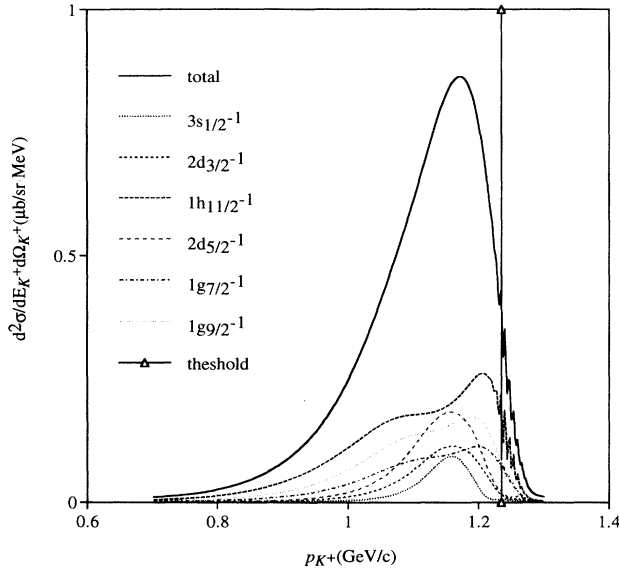


FIG. 2. The same spectrum from  $^{208}\text{Pb}(K^-, K^+)$  for  $p_{K^-} = 1.65$  GeV/c and  $\theta_{K^+} = 0^\circ$ . See also the caption of Fig. 1.

In Figs. 3 and 4 the calculations are compared with experimental data obtained at KEK by Iijima *et al.* [13]. The experimental data are averaged between  $\theta_{K^+} = 1.7^\circ$  and  $13.6^\circ$ . However, we show the result at  $\theta_{K^+} = 0^\circ$  because one at  $13.6^\circ$  is almost same. The magnitudes of the cross sections are fairly reproduced by our calculation in the  $\Xi^-$  quasifree production region. Since energies of  $\Xi^-$  are very high in the quasifree region, for example about 100 MeV at the quasifree peak,  $\Xi^-$  hardly feels the influ-

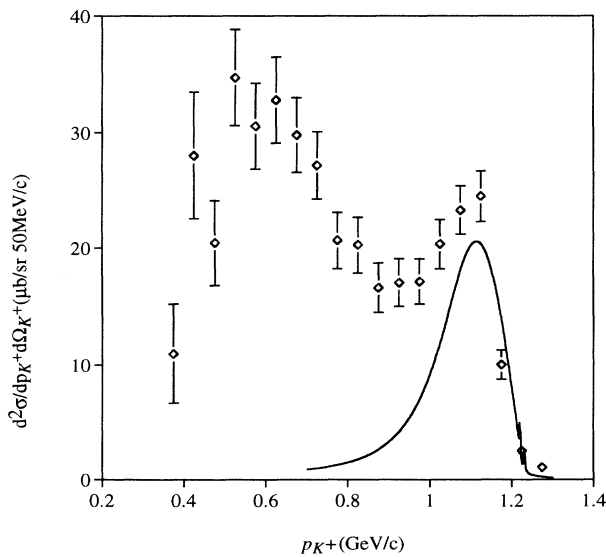


FIG. 3. Comparison between our calculated spectrum ( $\theta_{K^+} = 0^\circ$ ) for  $^{12}\text{C}$  target and the KEK experimental data taken from Ref. [13]. The experimental spectrum is averaged between  $\theta_{K^+} = 1.7^\circ$  and  $13.6^\circ$ .

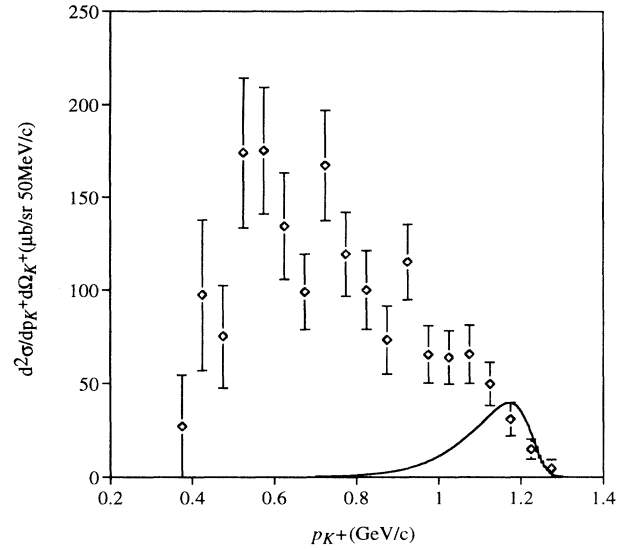


FIG. 4. The same as Fig. 3 but for the  $^{208}\text{Pb}$  target case.

ence of the nucleus- $\Xi^-$  potential. Therefore, even if we change parameters of nucleus- $\Xi^-$  potential, the spectra in the quasifree region are almost unchanged.

#### A. $^{12}\text{C}(K^-, K^+)^{12}_{\Xi^-}\text{Be}$

Let us discuss in detail the spectra in the bound-state region of  $\Xi^-$ . Figure 5 shows the spectrum for

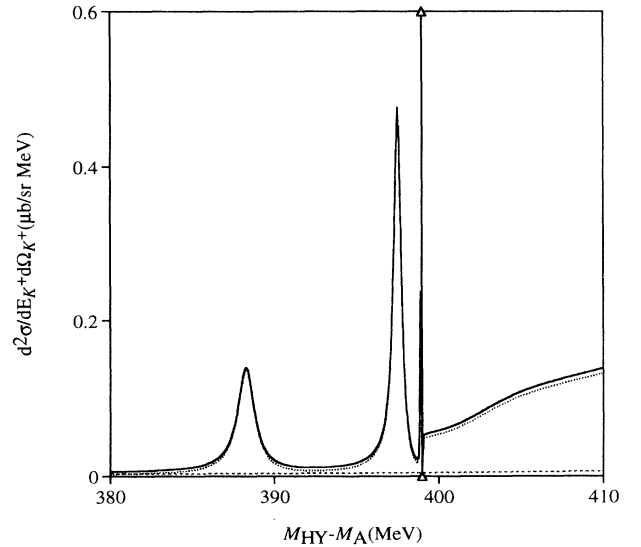


FIG. 5. The hypernuclear spectrum for  $^{12}\text{C}$  target in the bound-state region as a function of  $M_{\text{HY}} - M_A$ , where  $M_A$  and  $M_{\text{HY}}$  are target and hypernuclear masses, respectively. The spectrum is calculated with  $V_0 = -24$  MeV and  $W_0 = -1$  MeV. Energy resolution is taken to be 0 MeV.

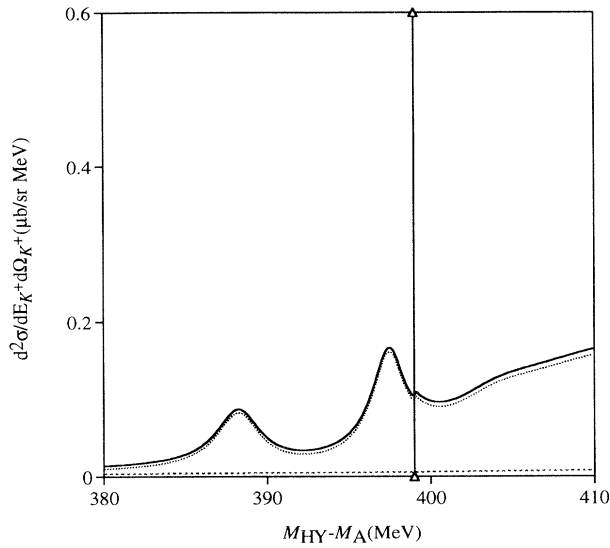


FIG. 6. The same as Fig. 5 but it is smeared with  $\Delta E = 2$  MeV.

the  $^{12}\text{C}$  target calculated with  $V_0 = -24$  MeV and  $W_0 = -1$  MeV, where the energy resolution is taken to be zero in order to compare it with the previous result by Ikeda *et al.* [6]. Two separated peaks appear below the  $\Xi^- + ^{11}\text{B}$  threshold at 399 MeV. The lower and upper peaks correspond to the excitation of hypernuclear states  $[p1p_{3/2}^{-1} \otimes \Xi s_{1/2}]^{J=1}$  and  $[p1p_{3/2}^{-1} \otimes \Xi p]^{J=0,2}$ , of which decay widths are 1.2 MeV and 0.5 MeV, respectively. The widths are in good agreement with what Ikeda *et al.* obtained from the Nijmegen model-*D* potential. In the quasifree production region, however, our spectrum

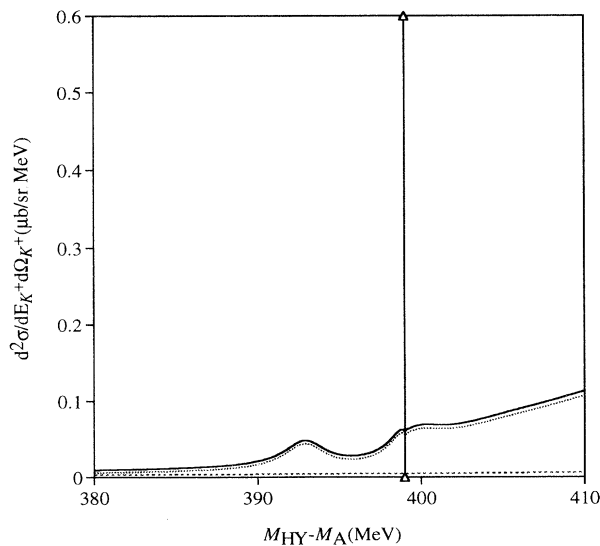


FIG. 7. The same as Fig. 6 but it is calculated with  $V_0 = -16$  MeV.

is smooth, while their spectrum is rugged even though there is no resonance. This indicates that the Green's function method is easier to get a good description of the continuum spectrum than the Kapur-Peierls method used by them. We introduce into the spectrum the effect of energy resolution of a spectrometer. The result with 2 MeV smearing is shown in Fig. 6. The heights of two peaks at  $M_{HY} - M_A \sim 387$  MeV and 398 MeV become about 0.09 and 0.17  $\mu\text{b}/\text{sr MeV}$ , respectively. The effective proton numbers are estimated to be 0.012 and 0.017.

There is a possibility that the depth of the nucleus- $\Xi$  potential depends on the mass number of core nucleus, as stated in Secs. I and III. Figure 7 shows the spectrum when the shallow potential of  $V_0 = -16$  MeV is used. In this case there remains only one peak which corresponds to the  $[p1p_{3/2}^{-1} \otimes \Xi s_{1/2}]^{J=1}$  configuration.

### B. $^{208}\text{Pb}(K^-, K^+) \Xi^- ^{208}\text{Hg}$

Figures 8(a) and 8(b) show the spectra for the  $^{208}\text{Pb}$  target calculated with  $V_0 = -24$  MeV and  $W_0 = -1$  MeV in the cases of energy resolution  $\Delta E = 0$  MeV and of  $\Delta E = 2$  MeV, respectively. In the spectra we can see a series of  $\Xi^-$  bound-state peaks at  $M_{HY} - M_A \sim 358, 362, 368, 374, 381,$  and  $387$  MeV below the  $\Xi^- + ^{207}\text{Tl}$  threshold at 391 MeV. The prominent ones at 374, 381, and 387 MeV correspond to the excitation of spin-stretched states  $[p1h_{11/2}^{-1} \otimes \Xi h_{9/2}]^{J=10}$ ,  $[p1h_{11/2}^{-1} \otimes \Xi i_{11/2}]^{J=11}$  and  $[p1h_{11/2}^{-1} \otimes \Xi j_{13/2}]^{J=12}$ , respectively, as explained later. The selective excitation of spin-stretched states in the high momentum transfer reaction was pointed out many years ago by Dover *et al.* [10]. Hole states with small angular momentum such as  $3s_{1/2}^{-1}$ ,  $2d_{3/2}^{-1}$ ,  $2d_{5/2}^{-1}$  are hardly excited in the  $\Xi^-$  bound-state region by the  $(K^-, K^+)$  reaction with large momentum transfer, though they give sizable contributions to the spectrum in the  $\Xi^-$  quasifree production region. Furthermore the second largest- $l$  hole states,  $1g_{9/2}^{-1}$  and  $1g_{7/2}^{-1}$ , show no sharp-peak structures because of their broad spreading widths [15]. Thus, the spectrum of  $^{208}\text{Pb}(K^-, K^+) \Xi^-$  has the structure determined dominantly by the excitation of  $p1h_{11/2}^{-1} \otimes \Xi^-$  hypernuclear states.

Let us discuss the structure of the prominent peaks in detail. Figure 9 shows the excitation spectra of the hypernuclear states with the configuration  $p1h_{11/2}^{-1} \otimes \Xi^-$ . A selection rule

$$l_p + l_\Xi + J = \text{even} \quad (23)$$

follows from the factor  $[1 + (-1)^{l_N + l_Y + J}]/2$  in Eq. (17) which comes from the matrix element  $\langle (l_Y 1/2) j_Y || \mathbf{Y}^J || (l_N 1/2) j_N \rangle$ . For example, although  $\Xi h_{9/2}$ ,  $\Xi h_{11/2}$  and  $\Xi j_{13/2}$  can contribute to the  $J=10$  hypernuclear state, the first one dominates the others by forming a spin-stretched state together with  $p1h_{11/2}^{-1}$ . Similarly,  $\Xi j_{13/2}$  and  $\Xi j_{15/2}$  are relevant in the case of the  $J=12$  state, and the former gives the main component again forming a spin-stretched state. Figure 10 compares

contributions from  $ls$  splitting partners. One of them is so weakly excited due to angular-momentum coupling coefficients of Eq. (17) that the  $ls$  splitting of  $\Xi^-$  cannot be observed but, fortunately, does not destroy the peak structures in Fig. 8. Thus, it is known that the  $J=12$  peak almost purely consists of the  $[p1h_{11/2}^{-1} \otimes \Xi j_{13/2}]^{J=12}$  state.

It is noted that  $\Xi^-$  single-particle bound states exist at least up to  $l_{\Xi}=7$  with the assistance of the Coulomb attraction between  $\Xi^-$  and  $^{207}\text{Tl}$ . Figure 11(a) shows  $\Xi^-$  density distributions together with the net potential which is a sum of strong, Coulomb and centrifugal poten-

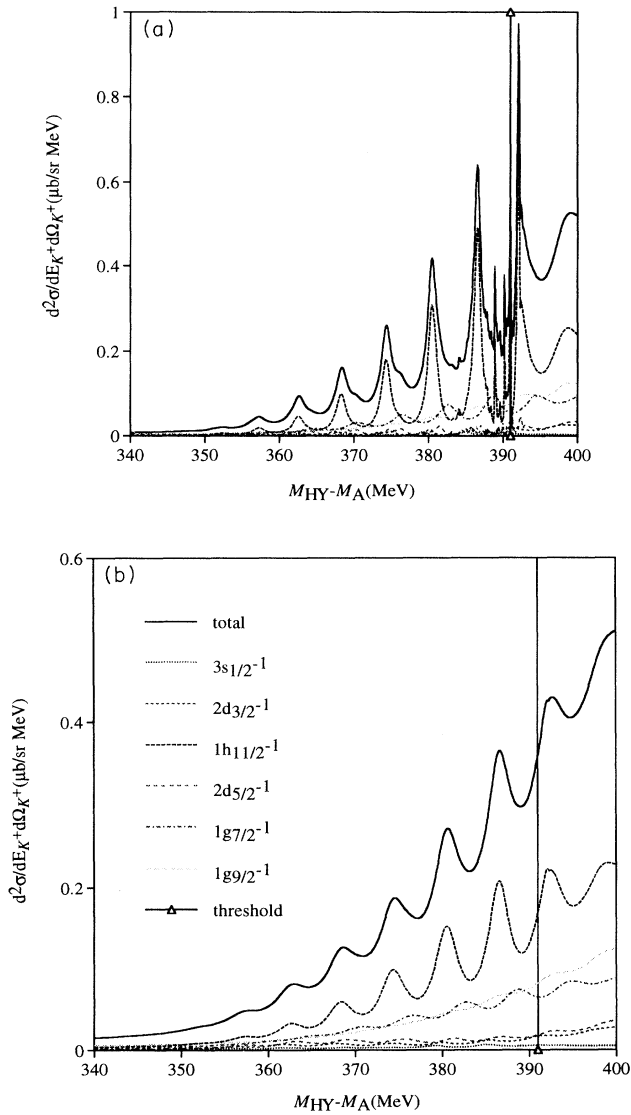


FIG. 8. The spectrum from  $^{208}\text{Pb}(K^-, K^+)$  for  $p_{K^-} = 1.65$  GeV/c and  $\theta_{K^+} = 0^\circ$  calculated with  $V_0 = -24$  MeV and  $W_0 = -1$  MeV. (a) The case of resolution  $\Delta E = 0$  MeV. (b) The case of  $\Delta E = 2$  MeV.

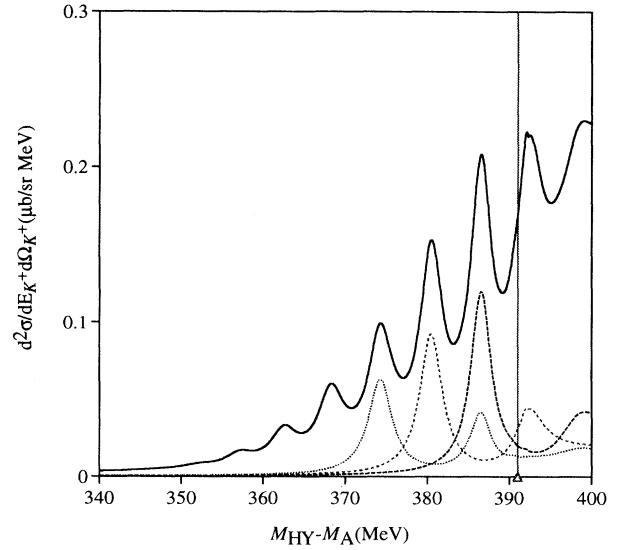


FIG. 9. The total excitation spectrum of hypernuclear states with the  $p1h_{11/2}^{-1} \otimes \Xi^-$  configuration (solid curve), and the contributions from spin-stretched hypernuclear states  $[p1h_{11/2}^{-1} \otimes \Xi h_{9/2}]^{J=10}$  (dotted curve),  $[p1h_{11/2}^{-1} \otimes \Xi i_{11/2}]^{J=11}$  (dashed curve) and  $[p1h_{11/2}^{-1} \otimes \Xi j_{13/2}]^{J=12}$  (long-dashed curve).

tials. The dotted line is for the  $l_{\Xi}=7$  state with energy  $-5.8$  MeV and the dot-dashed line is for a  $l_{\Xi}=0$  state with energy  $-5.4$  MeV. We can see that the  $l_{\Xi}=7$  state is well confined in the nuclear surface region by the centrifugal potential and is essentially different from the  $l_{\Xi}=0$  Coulomb atomic state. As a result, the final  $\Xi j_{13/2}$  state has a remarkably good overlap with the initial  $p1h_{11/2}$

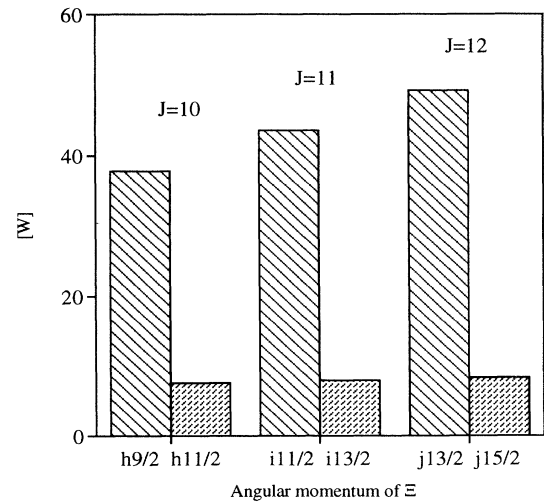


FIG. 10. Weight factors of  $ls$  splitting partners for the same  $J$ .

state shown in Fig. 11(b). Furthermore partial waves  $j_{J=12}(p_{K^-}, p_{K^+}; r)$  in Eq. (18) is peaked at  $r \sim 6$  fm due to the large momentum transfer. Therefore the  $J=12$  peak gets prominent. Thus the centrifugal force plays an essential role in strongly exciting the spin-stretched high- $J$  states. It was discussed by Bandō and Motoba that pairs with nodeless single-particle wave functions acquire a particularly strong population in the case of the  $(\pi^+, K^+)$  reaction [16]. The present result provides another typical example of their statement with a physical explanation.

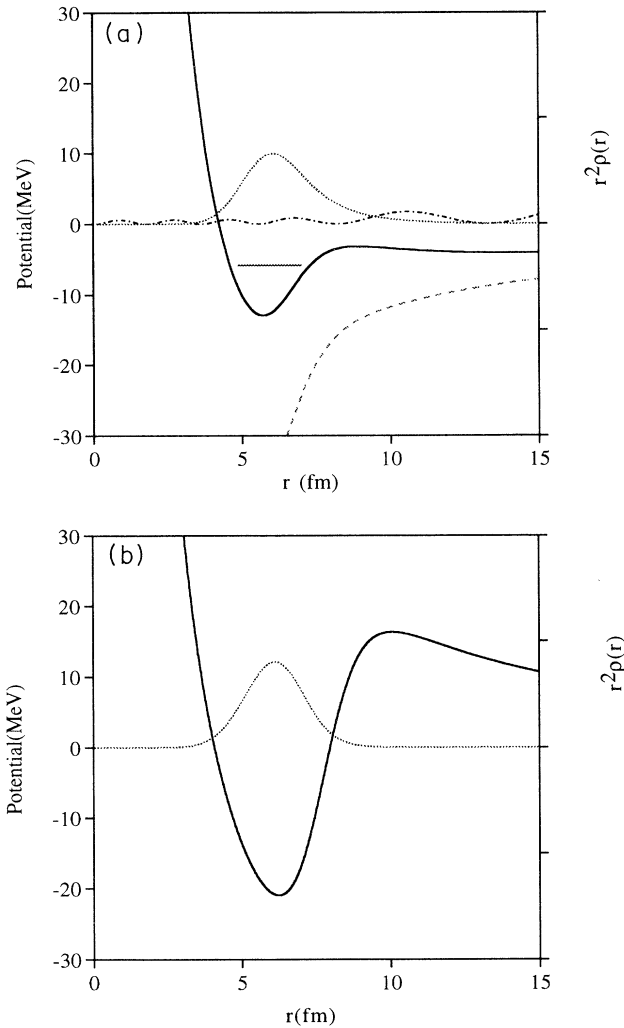


FIG. 11. Probability distributions and potentials for  $\Xi^-$  and  $p$ . (a) The  $\Xi^-$  case: The solid curve is for the net potential of strong, Coulomb and centrifugal ones with  $l_{\Xi}=7$ . The dashed curve is for the potential without the centrifugal one. The dotted and dot-dashed curves are for density distributions of the  $\Xi 1j$  state with energy  $-5.8$  MeV and of the  $\Xi 6s$  state with energy  $-5.4$  MeV, respectively. (b) The proton case: The solid curve is for the net potential with  $l=5$ . The dotted curve is for the density distribution of the  $p 1h_{11/2}$  state.

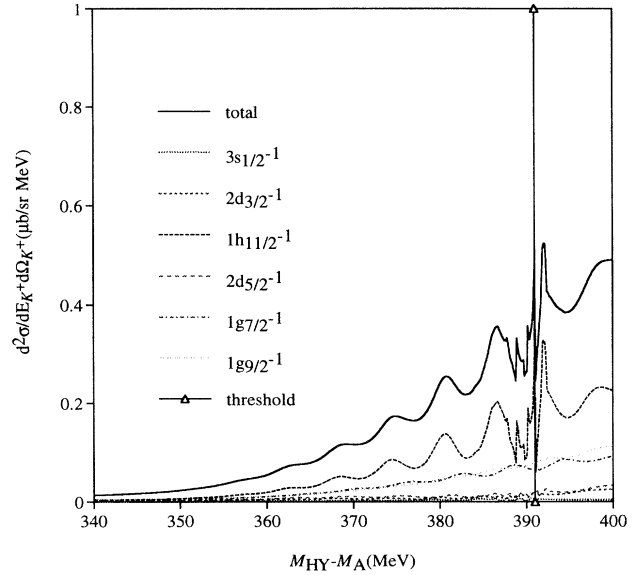


FIG. 12. The same spectrum as Fig. 8(a) but calculated with  $W_0 = -3$  MeV and  $V_0 = -24$  MeV. Energy resolution is  $\Delta E = 0$  MeV.

We examine the effect of the imaginary strength of the nucleus- $\Xi^-$  potential. Figure 12 shows the spectrum which is calculated with  $W_0 = -3$  MeV and  $V_0 = -24$  MeV in the case of zero resolution. The peaks at  $M_{HY} - M_A \sim 374, 381,$  and  $387$  MeV, which are clearly seen in Fig. 8, become broad but still remain. Many sharp peaks around the  $\Xi^-$  threshold are due to the excitation of  $\Xi^-$  atomic states. If a very high resolution spectrometer would become available, several hypernuclear peaks can be observed even in the strong imaginary case of  $W_0 = -3$  MeV which gives a width of 6 MeV in nuclear matter. On the contrary, we could consider the case of a weaker strength than  $W_0 = -1$  MeV for high- $J$   $\Xi$  states. Such possibility of width suppression will be investigated elsewhere.

Finally it should be mentioned that the  $p 1h_{11/2}^{-1}$  state of  $^{207}\text{Tl}$  is not dispersed as seen from the  $^{208}\text{Pb}(e, e'p)$  experiment [15]. This may provide a justification of our treatment of  $p 1h_{11/2}^{-1} \otimes \Xi$  states based on the single-particle models. In sum, the characteristics of the  $(K^-, K^+)$  reaction makes it possible to observe some isolated peaks due to the excitation of  $\Xi^-$ -hypernuclear bound states even in the case of  $^{208}\text{Pb}$ . The most prominent peak at  $M_{HY} - M_A \sim 387$  MeV in Fig. 8(b) has a height of  $0.4 \mu\text{b}/\text{sr MeV}$  and an effective proton number of 0.024.

## V. SUMMARY AND CONCLUSIONS

The  $K^+$  spectra for  $^{12}\text{C}$  and  $^{208}\text{Pb}(K^-, K^+)$  reactions are calculated at forward-angle  $\theta_{K^+} = 0^\circ$  with  $p_{K^-} = 1.65$  GeV/c by using the Green's function method. The nucleus- $\Xi$  potential is assumed to be of Woods-

Saxon form with complex strengths,  $V_0 = -24$  MeV and  $W_0 = -1$  MeV. The calculated spectra are folded with an energy resolution of  $\Delta E = 2$  MeV.

In the spectrum of  $^{12}\text{C}$  there appear two narrow peaks which correspond to the  $[p1p_{3/2}^{-1} \otimes \Xi_{s_{1/2}}]^{J=1}$  and  $[p1p_{3/2}^{-1} \otimes \Xi p]^{J=0,2}$  hypernuclear states, as has been shown by Ikeda *et al.* [6]. The double differential cross sections to the lower and upper states are estimated to be 0.09 and 0.17  $\mu\text{b}/\text{sr MeV}$ , respectively, when a spectrometer of  $\Delta E = 2$  MeV resolution is used. Since it is suggested that the nucleus- $\Xi$  potential depth is shallower for  $^{12}\text{C}$  than for  $^{208}\text{Pb}$ , the spectrum is also calculated with  $V_0 = -16$  MeV for  $^{12}\text{C}$ . In this case, there remains only one peak in the spectrum.

In the spectrum of  $^{208}\text{Pb}$  several well-separated peaks are observed in spite of many possible excitations, as seen from our main result Fig. 8(b). The peaks have the spin-stretched high- $J$  structure, and the most prominent one is the  $[p1h_{11/2}^{-1} \otimes \Xi_{j_{13/2}}]^{J=12}$  state bound by about 6 MeV below the  $\Xi^-$  threshold. The  $(K^-, K^+)$  reaction populates selectively high angular-momentum states with no node. The configuration  $p1h_{11/2}^{-1} \otimes \Xi$  essentially determines the shape of the spectrum from the  $^{208}\text{Pb}(K^-, K^+)$  reaction. The high- $l_{\Xi}$  single-particle state can be bound with the assistance of strong Coulomb attraction between  $\Xi^-$  and  $^{207}\text{Tl}$ . The centrifugal force also plays an essential role. Since the force confines  $\Xi^-$

in the nuclear surface region, the  $\Xi^-$  single-particle state is not an atomic but a nuclear state. Because of a remarkably good overlap between the initial  $p1h_{11/2}$  and the final  $\Xi_{j_{13/2}}$  wave functions, the  $J=12$  hypernuclear state is strongly excited with a double-differential cross section of 0.4  $\mu\text{b}/\text{sr MeV}$  and an effective proton number of 0.024.

It is investigated how the spectrum changes if a stronger imaginary strength of  $W_0 = -3$  MeV is adopted. The peaks in  $^{208}\text{Pb}$  become broad and are hardly distinguished from each other with a  $\Delta E = 2$  MeV resolution detector. However, if we could use a high-resolution spectrometer, some of individual peaks can be separated out even in this strong imaginary case.

By carrying out  $(K^-, K^+)$  experiments on both light  $^{12}\text{C}$  and heavy  $^{208}\text{Pb}$  targets, we can expect to clarify whether or not the mass-number dependence of the nucleus- $\Xi$  potential depth exists. An experimental observation of  $\Xi^-$  states in  $^{208}\text{Pb}$  is awaited.

#### ACKNOWLEDGMENTS

The authors would like to thank Professor Y. Yamamoto, Professor T. Fukuda, and Professor S.W. Hong for valuable discussions. This work was supported by the Grants-in-Aid for Scientific Research of the Japanese Ministry of Education, No. 06640377.

- 
- [1] M. Danysz *et al.*, Phys. Rev. Lett **11**, 29 (1963); D.J. Prowse, *ibid.* **17**, 782 (1966).
  - [2] S. Aoki *et al.*, Prog. Theor. Phys. **85**, 1287 (1991).
  - [3] C.B. Dover and A. Gal, Ann. Phys. (NY) **146**, 309 (1983).
  - [4] S. Aoki *et al.*, Prog. Theor. Phys. **89**, 493 (1993).
  - [5] O. Morimatsu and K. Yazaki, Nucl. Phys. **A435**, 727 (1985); **A483**, 493 (1988).
  - [6] K. Ikeda, T. Fukuda, T. Motoba, M. Takahashi, and Y. Yamamoto, Prog. Theor. Phys. **91**, 747 (1994).
  - [7] M.M. Nagels, T.A. Rijken, and J.J. de Swart, Phys. Rev. **D 12**, 744 (1975); **15**, 2547 (1977); **20**, 1633 (1979).
  - [8] Y. Yamamoto (private communication).
  - [9] E.H. Auerbach, A.J. Balitz, C.B. Dover, A. Gal, S.H. Kahana, L. Ludeking, and D.J. Millener, Ann. Phys. (NY) **148**, 381 (1983).
  - [10] C.B. Dover, L. Ludeking, and G.E. Walker, Phys. Rev. **C 22**, 2073 (1980).
  - [11] Particle Data Group, Phys. Rev. D **45**, 1 (1992).
  - [12] D.H. Wilkinson, S.J. St. Lorant, D.K. Robinson, and S. Lokanathan, Phys. Rev. Lett. **3**, 397 (1959); A. Bechdolf, G. Baumann, J.P. Gerber, and P. Cüer, Phys. Lett. **26B**, 174 (1968).
  - [13] T. Iijima *et al.*, Nucl. Phys. **A546**, 588 (1992).
  - [14] H. Tyrén, S. Kullander, O. Sundberg, R. Ramachandran, and P. Isacson, Nucl. Phys. **79**, 321 (1966).
  - [15] A.E.L. Dieperink and P.K.A. de Witt Huberts, Annu. Rev. Nucl. Part. Sci. **40**, 239 (1990).
  - [16] H. Bandō and T. Motoba, Prog. Theor. Phys. **76**, 1321 (1986).