Variational calculations of the Λ-separation energy of the $^{17}_{\Lambda}$O hypernucleus

A. A. Usmani*
Interdisciplinary Laboratory, SISSA, I-34014, Trieste, Italy

Steven C. Pieper
Physics Division, Argonne National Laboratory, Argonne, Illinois 60439

Q. N. Usmani
Department of Physics, Jamia Millia Islamia, New Delhi 110025, India
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Variational Monte Carlo calculations have been made for the $^{17}_{\Lambda}$O hypernucleus using realistic two- and three-baryon interactions. A two-pion exchange potential with spin- and space-exchange components is used for the ΛN potential. Three-body two-pion exchange and strongly repulsive dispersive ΛNN interactions are also included. The trial wave function is constructed from pair- and triplet-correlation operators acting on a single-particle determinant. These operators consist of central, spin, isospin, tensor, and three-baryon potential components. A cluster Monte Carlo method is developed for noncentral correlations and is used with up to four-baryon clusters in our calculations. The three-baryon ΛNN force is discussed.

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I. INTRODUCTION

In this paper we initiate a variational Monte Carlo study of hypernuclei using realistic two- and three-baryon interactions involving a Λ and nucleons. In the past, variational calculations of the s-shell hypernuclei [1,2], a few p-shell hypernuclei using appropriate models [3], and Λ binding to nuclear matter [2,4] were performed using mostly simplified and central nucleon-nucleon (NN) interactions. The aims of these calculations have largely been to deduce information about the Λ-nucleon (ΛN) and Λ-nucleon-nucleon (ΛNN) interactions. In addition, as a result of these studies, one could also explore the structure of hypernuclei.

The reason for using simplified NN interactions is the hope that the uncertainties in the NN interaction will largely cancel out in these calculations. This is because the Λ-separation energies B_Λ, which are the main experimental input in these calculations, are the differences of the energies of hypernuclei and their cores, i.e., $-B_\Lambda = {}^{A}_{\Lambda}E - {}^{A-1}E$, where ${}^{A}_{\Lambda}E$ is the total energy of the hypernucleus and ${}^{A-1}E$ is the ground-state energy of the core nucleus. However, interactions generate strong NN correlations in the nuclear wave function. A realistic NN interaction will generate central, spin, spin-isospin, tensor and other two-nucleon correlations [5,6]. In addition, there are significant three-nucleon correlations. In a hypernucleus, because of the operator dependence, these correlations may interact in a complicated manner with the ΛN and ΛNN correlations. This whole group of correlations then interacts with the two- and three-body operators of the two- and three-baryon interactions. This can result in important contributions to the Λ-binding energy compared to the use of only central NN correlations generated by purely central NN interactions.

In this study, we use realistic two- and three-nucleon interactions and wave functions to see their effects on hypernuclei. There is another important aspect of the present study, the development of a methodology and variational program for hypernuclei. For the p-shell hypernuclei, we adopt the cluster Monte Carlo (CMC) technique developed in Ref. [7] by Pieper, Wiringa, and Pandharipande which we refer to as PWP. As a first step, we study $^{17}_{\Lambda}$O. Generalization to $^{16}_{\Lambda}$O will be straightforward. Further development of the one-body part of the nuclear wave functions will be needed for other p-shell hypernuclei. We intend to cover a wide range of hypernuclei in order to have reliable information on three-baryon forces, because in this study we find that the role of the three-body ΛNN interaction is greatly altered from that found in some previous studies.

There are a few calculations of hypernuclei in which realistic NN forces have been used. One such calculation is by Carlson [8] in which he explicitly considers the $\Lambda N \to \Sigma N$ channel in $^{5}_{\Lambda}$He and $^{4}_{\Lambda}$He using the Nijmegen interaction. This study shows that the Nijmegen interaction underbinds the four-body hypernuclei and that the five-body hypernucleus is unbound relative to a separated α and Λ particle. Also, it does not reproduce the spin splitting in the four-body hypernuclei. To resolve the classical overbinding [1,9,10] problem of $^{5}_{\Lambda}$He, Bando and Shimodaya [11], and Shinmura *et al.* [12] have also performed calculations with Reid soft core and Hamada-Johnston NN potentials by calculating effective interac-

*On leave from Department of Physics, Jamia Millia Islamia, New Delhi 110025, India.

tions using a G-matrix approach. All these calculations of the Λ-seperation energies B_Λ are confined to s-shell hypernuclei.

Since no experimental data for ${}^{17}_{\Lambda}$O exist, we generate "pseudoexperimental" or semiempirical data for this system. This is done in Sec. II where we also briefly discuss the experimental status of Λ-seperation energy data. In Sec. III we review the Λ and nucleon two- and three-body potentials. Section IV deals with the variational wave function. In Sec. V we describe the techniques of our calculations for ${}^{17}_{\Lambda}$O. In Sec. VI we present our results, and Sec. VII contains our conclusions.

II. Λ-SEPARATION ENERGIES

If we combine the results of previous experiments summarized in Ref. [13], the in-flight reaction (K^-,π^-), [14–16] and the associated production [17] (π^+,K^+), we have now almost 30 well-established hypernuclei with a wide range of baryon numbers $A \leq 81$ and orbital angular momentum $\ell_\Lambda \leq 3$. The hypernuclei that are relevant for the empirical determination of the B_Λ value of ${}^{17}_{\Lambda}$O are ${}^{11}_{\Lambda}$C, ${}^{12}_{\Lambda}$C, ${}^{13}_{\Lambda}$C, ${}^{16}_{\Lambda}$O, ${}^{28}_{\Lambda}$Si, ${}^{32}_{\Lambda}$S, ${}^{40}_{\Lambda}$Ca, ${}^{51}_{\Lambda}$V, and ${}^{89}_{\Lambda}$Y.

We use here three approaches for the empirical B_Λ value of ${}^{17}_{\Lambda}$O. In the first approach [18] microscopic calculations of B_Λ of the above hypernuclei were carried out with phenomenological two- and three-body ΛN and ΛNN interactions that were previously obtained from studies of Λp scattering, the s-shell hypernuclei, ${}^{9}_{\Lambda}$Be as a representative of p-shell hypernuclei in the $2\alpha + \Lambda$ model, and the Λ binding to nuclear matter [2–4]. The Λ-separation energies B_Λ are obtained from a Schrödinger equation with a Λ-nucleus potential U_Λ and an effective mass m^*_Λ which are obtained in the local-density approximation using the Fermi hypernetted chain technique for the Λ binding to nuclear matter. Such a procedure gives a good account of the B_Λ data of the above hypernuclei for $\ell_\Lambda \leq 3$. Using this procedure, we calculate the difference, ΔB_Λ, of the Λ-separation energies of ${}^{17}_{\Lambda}$O and ${}^{16}_{\Lambda}$O. This gives $\Delta B_\Lambda = 0.30$ MeV.

In the second approach, we use the purely phenomenological technique adopted by Millener *et al.* [19]. Here we use a Woods-Saxon Λ-nucleus potential whose parameters were fitted to the B_Λ data of the above-mentioned hypernuclei,

$$U_\Lambda(r) = \frac{V_{0\lambda}}{1+\exp(\frac{r-c}{a})}\ , \tag{2.1}$$

where $V_{0\lambda} = -28.0$ MeV, $c = (1.128 + 0.439A^{-2/3})A^{1/3}$, and $a = 0.6$ fm give a very good account of the B_Λ data. This procedure gives ΔB_Λ=0.40 MeV.

In the third approach, we consider a density-dependent Λ-nucleus potential [19–21] of the form

$$U_\Lambda(r) = A\rho(r) + B\rho^{4/3}(r), \tag{2.2}$$

where $\rho(r)$ represents the nucleon density. These are taken from Ref. [22]. We choose s and p orbits in ${}^{16}_{\Lambda}$O and the p and f orbits in ${}^{89}_{\Lambda}$Y to fix the parameters A and B. The experimental binding energies are 12.5±0.35 and 2.5±0.5 MeV in ${}^{16}_{\Lambda}$O and 16.0±1.0 and 2.5±1.0 MeV in ${}^{89}_{\Lambda}$Y. Fitting these energies results in a reasonable fit to all binding energies from ${}^{11}_{\Lambda}$C to ${}^{89}_{\Lambda}$Y for $\ell_\Lambda \leq 3$. The resulting ΔB_Λ is 0.76 MeV.

If we combine the results of the above three approaches and at the same time bear in mind that the experimental uncertainty in the B_Λ value of ${}^{16}_{\Lambda}$O is ±0.35 MeV, we may reliably fix the empirical B_Λ value of ${}^{17}_{\Lambda}$O as 13.0 ± 0.4 MeV. We shall make use of this value in our calculations. Other approaches, such as relativistic mean-field theories or the local-density approximation using a Skyrme interaction, can also be employed in the empirical determination of the B_Λ value of ${}^{17}_{\Lambda}$O. But, since these approaches are consistent with the approaches that we have adopted above, we do not feel that their inclusion will affect our results.

III. HAMILTONIAN

In an A-baryon hypernucleus, we will consider the first $A-1$ baryons to be nucleons. We will use Ψ to refer to the full wave function of the hypernucleus and Ψ_N to refer to the ground-state wave function of the $A-1$ nucleons. The full Hamiltonian H can be written as

$$H = H_N + H_\Lambda\ , \tag{3.1}$$

where H_N is the nucleon Hamiltonian:

$$H_N = -\sum_{i=1}^{A-1}\frac{\hbar^2}{2m}\nabla_i^2 + \sum_{i<j}^{A-1} v_{ij} + \sum_{i<j<k}^{A-1} V_{ijk} \tag{3.2}$$

and

$$H_\Lambda = -\frac{\hbar^2}{2m_\Lambda}\nabla_\Lambda^2 + \sum_{i=1}^{A-1} v_{i\Lambda} + \sum_{i<j}^{A-1} V_{ij\Lambda} \tag{3.3}$$

is the part of the full Hamiltonian due to the Λ particle.

The Λ-separation energy, B_Λ, of a hypernucleus is then given by

$$B_\Lambda = \frac{\langle\Psi_N|H_N|\Psi_N\rangle}{\langle\Psi_N|\Psi_N\rangle} - \frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle}\ . \tag{3.4}$$

Our goal is to calculate B_Λ using a variational principle for the two components of Eq. (3.4). In this section we briefly describe the two- and three-body baryon interactions that we have used in this study.

A. ΛN potential

Two-pion exchange (TPE) is a dominant part of the ΛN potential that in turn is mainly determined by the strong tensor one-pion-exchange (OPE) component acting twice. Moreover, there is the K-exchange interaction that primarily contributes to the ΛN-exchange potential. The tensor part of the ΛN interaction is very weak because the shorter range $\bar{K}$ and K^* exchanges that are responsible for this are of opposite sign and nearly cancel each other. (In the case of the NN interaction the π-exchange and ρ-exchange tensor components do not cancel so completely, because their masses are quite different.)

We use an Urbana-type [23] potential with spin- and space-exchange components and a TPE tail which is consistent with Λp scattering below the Σ threshold,

$$v_{\Lambda N}(r) = v_0(r)(1-\epsilon+\epsilon P_x) + \frac{1}{4} v_\sigma T_\pi^2(r) \boldsymbol{\sigma}_\Lambda \cdot \boldsymbol{\sigma}_N, \quad (3.5)$$

$$v_0(r) = v_c(r) - \bar{v} T_\pi^2(r), \quad (3.6)$$

$$v_c(r) = \frac{W_c}{1+\exp(\frac{r-R}{a})}, \quad (3.7)$$

where $T_\pi(r)$ is the OPE tensor potential

$$T_\pi = \left(1+\frac{3}{x}+\frac{3}{x^2}\right)\frac{e^{-x}}{x}\left(1-e^{-cr^2}\right)^2, \quad (3.8)$$

$x=\mu r$, μ=0.7 fm^{-1} is the pion mass, and the cutoff parameter $c = 2.0$ fm^{-2}. P_x is the space-exchange operator and ϵ is the corresponding exchange parameter. The $\bar{v} \equiv (v_s + 3v_t)/4$ and $v_\sigma \equiv v_s - v_t$ are, respectively, the spin-average and spin-dependent strengths, where v_s and v_t denote the singlet and triplet state depths, respectively. (Note that following the convention of Ref. [2], the Hamiltonian effectively contains $+v_c$, $-\bar{v}$, $+v_\sigma$, $-v_s$, and $-v_t$.) Finally, $v_c(r)$ is a short range Woods-Saxon repulsive potential. The various parameters are

$$v_s = 6.33\,\text{MeV},\ v_t = 6.1\,\text{MeV},\ \epsilon = 0.3,$$
$$W_c = 2137\,\text{MeV},\ R = 0.5\,\text{fm},\ a = 0.2\,\text{fm}. \quad (3.9)$$

These parameters are consistent with the low-energy Λp scattering data that essentially determine the spin-average potential $\bar{v}$. The parameter ϵ for the space-exchange strength is fairly well determined from the Λ single-particle scattering data [18]. For a detailed account of the determination of the other parameters see Ref. [2]. We point out that because of the noncentral ΛN and ΛNN correlations introduced in the next section, the ΛN spin-spin potential will have a nonzero contribution even in a closed-shell system such as $^{17}_{\Lambda}$O.

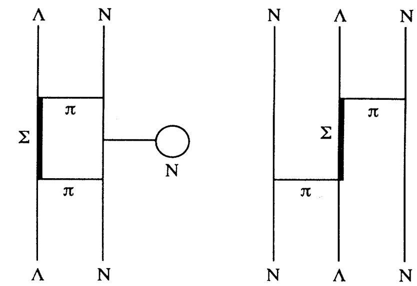

FIG. 1. Terms contributing to $V^D_{\Lambda NN}$ and $V^{2\pi}_{\Lambda NN}$.

B. ΛNN potential

When a ΛN potential that fits the Λp scattering is used, the B_Λ for hypernuclei with $A \geq 5$ are almost a factor of 2 too large. This is an old result that has been confirmed by various analyses. In the present work we also find that the use of a realistic NN interaction does not alleviate this overbinding problem. As in the previous studies, to resolve the overbinding problem, we incorporate a three-body ΛNN interaction.

We consider here two types of ΛNN potentials that arise from projecting out Σ, Δ, etc., degrees of freedom from a coupled channel formalism. These are the dispersive and the TPE ΛNN potentials designated as $V^D_{\Lambda NN}$ and $V^{2\pi}_{\Lambda NN}$, respectively (see Fig. 1). $V^D_{\Lambda NN}$ is expected to be repulsive, and, following Ref. [2], we assume the phenomenological form

$$V^D_{\Lambda NN} = W_0 T_\pi^2(r_{1\Lambda}) T_\pi^2(r_{2\Lambda}), \quad (3.10)$$

where W_0 is the strength of the potential and $T_\pi(r_{i\Lambda})$ is given by Eq. (3.8). $V^{2\pi}_{\Lambda NN}$ consists of two parts corresponding to p- and s-wave $\pi\Lambda$ interactions [24]

$$V^{2\pi}_{\Lambda NN} = W_p + W_s, \quad (3.11)$$

where

$$W_p = -\left(\frac{C_p}{6}\right)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\{X_{1\Lambda}, X_{2\Lambda}\}, \quad (3.12)$$

$$W_s = \frac{C_s(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)(\boldsymbol{\sigma}_1\cdot\mathbf{r}_{1\Lambda})(\boldsymbol{\sigma}_2\cdot\mathbf{r}_{2\Lambda})(\mu r_{1\Lambda}+1)(\mu r_{2\Lambda}+1)Y_\pi(r_{1\Lambda})Y_\pi(r_{2\Lambda})}{(\mu r_{1\Lambda} r_{2\Lambda})^2}, \quad (3.13)$$

$$Y_\pi(r) = \frac{e^{-\mu r}}{\mu r}\left(1-e^{-cr^2}\right), \quad (3.14)$$

$$\{A,B\} = AB + BA, \quad (3.15)$$

and

$$X_{i\Lambda} = (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_\Lambda) Y_\pi(r_{i\Lambda}) + S_{i\Lambda} T_\pi(r_{i\Lambda}). \tag{3.16}$$

Here $X_{i\Lambda}$ is the one-pion-exchange operator, and $S_{i\Lambda}$ is the tensor operator. The component W_s is quite weak and, as in previous studies, we neglect it here; we feel that its effect should be studied in future work.

There are theoretical as well as phenomenological estimates for C_p; but for W_0 the estimates are purely phenomenological. For example, for $W_0 \approx 0.02$, the reduction in the Λ binding to nuclear matter (using central correlations) is approximately in accord with the suppression obtained in coupled channel ($\Lambda N \to \Sigma N$) reaction matrix calculations. For C_p theoretical estimates give 1–2 MeV; however, the phenomenological values may not lie in this region as the results depend sensitively on the cutoff parameter c that appears in Eq. (3.8). In the present study, we have obtained results as a function of the values of these parameters.

C. Two- and three-nucleon potentials

For the nuclear part of the Hamiltonian, we use NN and NNN potentials that have been previously used to study various nuclei, including ^{16}O [7]. The NN potential contains the first six terms of the Argonne v_{14} [25] potential and a Coulomb term:

$$v_{ij} = \sum_{p=1}^{6} v^p(r_{ij}) O_{ij}^p + v_{\rm Coul}(r_{ij}) O_{ij}^{\rm Coul}, \tag{3.17}$$

where the operators are

$$\begin{aligned} O_{ij}^{p=1,6} &= 1, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \\ &\quad \times (\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j), S_{ij}, S_{ij}(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j), \\ O_{ij}^{\rm Coul} &= \frac{1}{4}(1+\tau_{z,i})(1+\tau_{z,j}). \end{aligned} \tag{3.18}$$

We shall also refer to these operators by the abbreviations c, τ, σ, $\sigma\tau$, t, and $t\tau$. In PWP it was found that the $7 \leq p \leq 14$ terms of Argonne v_{14} and the corresponding $p = 7,8$ correlation operators gave a net contribution of only -0.45 MeV/nucleon. We assume that the presence of a Λ will not significantly modify this result and hence we can safely omit all potential and correlation operators for $p \geq 7$ when computing $B_\Lambda(^{17}_{\Lambda}\text{O})$.

The NNN potential is of the Urbana type, which consists of dispersive and two-pion-exchange terms:

$$V_{ijk} = V_{ijk}^D + V_{ijk}^{2\pi}, \tag{3.19}$$

$$V_{ijk}^D = \sum_{\rm cyc} U_0 T_\pi^2(r_{ij}) T_\pi^2(r_{jk}), \tag{3.20}$$

$$V_{ijk}^{2\pi} = \sum_{\rm cyc} A_0 \left(\{X_{ij}, X_{jk}\}\{\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j, \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k\} + \frac{1}{4}[X_{ij}, X_{jk}][\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j, \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k] \right), \tag{3.21}$$

where the square brackets represent the commutator $[A,B] = AB - BA$. The constants A_0 and U_0 have the values -0.0333 and 0.0038 in Urbana model VII [26], which we use here.

IV. THE VARIATIONAL WAVE FUNCTIONS

We assume that a good variational wave function for a hypernucleus with a closed-shell nuclear core and a Λ particle can be written as

$$|\Psi\rangle = \left[\prod_{\rm IT} (1 + U_{ij\Lambda} + U_{ijk}) \right] \left[\prod_{i=1}^{A-1} (1 + U_{i\Lambda}) \right] \times \left[S \prod_{i<j}^{A-1} (1 + U_{ij}) \right] |\Psi_J\rangle, \tag{4.1}$$

$$|\Psi_J\rangle = \prod_{i=1}^{A-1} f_c^\Lambda(r_{i\Lambda}) \prod_{i<j}^{A-1} f_c(r_{ij}) \phi_\Lambda(r_\Lambda) \mathbf{A} |\Phi^{A-1}\rangle. \tag{4.2}$$

Here U_{ijk} represents a three-baryon correlation operator that has the same structure as V_{ijk},

$$U_{ijk} = \delta \tilde{V}_{ijk}, \tag{4.3}$$

$$U_{ij\Lambda} = \delta_\Lambda \tilde{V}_{ij\Lambda}. \tag{4.4}$$

The $\tilde{V}_{ijk}$ differs from V_{ijk} through the range c of the cutoff functions of $Y_\pi(r)$ and $T_\pi(r)$. The parameter δ is referred to as ϵ in PWP; we use δ here to avoid confusion with the ϵ of the space-exchange potential in Eq. (3.5). The parameters δ, δ_Λ, c, and c_Λ are determined variationally. The label IT stands for independent triplet product of $1 + U_{ijk}$. Thus,

$$\prod_{\rm IT} (1 + U_{ijk} + U_{ij\Lambda}) = 1 + \sum_{i<j} U_{ij\Lambda} + \sum_{\substack{i<j \\ i'<j'<k' \neq i,j}} U_{ij\Lambda} U_{i'j'k'} + \sum_{i<j<k} U_{ijk} + \sum_{\substack{i<j<k \\ i'<j'<k' \neq i,j,k}} U_{ijk} U_{i'j'k'} + \cdots. \tag{4.5}$$

The neglected terms are of the type $U_{ijk} U_{i'j'k}$. This restriction makes the three-body correlations much simpler to use. As is discussed in PWP, the U_{ijk} and $U_{ij\Lambda}$ should ideally act last as in Eq. (4.1). However, this requires considerably more computer time. The improvement in the energy of ^{16}O obtained by this was found to be only $-0.19(7)$ MeV/nucleon. In the present work we ignore this correction and compute the energies of both $^{17}_{\Lambda}$O and

^{16}O with the three-baryon correlations acting first.

Each operator in the two-baryon interaction can induce the corresponding correlation. The f_c^Λ and the f_c are central correlations that are primarily generated by the repulsive cores in the two-baryon interactions. For U_{ij} and $U_{i\Lambda}$, we make the following choice:

$$U_{ij} = \sum_{p=2}^{n} \beta_p u_p(r_{ij}) O_{ij}^p \tag{4.6}$$

and

$$U_{i\Lambda} = \sum_{p=2}^{m} u_p^\Lambda(r_{i\Lambda}) O_{i\Lambda}^p . \tag{4.7}$$

The notation $S\prod$ in Eq. (4.1) represents a symmetrized product of the noncommuting operators $U_{ij}U_{jk}\ldots$. Previous studies [6,7] on few-body nuclei and ^{16}O demonstrate that it is probably sufficient to use $2 \le p \le 6$ in Eq. (4.6). The pair-correlation functions f_c and u_p are generated by minimizing the two-body cluster energy using a quenched potential:

$$\tilde{v}_{ij} = \sum_{p=2}^{6} \alpha v_p(r_{ij}) O_{ij}^p . \tag{4.8}$$

The two-body cluster contribution has been minimized for infinite nuclear matter at Fermi momentum k_F, with the boundary conditions $f_c(r > d) = 1$, and $u_p(r > d) = 0$, for $p = \tau, \sigma$, and $\sigma\tau$, and $u_p(r > d_t) = 0$, for $p = t$ and $t\tau$ with their first derivatives zero at $r = d$ or d_t.

For the $U_{i\Lambda}$ we consider

$$U_{i\Lambda} = u_\sigma^\Lambda(r_{i\Lambda}) \boldsymbol{\sigma}_\Lambda \cdot \boldsymbol{\sigma}_i + u_{P_x}(r_{i\Lambda}) P_x . \tag{4.9}$$

In the present study, we have omitted the second, i.e., the exchange correlation term in Eq. (4.9). Inclusion of this term increases the computation effort by several fold and preliminary results indicate that it gives a small contribution. This will be the subject of a future publication. The spin correlation is

$$u_\sigma^\Lambda = \frac{f_t^\Lambda - f_s^\Lambda}{f_c^\Lambda} , \tag{4.10}$$

where f_s^Λ and f_t^Λ are the solutions of Schrödinger equations with quenched ΛN potentials in singlet and triplet states, respectively:

$$\left[-\frac{\hbar^2}{2\mu_{\Lambda N}} \nabla^2 + \tilde{v}_{s(t)}(r_{\Lambda N}) \right] f_{s(t)}^\Lambda = 0 . \tag{4.11}$$

The potentials $\tilde{v}_{s(t)}$ are quenched in the two-pion and spin-exchange parts of the central and spin channels:

$$\tilde{v}_s(r) = v_c(r) - (\alpha_{2\pi}\, \bar{v} + \tfrac{3}{4}\, \alpha_\sigma v_\sigma) T_\pi^2(r) , \tag{4.12}$$

$$\tilde{v}_t(r) = v_c(r) - (\alpha_{2\pi}\, \bar{v} - \tfrac{1}{4}\, \alpha_\sigma v_\sigma) T_\pi^2(r) . \tag{4.13}$$

The spin-averaged correlation function f_c^Λ is given by

$$f_c^\Lambda = \frac{f_s^\Lambda + 3 f_t^\Lambda}{4} . \tag{4.14}$$

The f_s^Λ, f_t^Λ, and f_c^Λ have been obtained by minimizing the two-body cluster energy for the Λ binding to nuclear matter with the asymptotic condition $f_c^\Lambda(r > d_\Lambda) = 1$.

The ϕ_Λ represents a bound-state wave function of a Λ particle of mass m_Λ moving in a Woods-Saxon potential that is bound to a nucleus of mass $(A-1)m$:

$$V_\Lambda(r_\Lambda) = \frac{V_\Lambda}{1 + \exp\left(\frac{r_\Lambda - R_\Lambda}{a_\Lambda}\right)} . \tag{4.15}$$

The parameters V_Λ, R_Λ, and a_Λ are determined variationally. The Slater determinant $\mathbf{A}\left|\Phi^{A-1}\right\rangle$ consists of orbitals of nucleons of mass m bound to a hypernucleus of mass $(A-2)m + m_\Lambda$ moving in a Woods-Saxon wine-bottle potential

$$V(r) = V_s \left(\frac{1}{1 + e^{(r - R_s)/a_s}} - \alpha_s e^{-(r/\rho_s)} \right) , \tag{4.16}$$

with V_s, R_s, a_s, α_s, and ρ_s as variational parameters. The coordinates of all the one-body orbitals are measured from the center of mass of the whole system, thus making Ψ_J and Ψ translationally invariant, i.e.,

$$\tilde{\mathbf{r}}_i = \mathbf{r}_i - \mathbf{R}_{\text{c.m.}} , \tag{4.17}$$

$$\mathbf{R}_{\text{c.m.}} = \frac{m \sum_{i=1}^{A-1} \mathbf{r}_i + m_\Lambda \mathbf{r}_\Lambda}{m(A-1) + m_\Lambda} . \tag{4.18}$$

V. THE CLUSTER EXPANSION

We briefly outline the general framework for the cluster expansion of PWP to calculate the expectation values of various operators. These expectation values are needed in the evaluation of the energy using the variational wave function (4.1). We demonstrate the cluster expansion for the two-body NN and ΛN potentials:

$$\frac{\langle \Psi | \sum v_{ij} + \sum v_{i\Lambda} | \Psi \rangle}{\langle \Psi | \Psi \rangle} = \frac{N}{D} . \tag{5.1}$$

The N and D can be expanded as a sum of n-body contributions

$$N = \sum_{i<j}^{A-1} n_{ij} + \sum_{i=1}^{A-1} n_{i\Lambda} + \sum_{i<j<k}^{A-1} n_{ijk} + \sum_{i<j}^{A-1} n_{ij\Lambda} + \cdots , \tag{5.2}$$

$$D = 1 + \sum_{i<j}^{A-1} d_{ij} + \sum_{i=1}^{A-1} d_{i\Lambda} + \cdots . \tag{5.3}$$

The expressions for the purely nuclear n_{ij}, d_{ij}, n_{ijk}, etc., may be found in PWP. The contributions of clusters containing a Λ are similar, e.g.,

$$n_{i\Lambda} = \left\langle (1 + U_{i\Lambda})^\dagger\, v_{i\Lambda} (1 + U_{i\Lambda}) \right\rangle , \tag{5.4}$$

$$d_{i\Lambda} = \left\langle (1+U_{i\Lambda})^\dagger (1+U_{i\Lambda}) \right\rangle - 1\,, \quad (5.5)$$

$$n_{ij\Lambda} = \langle\, (1+U^\dagger_{ij\Lambda})(1+U^\dagger_{ij})P\left[(1+U^\dagger_{i\Lambda})(1+U^\dagger_{j\Lambda})\right] \times [v_{ij}+v_{i\Lambda}+v_{j\Lambda}]\,P'\,[(1+U_{i\Lambda})(1+U_{j\Lambda})] \times (1+U_{ij})(1+U_{ij\Lambda})\,\rangle - n_{ij} - n_{i\Lambda} - n_{j\Lambda}\,, \quad (5.6)$$

where P and P' are permutation operators. In these expressions

$$\langle\theta\rangle = \frac{\langle\Psi_J|\theta|\Psi_{J'}\rangle}{\langle\Psi_{J'}|\Psi_{J'}\rangle}\,, \quad (5.7)$$

where $\Psi_{J'}$ denotes the Ψ_J of Eq. (4.2) without the antisymmetrization operator.

The expansions (5.2) and (5.3) for N and D are divergent [7]. We obtain a convergent linked-cluster expansion by expressing

$$\frac{N}{D} = \sum_{i<j}^{A-1} c_{ij} + \sum_{i=1}^{A-1} c_{i\Lambda} + \cdots. \quad (5.8)$$

The various c_{ij} and $c_{i\Lambda}$, etc., are obtained from the equation

$$N = \left[\sum_{i<j}^{A-1} c_{ij} + \sum_{i=1}^{A-1} c_{i\Lambda} + \cdots\right] D\,, \quad (5.9)$$

by equating terms with the same ij, $i\Lambda$, ijk, $ij\Lambda$, etc. Thus,

$$c_{i\Lambda} = \frac{n_{i\Lambda}}{1+d_{i\Lambda}} \quad (5.10)$$

and

$$c_{ij\Lambda} = \frac{n_{ij\Lambda} - c_{ij}(d_{j\Lambda}+d_{i\Lambda}) - c_{i\Lambda}(d_{ij}+d_{j\Lambda}) - c_{j\Lambda}(d_{ij}+d_{i\Lambda}) - (c_{ij}+c_{i\Lambda}+c_{j\Lambda})d_{ij\Lambda}}{1+d_{ij}+d_{i\Lambda}+d_{j\Lambda}+d_{ij\Lambda}}\,. \quad (5.11)$$

In the present work, we have used the CEA expansion of PWP so that all clusters of a given spin, isospin, and Λ content are averaged together.

VI. RESULTS

A. Variational parameters

We made detailed variational parameter searches for two cases: (1) with no ΛNN potential and hence no $U_{ij\Lambda}$ correlation, and (2) using a ΛNN potential with $C_p = 0.7$ MeV and $W_0 = 0.015$ MeV. The rest of the Hamiltonian was as described in Sec. III for both cases; in particular the NNN potential and U_{ijk} correlation were used in both cases.

For the case with no ΛNN potential, we found that the optimal values of all of the nucleon correlation parameters are the same as was found in PWP for ^{16}O, except that the well depth, V_s, of the Woods-Saxon potential changes from -49.1 MeV to -48.9 MeV to maintain the same p-wave separation energy, 14.0 MeV, with the 17-body instead of 16-body reduced mass. The reader is referred to PWP for these parameter values. The optimal parameters for correlation terms involving the Λ are, for the Λ Woods-Saxon well,

$$V_\Lambda = -28.3\,\text{MeV},\ R_\Lambda = 3.2\,\text{fm},\ a_\Lambda = 0.5\,\text{fm}\,, \quad (6.1)$$

which gives an s-wave separation energy of 15.0 MeV; for the $U_{i\Lambda}$

$$\alpha_{2\pi} = \alpha_\sigma = 1.0\,,\ d_\Lambda = 2.8\,\text{fm}\,. \quad (6.2)$$

In the presence of the ΛNN potential with $C_p = 0.7$ MeV, $W_0 = 0.015$ MeV, we found that only two of the above optimal values had to be changed. These are the quenching parameters α in the NN correlation and $\alpha_{2\pi}$ in the $N\Lambda$ correlation. The variational energy is sensitive to α and we made several searches at other values of C_p to determine

$$\alpha = 0.94 - 0.1\,C_p\,,\ 0 \le C_p \le 1.2\ \text{MeV}\,. \quad (6.3)$$

The sensitivity to $\alpha_{2\pi}$ is weak and we used

$$\alpha_{2\pi} = 0.95\,,\ 0.7 \le C_p \le 1.2\ \text{MeV}\,. \quad (6.4)$$

In addition to these parameters we found for the $U_{NN\Lambda}$

$$\delta_\Lambda = -0.0013\,,\ c_\Lambda = 1.6\ \text{fm}^{-2}\,, \quad (6.5)$$

for all C_p and W_0 considered.

B. Variational energies

Tables I and II show various components of the $^{17}_{\Lambda}$O energy for the cases of no ΛNN potential and $C_p = 0.7$ MeV, $W_0 = 0.015$, respectively. The cluster expansion for terms involving the Λ is converging well and it appears that it is not necessary to extrapolate beyond the four-body clusters for these terms. To get an accurate total energy of the $^{17}_{\Lambda}$O nucleus, it would be necessary to extrapolate the values of V_{NNN} as was done in PWP. However since we are mainly interested in B_Λ, we do not do that here and instead subtract an unextrapolated ^{16}O energy.

One important result shown in Table II is that the expectation of $V^{2\pi}_{\Lambda}$ is substantial and negative. This arises from the noncentral correlations in the wave function. A purely Jastrow wave function ($U_{ij} = U_{ijk} = U_{i\Lambda} =$

TABLE I. Variational energies for $C_p = W_0 = 0$. All values are in MeV. Numbers in parentheses are statistical errors in the last digit.

Clusters		One-body	Two-body	Three-body	Four-body	Total
Kinetic energy[a]		20.2(6)	0.1(1)	1.1(5)	−0.7(3)	20.7(9)
ΛN potential	$v_0(r)(1-\epsilon)$		−44.9(9)	−3.7(3)	1.4(7)	−47.3(12)
	$v_0(r)\epsilon P_x$		−13.4(4)	0.4(1)	0.7(3)	−12.2(5)
	$\frac{1}{4}v_\sigma T_\pi^2 \boldsymbol{\sigma}_\Lambda \cdot \boldsymbol{\sigma}_N$		0.34(3)	−0.06(1)	−0.05(2)	0.22(3)
Λ energy		20.2(6)	−57.9(11)	−2.3(7)	1.4(11)	−38.6(10)
Nuclear kinetic		317.(2)	269.(2)	−19.(3)	11.(3)	556.(5)
NN potential	v_{ij}		−737.(4)	111.(2)	4.(4)	−623.(5)
NNN potential	V_{ijk}			−59.8(8)	35.8(8)	−23.9(8)
Nuclear energy		317.(2)	−468.(2)	32.(2)	29.(3)	−90.(2)
Total energy		337.(2)	−526.(3)	30.(3)	30.(1)	−128.5(18)

[a]Includes nucleon kinetic energy from ΛN correlations.

$U_{ij\Lambda} = 0$) gives $\langle V^{2\pi}_{\Lambda NN}\rangle = 0.9$ MeV; including U_{ij} (with $\alpha = 0.94$), U_{ijk}, and $U_{i\Lambda}$ (with $\alpha_{2\pi} = 1.0$), but no $U_{ij\Lambda}$, results in $\langle V^{2\pi}_{\Lambda NN}\rangle = -4.2$ MeV. Including $U_{ij\Lambda}$ also but still keeping $\alpha = 0.94$, $\alpha_{2\pi} = 1.0$, gives $\langle V^{2\pi}_{\Lambda NN}\rangle = -7.9$ MeV, while lowering α to the optimal value of 0.87 and $\alpha_{2\pi}$ to 0.95 reduces this to −5.0 MeV (this loss of binding is offset by changes in the expectation values of other parts of the Hamiltonian). These results can be understood as follows: By using the relation

$$\boldsymbol{\sigma}\cdot\mathbf{A}\,\boldsymbol{\sigma}\cdot\mathbf{B} = \mathbf{A}\cdot\mathbf{B} + \mathbf{i}\boldsymbol{\sigma}\cdot(\mathbf{A}\times\mathbf{B})\,, \tag{6.6}$$

the $\{X_{1\Lambda}, X_{2\Lambda}\}$ appearing in $V^{2\pi}_{\Lambda NN}$ can be expressed in terms of the operators $\boldsymbol{\sigma}_1\cdot\mathbf{r}_{1\Lambda}\,\boldsymbol{\sigma}_2\cdot\mathbf{r}_{2\Lambda}$, $\boldsymbol{\sigma}_1\cdot\mathbf{r}_{1\Lambda}\,\boldsymbol{\sigma}_2\cdot\mathbf{r}_{1\Lambda}$, $\boldsymbol{\sigma}_1\cdot\mathbf{r}_{2\Lambda}\,\boldsymbol{\sigma}_2\cdot\mathbf{r}_{2\Lambda}$, and $\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$ and hence is a generalization of the tensor operator S_{12}. However the expectation value of S_{12} in a Jastrow wave function for a closed-shell nuclear system is zero, while the expectation value of S_{12}^2 is nonzero. Hence the S_{12} operator in U_{12} completely changes $\langle V^{2\pi}_{\Lambda 12}\rangle$. Of course the $U_{ij\Lambda}$ further enhances its contribution.

The ^{16}O energy that corresponds to the present calculation (U_{ijk} acts first, use of only the first six operators in Argonne v_{14}, and no extrapolation) is −101.0(9) MeV. We emphasize that, because of the above approximations, this energy is to be used only in comparison with the $^{17}_{\Lambda}$O energies. The resulting $B_\Lambda(^{17}_{\Lambda}\mathrm{O})$ are 27.5(2.0) MeV for no $V_{\Lambda NN}$ and 13.5(1.8) MeV for $C_p = 0.7$, $W_0 = 0.015$, which are to be compared with the empirical value of 13.0(4) found in Sec. II. Thus even with realistic NN potentials and correlations, $^{17}_{\Lambda}$O is very overbound if no $V_{\Lambda NN}$ is used. However a reasonable $V_{\Lambda NN}$ results in a B_Λ consistent with the empirical value.

To study the dependence of B_Λ on the strength of the $V_{\Lambda NN}$, we made a number of calculations with different values of C_p and W_0. In each case the NN quenching parameter was chosen according to Eq. (6.3). To minimize statistical errors, we made correlated difference calculations using the $C_p = 0.7$, $W_0 = 0.015$ random walk. Table III presents the resulting changes, δB_Λ, in B_Λ. All of these values of B_Λ and δB_Λ are well fit by the formula

$$B_\Lambda = 27.3 - 8.9\,C_p + 11.2\,C_p^2 - 870.\,W_0\,, \tag{6.7}$$

the statistical error of B_Λ is ± 1.6 MeV. The quadratic dependence on C_p comes from $U_{ij\Lambda}$; the contribution of the dispersive term in $U_{ij\Lambda}$ (and also in U_{ijk}) is very small. When comparable calculations of other hypernuclei are available, Eq. (6.7) and the empirical value of $B_\Lambda(^{17}_{\Lambda}\mathrm{O}) = 13.0(4)$ can be used to uniquely determine

TABLE II. Variational energies for $C_p = 0.7$ MeV, $W_0 = 0.015$ MeV. All values are in MeV. Numbers in parentheses are statistical errors in the last digit.

Clusters		One-body	Two-body	Three-body	Four-body	Total
Λ kinetic energy[a]		17.1(5)	0.1(1)	2.4(5)	−1.0(4)	18.6(6)
ΛN potential	$v_0(r)(1-\epsilon)$		−39.5(7)	−2.0(2)	0.6(5)	−40.9(8)
	$v_0(r)\epsilon P_x$		−12.5(3)	0.5(1)	−0.2(4)	−12.2(4)
	$\frac{1}{4}v_\sigma T_\pi^2 \boldsymbol{\sigma}_\Lambda \cdot \boldsymbol{\sigma}_N$		0.26(2)	−0.06(1)	0.00(2)	0.20(2)
ΛNN potential	$V^D_{\Lambda NN}$			14.1(4)	−0.1(1)	14.0(4)
	$V^{2\pi}_{\Lambda NN}$			−6.5(3)	1.5(2)	−5.0(3)
Λ energy		17.1(5)	−51.7(10)	8.4(6)	0.9(9)	−25.3(7)
Nuclear kinetic		309.(2)	229.(2)	−7.(2)	−12.(2)	520.(4)
NN potential	v_{ij}		−682.(3)	85.(2)	10.(3)	−587.(4)
NNN potential	V_{ijk}			−50.2(6)	27.9(6)	−22.3(7)
Nuclear energy		309.(2)	−453.(2)	28.(2)	27.(2)	−90.(2)
Total energy		326.(2)	−505.(2)	36.(2)	38.(2)	−114.5(16)

[a]Includes nucleon kinetic energy from Λ correlations.

TABLE III. Differences $B_\Lambda(C_p, W_0) - B_\Lambda(C_p = 0.7, W_0 = 0.015)$. The NN quenching parameter, α, is also given.

C_p	W_0	α	δB_Λ (MeV)
0.7	0.01	0.87	$+4.4 \pm 0.3$
0.7	0.017	0.87	-1.7 ± 0.1
0.7	0.02	0.87	-4.4 ± 0.3
0.9	0.01	0.85	$+6.3 \pm 0.7$
0.9	0.015	0.85	$+1.9 \pm 0.5$
0.9	0.02	0.85	-2.4 ± 0.5
1.0	0.015	0.84	$+2.8 \pm 0.8$
1.0	0.02	0.84	-1.5 ± 0.7

the values of C_p and W_0 (or to show that a different Hamiltonian is needed if no fit can be found).

C. Densities and polarization of ^{16}O core

Figure 2 shows point nucleon and Λ densities for the calculations of Tables I and II. The density of ^{16}O is also shown. For the case with no $V_{\Lambda NN}$, the nuclear correlation parameters were not changed from those used in ^{16}O. The resulting ^{16}O density is, however, reduced near the origin and somewhat more peaked at $r = 1.4$ fm. This is presumably due to the repulsive $f_{\Lambda N}$ which pushes the nucleons away from the Λ which is strongly localized near the origin.

With $V_{\Lambda NN}$, the NN quenching parameter was significantly reduced. This results in a slightly more repulsive f_c (α does not quench the central part of V_{NN}) and so the nuclear density is reduced for $r \lesssim 2.2$ fm. The nuclear kinetic and potential energies (see Tables I and II) for the no $V_{\Lambda NN}$ case are separately larger in magnitude due to the higher density. It is probably accidental that the total nucleon energies for the two cases are so nearly the same: $-90(2)$ MeV. This value is 11 MeV less than the corresponding binding energy of ^{16}O, showing that the Λ significantly reduces the binding of the nucleons.

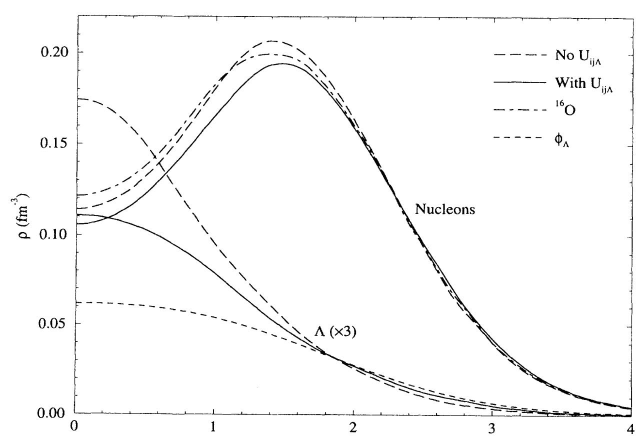

FIG. 2. One-body densities for the nucleons and Λ in $^{17}_{\Lambda}$O, and for ^{16}O. The short-dashed curve is the density corresponding to just ϕ_Λ.

The density profile of the Λ for the two cases is also shown in Fig. 2, along with the density corresponding to the one-body part of Ψ, i.e., $|\phi_\Lambda(r)|^2/4\pi$. In both cases the Jastrow part of Ψ, i.e., $\Pi f_c^\Lambda(r_{i\Lambda})$, significantly increases the Λ density at the origin. This is presumably because f_c^Λ is small for $r_{i\Lambda} \to 0$ and hence pushes the Λ away from the high nuclear density around $r = 1.4$ fm. Because this density is larger for $V_{\Lambda NN} = 0$, the central Λ density is also larger in this case.

VII. CONCLUSIONS

In this study, we have extended the cluster Monte Carlo technique developed in PWP for ^{16}O to $^{17}_{\Lambda}$O. The cluster contributions that involve the Λ seem to converge well. It thus seems sufficient to include terms up to four-body clusters in the calculation. These calculations have been performed for a number of sets of W_0 and C_p which will be helpful in determining the parameters of the three-body interaction $V_{ij\Lambda}$ by fitting the B_Λ values of $^{17}_{\Lambda}$O and other hypernuclei. The present calculations show that the use of noncentral NN, NNN, $N\Lambda$, and $NN\Lambda$ correlations completely change the expectation value of the three-baryon ΛNN interaction found with central wave functions, and thus have a strong effect on the choice of the parameters W_0 and C_p. With such correlations, reasonable values of C_p and W_0 give the correct $B_\Lambda(^{17}_{\Lambda}\mathrm{O})$. We also find that the Λ significantly changes the density profile and energy of the 16 nucleons in $^{17}_{\Lambda}$O; the ΛNN potential is particularly significant in this regard.

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