Threshold parameters of the $K\overline{K}$ and $\pi\pi$ scalar-isoscalar interactions

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Threshold expansions of the $K\overline{K}$ and $\pi\pi$ spin-0 isospin-0 scattering amplitudes are performed. Scattering lengths, effective ranges, and so-called volume parameters are evaluated in both channels. It is shown that the M-matrix expansion near the $K\overline{K}$ threshold has a larger convergence radius than the usually used effective-range expansion. The importance of future accurate measurements of the $K\overline{K}$-threshold parameters is stressed.

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Kaon-antikaon interactions are very poorly known. A characteristic feature of $K\overline{K}$ interactions is the presence of the annihilation processes in which the creation of $\pi\pi$ pairs plays a very important role. Thus the $K\overline{K}$ and $\pi\pi$ channels are coupled together and should be treated simultaneously. Our knowledge of the meson-meson interactions is based mainly on those reactions in which kaon pairs or pion pairs are produced. The production processes of the scalar mesons $f_0(975)$ and $a_0(980)$ (which both decay into $K\overline{K}$ pairs) have been studied in many experiments [1,2] and new experiments like those at COSY (Jülich) [3], DAΦNE (Frascati) [4,5], and CEBAF (Newport News) are planned. Unfortunately the existing $K\overline{K}$ and $\pi\pi$ data are not sufficiently precise to construct a unique model explaining the nature of the poorly known scalar mesons. Therefore different theoretical approaches to this question exist (see, for example, Refs. [6–12]).

In order to compare different models of the $K\overline{K}$ interactions we propose to calculate in future for each theoretical framework the low-energy $K\overline{K}$ parameters using the effective-range approximation known, for example, from the studies of the nucleon-nucleon interactions [13]. A natural and very advantageous generalization of the effective-range approximation in the single channel is the expansion of the M-matrix elements describing the coupled $K\overline{K}$ and $\pi\pi$ channels. The threshold parameters are crucial in understanding the nature of the K-$\overline{K}$ interactions. They could be very helpful to interpret future experimental results on the $K\overline{K}$ production just above the threshold. The importance of computing the threshold parameters has also been recently stressed by Törnqvist [14].

The masses of the $f_0(975)$ and $a_0(980)$ mesons are very close to the $K\overline{K}$ threshold. Therefore these mesons are frequently interpreted as the quasibound states of the $K\overline{K}$ pairs [15–19]. In Ref. [8] the $K\overline{K}$ scalar-isoscalar scattering length has been calculated already using a separable potential formalism. Then in Ref. [20] its value has been used to discuss the kaonium production. More recently we have extended the calculations of the scalar-isoscalar $K\overline{K}$ and $\pi\pi$ scattering amplitudes using the relativistic approach [9]. A simple rank-1 separable potential has been used to describe the $K\overline{K}$ interaction and a rank-2 potential in the $\pi\pi$ channel. Choosing the rank-2 potential responsible for the coupling of two channels, we have obtained very good fits to the data starting from the $\pi\pi$ threshold up to 1400 MeV, thus fully covering the interesting region of the $K\overline{K}$ threshold near 1 GeV [21]. In this procedure we have been able to fix the parameters of the meson-meson interactions. In such a way the two-channel unitary model has been completed. Then analyzing the analytical structure of the $\pi\pi$ amplitude we have obtained the parameters of the scalar mesons $f_0(975)$ and $f_0(1400)$.

As a next step we report the calculations of the threshold parameters for the $K\overline{K}$ and $\pi\pi$ interactions in the spin- and isospin-zero state. Our main aim is to obtain the $K\overline{K}$ parameters largely unknown experimentally, but as a further test of our model we also perform the calculations near the $\pi\pi$ threshold where more experimental data already exist [22]. There are different methods to parametrize the low-momentum behavior of the scattering amplitude. We use the effective-range expansion in the $\pi\pi$ and $K\overline{K}$ channels:

$$k\cot\delta = \frac{1}{a} + \frac{1}{2}rk^2 + vk^4 + O(k^6), \tag{1}$$

where δ is the scattering phase shift, k is the meson momentum in the c.m. system, a is the scattering length, r is the effective range, and the so-called volume parameter v can be related to the shape of the intermeson potentials.

The low-momentum expansion of the phase shift has a polynomial form:

$$\delta = \alpha k + \beta k^3 + \gamma k^5 + O(k^7). \tag{2}$$

The coefficients α, β, γ can be obtained from the low-momentum expansion of the scattering amplitudes calculated in Ref. [9].

Above the $K\overline{K}$ threshold we define the complex $K\overline{K}$ phase shift $\delta = \delta_K + i\rho$, where δ_K is the $K\overline{K}$ phase shift and ρ is related to the inelasticity parameter

$$\eta = e^{-2\rho}. \tag{3}$$

In the $K\overline{K}$ channel the expansions (1) and (2) can still be valid if we make the parameters a, r, v and α, β, γ complex.

The $K\overline{K}$ effective-range parameters are given in Table I corresponding to two fits to two sets of experimental data analyzed in [9]. These data sets differ qualitatively

TABLE I. Low-momentum parameters of the $K\overline{K}$ scalar, $I = 0$ scattering.

Set no.	a_K (fm)	r_K (fm)	v_K (fm^3)	R_K (fm)	V_K (fm^3)
1	$-1.73 + i\,0.59$	$-0.057 + i\,0.032$	$0.016 - i\,0.0044$	0.38	-0.66
2	$-1.58 + i\,0.61$	$-0.352 + i\,0.043$	$0.028 - i\,0.0057$	0.20	-0.83

in the vicinity of the KK threshold as shown in Fig. 3 of [9]. The $K\overline{K}$ phase shifts tend to decrease at threshold for the set 1 [23] and increase for the set 2 [24]. This apparent controversy between the different experimental analyses should be removed by future more accurate measurements. The model [9] describes the data set 1 better than the set 2. We present the results for two data sets in order to show the sensitivity of the threshold parameters to the experimental uncertainties. Let us notice that at least four real parameters have to be phenomenologically determined in the $K\overline{K}$ channel under the condition that one uses only two terms of the effective-range expansion (1). This is in contrast to the case of the low-energy proton-neutron scattering in the $^3S_1$ state (as discussed by Törnqvist in Ref. [14]) since in the latter case the scattering is purely elastic and two parameters are sufficient.

The $K\overline{K}$ scattering length is complex in the presence of the open annihilation channel. Its real part is negative and rather large. The imaginary part is positive and gets a value about 0.6 fm. As seen in Table I the expansion parameters r and v are rather small. This is not accidental and can be easily understood if one notices the fact that the S-matrix pole $f_0(975)$ is very close to the $K\overline{K}$ threshold. Its position in the $K\overline{K}$ momentum frame is $p_0 = (-34.7 + i\,100.3)$ MeV for the set 1 and $p_0 = (-36.1 + i\,100.2)$ MeV for the set 2. If we approximate the $K\overline{K}$ element of the S matrix by its dominant pole contribution,

$$S^{\text{pole}}_{K\overline{K}} = \frac{-k - p_0}{k - p_0}, \tag{4}$$

then the $K\overline{K}$ scattering length is $a_0 = (ip_0)^{-1}$ (see also Ref. [21]) and all other parameters of the threshold expansion of $k\cot\delta$ identically vanish since $k\cot\delta \equiv 1/a_0$. Therefore in the single $f_0(975)$ pole approximation the parameters r_K and v_K are zero. Their smallness in the full model calculation is a reflection of the $f_0(975)$ dominance near the $K\overline{K}$ threshold. The values a_0 are $(-1.76 + i\,0.61)$ fm for the set 1 and $(-1.74 + i\,0.63)$ fm for the set 2; they are quite close to the values a_K given in Table I especially for the set 1 preferred by our model. The negative sign of $\mathrm{Re}a_K$ is characteristic for the appearance of a bound $K\overline{K}$ state $f_0(975)$. We have studied the accuracy of the pole approximation (4) in comparison with the results calculated from the complete model. For the model parameters fitted to the data set 1, both the $K\overline{K}$ phase shifts and the inelasticity are reproduced with a precision better than 2% for the $K\overline{K}$ momenta as large as 380 MeV/c (or the effective mass as high as 1250 MeV). For the set 2 the inelasticity parameter is described within 3% up to 450 MeV/c but the phase shifts are less accurately reproduced (to 11% at the threshold and up to 17% at 400 MeV/c). At energies higher than 1250 MeV the $f_0(1400)$ resonance plays an important role and gives an additional contribution to the $f_0(975)$ term.

In Table I we have introduced two additional real parameters R_K and V_K entering into the familiar expansion valid for the real δ_K:

$$k\cot\delta_K = \frac{1}{\mathrm{Re}\,a_K} + \frac{1}{2}R_K k^2 + V_K k^4 + O(k^6). \tag{5}$$

These parameters are not independent of a_K, r_K, and v_K but have been introduced for a further comparison with the corresponding parameters in the $\pi\pi$ channel. The $K\overline{K}$ effective-range parameter R_K is relatively small in comparison with $|\mathrm{Re}\ a_K|$. The contribution of the $f_0(975)$ pole to R_K and the third parameter V_K shown in Table I is also dominant. In this approximation both parameters R_K and V_K can be expressed in terms of $\mathrm{Re}a_K$ and $\mathrm{Im}a_K$. If the kaon momentum increases then the higher terms in the threshold expansion become important. The convergence radius of the expansions (2) and (5) is equal to a distance $|p_0|$ to the nearest S-matrix pole. The energy corresponding to $k = |p_0|$ is 1014 MeV, which is only 23 MeV above the $K\overline{K}$ threshold [throughout this paper we use the average kaon mass $m_K = \frac{1}{2}(m_{K^\pm} + m_{K^0}) \approx 495.69$ MeV]. Therefore one can draw a severe limit on the experimental energy resolution needed in the determination of the $K\overline{K}$ threshold parameters when using Eqs. (1) or (5). In practice one should require an energy resolution of the order of 1 MeV.

According to our knowledge, experimental information about the $K\overline{K}$ threshold parameters is almost nonexistent. We are aware of only one pioneer experimental determination of the $K^0_S K^0_S$ scattering length by Wetzel *et al.* [25]. Although the values obtained by the authors of [25] ($|\ a\ | = 1.25 \pm 0.12$ fm, $\mathrm{Im}a = 0.27 \pm 0.03$ fm) are of the same order as our determinations, we think that their errors are too small. There are at least two reasons to believe that this observation is true: first, only two experimental points are used in the analysis for the $K\overline{K}$ effective mass smaller than 1.1 GeV, and secondly their parametrization of the $K\overline{K}$ phase shifts does not fulfill the general symmetry requirement $\delta_{K\overline{K}}(-k) = \delta_{K\overline{K}}(k)$. Nevertheless, these data seem to indicate the fact that the modulus of the $K\overline{K}$ scattering length is much larger than the $\pi\pi$ scattering one.

For a full description of the two coupled $\pi\pi$ and $K\overline{K}$ channels (including the $K\overline{K} \longrightarrow \pi\pi$ annihilation process) we introduce a real and symmetric matrix M related to the scattering matrix T by

$$M = T^{-1} + i\,\widehat{k}, \tag{6}$$

where $\widehat{k}$ is a diagonal 2×2 matrix of the $K\overline{K}$ and $\pi\pi$ momenta in the center-of-mass system. At the $K\overline{K}$ thresh-

TABLE II. M-matrix expansion parameters at the $K\overline{K}$ threshold.

Reaction channel	i	j	A_{ij} (fm^{-1})	B_{ij} (fm)	C_{ij} (fm^3)
$K\overline{K}$	1	1	-0.483	-8.10×10^{-2}	1.83×10^{-2}
$\pi\pi$	2	2	0.476	-1.58×10^{-1}	1.43×10^{-3}
$K\overline{K}\longleftrightarrow\pi\pi$	1	2	0.669	-1.57×10^{-2}	5.93×10^{-3}

old the M-matrix elements can be expanded as

$$M_{ij} = A_{ij} + \tfrac{1}{2}B_{ij}k_1^2 + C_{ij}k_1^4 + O(k_1^6), \qquad (7)$$

where A_{ij}, B_{ij}, and C_{ij} are real coefficients and k_1 is the $K\overline{K}$ momentum $(i,j=1,2)$. Every threshold parameter in the two channels introduced in Eq. (1) can be related to a set of the M_{ij} expansion parameters. For example, the complex $K\overline{K}$ scattering length is

$$a_K = \left(A_{11} - \frac{A_{12}^2}{A_{22}-iq}\right)^{-1}, \qquad (8)$$

where $q=(m_K^2-m_\pi^2)^{1/2}$ is the pion momentum at the $K\overline{K}$ threshold. We use the average pion mass $m_\pi = \frac{1}{2}(m_{\pi^\pm}+m_{\pi^0}) \approx 137.27$ MeV. The coefficients A_{ij}, B_{ij}, and C_{ij} are shown in Table II for the data set 1.

Let us stress the important fact that the expansion (7) of the M-matrix elements has a much larger convergence radius than the expansion (5). This radius in the kaon c.m. momentum space is equal to the kaon mass, which is a few times larger than the distance to the nearest pole $|p_0|$. Therefore the expansion (7) is particularly useful as a practical parametrization of the scattering matrix elements near the $K\overline{K}$ threshold.

For completeness let us discuss the $\pi\pi$ threshold parameters shown in Table III (theoretical errors of a_π correspond to an increase of the total χ^2 value by one unit as explained in [9]). The $\pi\pi$ scattering length is small and positive while the $\pi\pi$ effective-range is negative and much larger. This is in contrast with the $K\overline{K}$ case, where the real part of the scattering length is negative and the effective-range R_K is small. The above values point to an essential difference between the $\pi\pi$ and $K\overline{K}$ interactions. The third parameter (sometimes called the shape parameter) is positive in our model. In a recent analysis of the near threshold $\pi N \longrightarrow \pi\pi N$ data Počanić *et al.* [26] have provided the $\pi\pi$ scattering length $a=(0.177\pm0.006)m_\pi^{-1}$, which is in a very good agreement with our evaluations [9] (compare the second column of Table III). In earlier analyses, Lowe *et al.* [27] and Burkhardt and Lowe [28] have given the $\pi\pi$ scattering length values $(0.207\pm0.028)m_\pi^{-1}$ and $(0.197\pm0.01)m_\pi^{-1}$, respectively. Using the chiral perturbation theory, Gasser and Leutwyler [29] obtain a value $(0.20\pm0.01)m_\pi^{-1}$ which, if recalculated using our average pion mass $m_\pi = 137.27$ MeV, becomes $(0.184\pm0.008)m_\pi^{-1}$. In a recent paper by Roberts *et al.* [30] the calculated values of the scattering length are $0.16m_\pi^{-1}$ or $0.17m_\pi^{-1}$.

TABLE III. Low-momentum parameters of the $\pi\pi$ scalar, $I=0$ scattering.

Set no.	$a_\pi(m_\pi^{-1})$	$r_\pi(m_\pi^{-1})$	$v_\pi(m_\pi^{-3})$
1	0.172 ± 0.008	-8.60	3.28
2	0.174 ± 0.008	-8.51	3.25

The $\pi\pi$ effective range is not well determined experimentally. Belkov *et al.* [31] have obtained $r_\pi = (-9.6\pm19.1)m_\pi^{-1}$. Based on the analysis of the $\pi^-p\longrightarrow\pi^+\pi^-n$ data performed by Belkov and Bunyatov [32], we have derived the value of the effective range $r_\pi=-8.1m_\pi^{-1}$ with an estimated error at least 65%. Within the Weinberg approach [33] the parameter $r_\pi=-8.48m_\pi^{-1}$ which is very close to our values, about $-8.6m_\pi^{-1}$ or $-8.5m_\pi^{-1}$, given in Table III (the scattering length used in the Weinberg model was $0.157m_\pi^{-1}$). The effective range $(-7.4\pm2.5)m_\pi^{-1}$ can be obtained from two low-energy parameters a and b predicted in Ref. [29]. It is also possible to evaluate the effective range from the similar parameters fitted to the $\pi\pi$ phase shifts by Rosselet *et al.* [34] in the study of the K_{e4} decays $[a=0.28\pm0.05$, $b=0.19-(a-0.15)^2]$. Its value is $(-2.4\pm4.0)m_\pi^{-1}$, which differs considerably from the above cited value -8.5 fm. Another estimation based on the same data using a and b as free parameters leads to a different value $r_\pi=-0.5^{+5.7}_{-10.3}m_\pi^{-1}$. We infer from these numbers that the existing $\pi\pi$ data are not yet substantially accurate to determine the effective range with good precision.

In conclusion, we have determined the effective-range parameters of the $K\overline{K}$ and $\pi\pi$ scalar-isoscalar interactions. Good agreement with the existing experimental data for the $\pi\pi$ amplitude has been found. We have also calculated the M-matrix expansion parameters near the $K\overline{K}$ threshold, showing that this expansion has a particularly large convergence radius. Since the available information about the threshold behavior of the $K\overline{K}$ phase shifts is very controversial, it is clear that new precise data should be taken at energies sufficiently close to the $K\overline{K}$ threshold. This is important not only for a precise determination of the low-energy $K\overline{K}$ interactions but also for understanding the nature of the scalar mesons.

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