Relativistic nuclear matter with alternative derivative coupling models

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Effective Lagrangians involving nucleons coupled to scalar and vector fields are investigated within the framework of relativistic mean-field theory. The study presents the traditional Walecka model and different kinds of scalar derivative couplings suggested by Zimanyi and Moszkowski. The incompressibility (presented in an analytical form), scalar potential, and vector potential at the saturation point of nuclear matter are compared for these models. The real optical potential for the models are calculated and one of the models fits well the experimental curve from −50 to 400 MeV while also giving a soft equation of state. By varying the coupling constants and keeping the saturation point of nuclear matter approximately fixed, only the Walecka model presents a first-order phase transition for finite temperature at zero density.

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I. INTRODUCTION

To study the properties of hadronic matter, Walecka [1] has proposed a simple renormalizable model based on field theory which is often referred to as quantum hadrodynamics (QHD-I). In this model nucleons interact through the exchange of σ and ω mesons the σ simulating medium-range attraction and the ω simulating short-range repulsion. The usual approach to solve this model is the mean-field approximation, in which the meson fields are replaced by their expectation values. The Walecka model (W) has achieved important goals in the description of hadronic matter as, for example, some bulk properties of nuclear matter as well as some properties of finite nuclei. An interesting one was to show us the relativistic mechanism for nuclear matter saturation: it occurs at a density (ρ_0) at which the scalar (S) and the vector (V) potentials largely cancel one other. Extended to finite nuclei, this model predicts a reasonable noncentral spin-orbit splitting contribution given mainly by $V-S$. However the incompressibility (K) calculated from this model [2] indicates the equation of state to be stiffer than the expected one and at moderately high density and/or temperature the effective mass of the nucleons becomes very small. Many attempts have been made to improve this model, among them let us quote here: inclusion of one-nucleon-loop vacuum effects [3], inclusion of two-nucleon-loop effects [4], and addition of nonlinear cubic and quartic scalar meson interactions in the Lagrangian [5].

Recently Zimanyi and Moszkowski (ZM) have proposed models for hadronic matter differing from the Walecka model only in the form of the coupling of the nucleon to the scalar meson [6]. To avoid any confusion let us explain what we understand for ZM models. The usual one is referred in the literature as the ZM model or derivative scalar coupling model (DSC) and consists in including derivative coupling between nucleons and scalar mesons. It yields a $K = 224.49$ MeV and an effective mass $M^* = 797.64$ MeV at the nuclear matter saturation point. This model has already been applied to investigate the following problems: multilambda matter properties [7], neutron star [8], Δ-excited nuclear matter [9], and some thermodynamical properties of nuclear matter [10]. In the Appendix of [6], ZM pointed out that the specific form of the coupling between the nucleon and scalar meson is somewhat arbitrary and it would not be only possible to extend the scalar coupling to the nucleon derivative term, but also to the interaction between nucleon and vector meson. The different proposed scalar couplings to the vector meson we refer to in this paper as ZM2 and ZM3 models. In a very concise description of them, ZM models can be understwood as models which introduce a nonlinear effective scalar coupling constant $g_\sigma^* = g_\sigma m^* = g_\sigma(1 + g_\sigma \sigma/M)^{-1}$ in the Walecka model. This nonlinear effective coupling constant can then act: (a) on the nucleon derivative and the nucleon-vector coupling terms (ZM), (b) on the nucleon derivative term and on all terms involving the vector field (ZM2), and (c) only on the nucleon derivative term (ZM3). Constructed in that way, after an appropriate rescaling, all the Lagrangians will describe the motion of a baryon with mass $M^* = m^* M$ instead of the bare mass M. This is the main idea of the ZM models, where m^* has the particular form given above. This information manifests itself as different forms for the meson-baryon couplings in the above three models. In case (a), the rescaled Lagrangian presents an effective scalar-baryon coupling, and does not change the vector-baryon coupling at all. This is the usual ZM model. The motivation to study cases (b) and (c) is that, now the rescaled Lagrangians present not only effective scalar-baryon coupling but also, through m^*, different forms of effective vector-baryon coupling, which will couple the scalar field to the vector field.

In this paper we shall be committed to the standard Walecka model and the three above-mentioned ZM models. Each one of these four models is very simple since they have only two free parameters, the scalar (vector) coupling constants C_σ^2 (C_ω^2), adjusted to reproduce the binding energy (E_b) of the nuclear matter at $\rho = \rho_0$.

ZM2 and ZM3 models are not well known in the literature and it seems important to enhance the knowledge of hadronic models by investigating whether or not they bring some new features in comparison to the others. Besides, these models provide an excellent theoretical laboratory to analyze the sensitivity of nuclear observables on different kinds of field couplings. Therefore a kind of anatomy of these models is needed. First we put the four models in a unified rescaled Lagrangian form. The fitting of C_σ^2 and C_ω^2 is presented in the coupling-constant plane exhibiting the sensitivity of them under small variations of E_b and ρ_0. From this study it turns out that none of the ZM models presents phase transition for finite temperature at $\rho = 0$ as opposed to the Walecka model [11]. Furthermore, in order to establish a meaningful comparison we have also calculated S, V, and K. For finite nuclei $V - S$ gives us roughly the strength of the spin-orbit splitting while $V + S$ indicates the real part of the optical potential for zero three-momentum. Since these three quantities K, $V - S$, and $V + S$ are related directly to observables, none of them alone can establish the goodness of a particular model. Our results show a strong sensitivity of K and $V - S$ on change of the coupling. Let us also remark upon an important point regarding how $M^*(\rho_0)$ behaves as a function of K. The thought that the stiffest equation of state is obtained for the lowest effective ground-state mass suggested by many calculations [5,12] is not supported from our calculations with ZM2 and ZM3 models. It means that these two models are bringing a new qualitative feature regarding $M^*(\rho_0)$ versus K. For example, we find for ZM3 $M^*(\rho_0)$ = 671.25 MeV and K = 155.43 MeV while the ZM model gives $M^*(\rho_0)$ = 797.64 MeV and K = 224.49 MeV. We have also calculated the real part of the optical potential for the models and the calculations have shown that the ZM3 model fits the experimental curve quite well from −50 to 400 MeV.

II. THE MODELS

Since the models we are dealing with were presented in detail in [1,6], let us go to their main points. The degrees of freedom are baryon fields (ψ), scalar meson fields (σ), and vector meson fields (ω). The Lagrangian densities are given by

$$\begin{aligned}\mathcal{L}_{\mathrm{W}} = {} & \bar{\psi} i\gamma_\mu \partial^\mu \psi - \bar{\psi} M \psi + \tfrac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) \\ & + g_\sigma \sigma \bar{\psi}\psi - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + \tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu \\ & - g_\omega \bar{\psi}\gamma_\mu\psi\omega^\mu, \end{aligned} \tag{1}$$

$$\begin{aligned}\mathcal{L}_{\mathrm{ZM}} = {} & -\bar{\psi} M \psi + m^{*-1}[\bar{\psi} i\gamma_\mu \partial^\mu \psi - g_\omega \bar{\psi}\gamma_\mu\psi\omega^\mu] \\ & - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + \tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu \\ & + \tfrac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2), \end{aligned} \tag{2}$$

$$\begin{aligned}\mathcal{L}_{\mathrm{ZM2}} = {} & -\bar{\psi} M \psi + m^{*-1}[\bar{\psi} i\gamma_\mu \partial^\mu \psi - g_\omega \bar{\psi}\gamma_\mu\psi\omega^\mu \\ & - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + \tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu] \\ & + \tfrac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2), \end{aligned} \tag{3}$$

$$\begin{aligned}\mathcal{L}_{\mathrm{ZM3}} = {} & -\bar{\psi} M \psi + m^{*-1}\bar{\psi} i\gamma_\mu \partial^\mu \psi - g_\omega \bar{\psi}\gamma_\mu\psi\omega^\mu \\ & - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + \tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu \\ & + \tfrac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2), \end{aligned} \tag{4}$$

where

$$m^* = (1 + g_\sigma\sigma/M)^{-1}, \tag{5}$$

$F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$, and M is the bare nucleon mass. Now we proceed to rescale the fields ψ and ω_μ as follows: $\psi \to m^{*1/2}\psi$ for all ZM's models and $\omega_\mu \to m^*\omega_\mu$ for ZM2 and ZM3 models. Considering that m^* does not depend on space-time coordinates (nuclear matter case), the rescaled Lagrangian densities acquire a unified form:

$$\begin{aligned}\mathcal{L}_R = {} & \bar{\psi} i\gamma_\mu \partial^\mu \psi - \bar{\psi}(M - {m^*}^\beta g_\sigma\sigma)\psi \\ & + {m^*}^\alpha\,[-g_\omega \bar{\psi}\gamma_\mu\psi\omega^\mu - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + \tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu] \\ & + \tfrac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2), \end{aligned} \tag{6}$$

where α and β have the following association to the models:

$$\mathrm{W} : \alpha = 0, \beta = 0, \tag{7a}$$

$$\mathrm{ZM} : \alpha = 0, \beta = 1, \tag{7b}$$

$$\mathrm{ZM2} : \alpha = 1, \beta = 1, \tag{7c}$$

$$\mathrm{ZM3} : \alpha = 2, \beta = 1. \tag{7d}$$

From Eq. (6) we obtain the coupled equations of motion for the fields of nucleon and mesons:

$$[i\gamma_\mu\partial^\mu - (M - {m^*}^\beta g_\sigma\sigma) - {m^*}^\alpha g_\omega\gamma_\mu\omega^\mu]\psi = 0, \tag{8}$$

$$\partial_\nu F^{\nu\mu} + m_\omega^2\omega^\mu = g_\omega\bar{\psi}\gamma^\mu\psi, \tag{9}$$

$$\begin{aligned}(\partial_\mu\partial^\mu + m_\sigma^2)\sigma = {} & g_\sigma {m^*}^\beta[1 - \beta(1 - m^*)]\bar{\psi}\psi \\ & - \frac{\alpha}{M} g_\sigma {m^*}^{\alpha+1}[-g_\omega\bar{\psi}\gamma_\mu\psi\omega^\mu \\ & - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + \tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu]\ . \end{aligned} \tag{10}$$

When the meson fields are replaced by the constant classical fields σ_0 and ω_0 we arrive at the mean-field approximation with the equations

$$\omega_0 = \frac{g_\omega}{m_\omega^2}\langle\psi^+\psi\rangle = \frac{g_w}{m_\omega^2}\rho_B, \tag{11}$$

$$\sigma_0 = \frac{g_\sigma}{m_\sigma^2}{m^*}^{2\beta}\langle\bar{\psi}\psi\rangle + \frac{\alpha}{2}\left(\frac{m_\omega^2}{m_\sigma^2}\right)\frac{g_\sigma}{M}{m^*}^{\alpha+1}\omega_0^2 \tag{12}$$

$$= \frac{g_\sigma}{m_\sigma^2}m^{*2\beta}\langle\bar{\psi}\psi\rangle + \frac{\alpha}{2}\frac{C_\sigma^2 C_\omega^2}{g_\sigma M^5}{m^*}^{\alpha+1}\rho_B^2, \tag{13}$$

where we have introduced $C_\sigma^2 = g_\sigma^2 M^2/m_\sigma^2$ and $C_\omega^2 = g_\omega^2 M^2/m_\omega^2$, and also identified $[1-\beta(1-m^*)] = m^{*\beta}$ which is valid to $\beta = 0$ and $\beta = 1$. The constant classical fields σ_0 and ω_0 are thus directly related to the baryon sources. The source for ω_0 is simply the baryon density $\rho_B = \frac{B}{V}$, which is a constant of the motion for a uniform system of B baryons in a volume V. The source for σ_0 involves the expectation value of the Lorentz scalar density $\bar\psi\psi = \rho_s$. Notice however that for the cases where α is not identically zero (ZM2 and ZM3 models) σ_0 is also coupled to ρ_B. Any way σ_0 is, for any case a manifest scalar.

Now we proceed to define the scalar potential (S) and the vector potential (V). It can be done by looking at the Dirac equation for the models, Eq. (8), and by rewritting M^* in the form

$$M^* = M - g_\sigma\sigma\left(1+\frac{g_\sigma\sigma}{M}\right)^{-\beta} = M - m^{*^\beta} g_\sigma\sigma. \quad (14)$$

If we interpret M^* in the Dirac equation as an effective mass shifted by S ($M^* = M + S$), the natural way to define S is

$$S = -m^{*^\beta} g_\sigma\sigma = -m^{*^\beta} S_0 = -\frac{S_0}{(1+\frac{S_0}{M})^\beta},$$
$$S_0 = g_\sigma\sigma\ . \quad (15)$$

Here we call attention to the fact that Zimanyi and Moszkowski have defined S, (Eq. (12) of [6]) in a slightly different fashion; namely,

$$S = S_0 = g_\sigma\sigma\ ,\ m^* = \frac{1}{(1+\frac{S_0}{M})}. \quad (16)$$

Equations (15) and (16) agree at the first order in S_0, the case where the Walecka and ZM models are equivalents. In the next section we will give the values of S and S_0 for models ZM, ZM2, and ZM3 in order to see the impact of the higher order terms in the scalar coupling present in m^*. Still analyzing the Dirac equation we see that V can be defined as a quantity which shifts the energy,

$$V = m^{*^\alpha} g_\omega\omega_0 = \left(\frac{1}{1+\frac{g_\sigma\sigma}{M}}\right)^\alpha g_\omega\omega_0. \quad (17)$$

This definition recovers the usual one for the Walecka model and the ZM model ($\alpha = 0$). For ($\alpha \neq 0$) V is coupled to the scalar field. The expressions for the energy density and pressure at a given temperature T can be found as usual by the average of the energy-momentum tensor,

$$\mathcal{E} = \frac{C_\omega^2}{2M^2} m^{*^\alpha}\rho_B^2 + \frac{M^4}{2C_\sigma^2}\left(\frac{1-m^*}{m^{*\beta}}\right)^2$$
$$+\frac{\gamma}{(2\pi)^3}\int d^3k\, E^*(k)(n_k+\bar n_k) \quad (18)$$

and

$$p = \frac{C_\omega^2}{2M^2} m^{*^\alpha}\rho_B^2 - \frac{M^4}{2C_\sigma^2}\left(\frac{1-m^*}{m^{*^\beta}}\right)^2$$
$$+\frac{1}{3}\frac{\gamma}{(2\pi)^3}\int d^3k\,\frac{k^2}{E^*(k)}(n_k+\bar n_k), \quad (19)$$

where

$$\rho_B = \frac{\gamma}{(2\pi)^3}\int d^3k\,(n_k - \bar n_k). \quad (20)$$

Here γ is the degeneracy factor ($\gamma = 4$ for nuclear matter and $\gamma = 2$ for neutron matter), n_k and $\bar n_k$ stand for the Fermi-Dirac distribution for baryons and antibaryons with arguments $(E^* - \nu)/T$, respectively. $E^*(k)$ is given by

$$E^*(k) = (k^2 + M^{*2})^{\frac{1}{2}}, \quad (21)$$

while an effective chemical potential which preserves the number of baryons and antibaryons in the ensemble, is defined by

$$\nu = \mu - V, \quad (22)$$

where μ is the thermodynamical chemical potential. The expression for the entropy per volume (s) can be obtained from the thermodynamical potential (Ω),

$$\frac{\Omega}{V} = -p = \mathcal{E} - Ts - \mu\rho_B. \quad (23)$$

The gap equation already presented by Eqs. (11)–(13) is obtained explicitly through the minimization of $\mathcal{E}$ in relation to m^*. It reads

$$1 - m^* - \frac{C_\sigma^2\gamma}{2\pi^2} m^{*^{3\beta+1}}\int \frac{x^2dx}{(x^2+m^{*2})^{\frac{1}{2}}}(n_x+\bar n_x)$$
$$+\frac{\alpha}{2}\frac{C_\sigma^2 C_\omega^2}{M^6} m^{*^{\alpha+\beta+1}}\rho_B^2 = 0\ , \quad (24)$$

where we have introduced the dimensionless variable $x = \frac{k}{M}$. This equation has to be solved self-consistently and provides the basis for obtaining all kinds of thermodynamical quantities in the mean-field approach we are using.

At zero temperature limit the ground state is obtained by filling energy levels up to a Fermi surface K_F, and so n_k goes to $\Theta(K_F - k)$ and $\bar n_k$ vanishes. Now we are in a position to calculate the incompressibility. At $T = 0$ its definition is given by

$$K = 9\rho_0^2\frac{\partial^2}{\partial\rho^2}\left(\frac{\mathcal{E}}{\rho}\right)\bigg|_{\rho=\rho_0} = 9\rho_0\frac{\partial^2\mathcal{E}}{\partial\rho^2}\bigg|_{\rho=\rho_0}, \quad (25)$$

where ρ means the baryon density ρ_B. This expression can be analytically obtained for the models we are dealing with. Its derivation is straightforward but tedious and we give the details in the Appendix. The closed form is

$$K = 9\frac{C_\omega^2}{M^2}m^{*^\alpha}\rho|_{\rho=\rho_0} + 3\frac{K_F^2}{E^*(k)}\Bigg|_{\rho=\rho_0} + 9\left[\rho\frac{\partial m^*}{\partial\rho}\left(M^2\frac{m^*}{E^*(k)} + \alpha m^{*^{\alpha-1}}\frac{C_\omega^2}{M^2}\rho\right)\right]\Bigg|_{\rho=\rho_0}, \tag{26a}$$

or the equivalent one, by using the definitions of S and V,

$$K = 9V|_{\rho=\rho_0} + 3\frac{K_F^2}{[K_F^2+(M+S)^2]^{1/2}}\Bigg|_{\rho=\rho_0} + 9\left[\rho\frac{\partial S}{\partial\rho}\left(\frac{(M+S)}{[K_F^2+(M+S)^2]^{1/2}} + \alpha\frac{V}{(M+S)}\right)\right]\Bigg|_{\rho=\rho_0}, \tag{26b}$$

where

$$\frac{\partial m^*}{\partial\rho} = -\frac{\left(\frac{m^*}{E^*(k)}m^{*^{2\beta}} + \alpha\frac{C_\omega^2}{M^4}m^{*^{\alpha+1}}\rho\right)}{\left[\frac{M^2}{C_\sigma^2}\left(\frac{m^*+3\beta(1-m^*)}{m^{*\beta+1}}\right)\right] + \frac{\alpha}{2}(\alpha+1-2\beta)\frac{C_\omega^2}{M^4}m^{*\alpha}\rho^2 + m^{*^{2\beta}}3\left(\frac{\rho_s}{M^*} - \frac{\rho}{E^*(k)}\right)\Bigg]} \tag{27}$$

and

$$\rho_s = \frac{M^3}{C_s^2}\frac{\left(\frac{(1-m^*)}{m^{*\beta}} - \frac{\alpha}{2}\frac{C_\sigma^2C_\omega^2}{M^6}m^{*^{\alpha+1}}\rho^2\right)}{m^{*^{2\beta}}}. \tag{28}$$

Let us define K_1, K_2, and K_3 as the first, second, and third terms of the incompressibility, respectively. Notice that K_1 and K_2 are always positive while K_3 is negative due the derivative of m^* with respect to ρ. It is really very interesting to see how the different contributions sum up to balance the final result of the incompressibility. In particular K_3 exhibits in a very clear form that it depends essentially on $\frac{\partial m^*}{\partial\rho}$ at the saturation point of the nuclear matter. Of course as we can see from Eqs. (26a) and (26b), this term also involves coupling constants and other quantities so that a smaller value of $\frac{\partial m^*}{\partial\rho}$ does not necessarily indicate a smaller value for the incompressibility. K_1 is directly proportional to V and K_2 comes from the baryon gas.

III. THE REAL PART OF THE OPTICAL POTENTIAL

Since we have already S and V, the natural way to proceed further comparing the models is the construction of an optical potential. Let us start defining the real part of the optical potential as the difference between the total (E) and kinetic energies of a nucleon traveling in nuclear matter with momentum $\mathbf{p}$:

$$U_{\rm OPT} = E - (\mathbf{p}^2 + M^2)^{1/2}. \tag{29}$$

The equation of motion for the nucleon [Dirac equation, Eq. (8)] supports the understanding that in the nuclear medium this nucleon acquires an effective mass $M^* = M + S$ with a shifted energy $E - V$. Therefore

$$[\mathbf{p}^2 + (M+S)^2]^{1/2} = E - V. \tag{30}$$

If we now substitute the value of $\mathbf{p}^2$ given by the above constraint into Eq. (29) we end up with

$$U_{\rm OPT} = E - [(E-V)^2 - S(2M+S)]^{1/2}. \tag{31}$$

This definition of $U_{\rm OPT}$ is exactly the same used by Feldmeier and Lindner [12] . For zero momentum $U_{\rm OPT}$ goes to $S+V$ while in the ultrarelativistic energy regime $U_{\rm OPT}$ goes to V. Notice that this definition differs slightly from the linear definition widely used in the literature (see [1] and references therein):

$$U_{\rm LIN} = (1/2M)[V(2E-V) + S(2M+S)]. \tag{32}$$

These definitions can be related through

$$U_{\rm OPT} = E\left\{1 - [1 - (2M/E^2)U_{\rm LIN}]^{1/2}\right\}. \tag{33}$$

For zero momentum, both definitions give the same result. In this paper we will assume $U_{\rm OPT}$ defined by Eq. (31). Nevertheless we will present in the next section a sample of numerical comparisons between $U_{\rm OPT}$ and $U_{\rm LIN}$ for a specific model. A more detailed theoretical discussion regarding both definitions is well presented in Ref. [12].

IV. RESULTS AND DISCUSSIONS

The coupling constants for the models are presented in Table I. For the Walecka model we have used the values given in [2], which are slightly different from those given in [1]. For the ZM model we have used those given in [6] and since this reference does not present the values for the models we named here as ZM2 and ZM3 models we have to calculate them. It has been done by solving Eq. (24) and fitting C_σ^2 and C_ω^2 to reproduce E_b and ρ_0, and these values are also given in Table I. Figure 1 presents how E_b varies with the density (ρ) for the models. The behavior of M^* in terms of ρ is given in Fig. 2. In order to have a qualitative idea about how stiff or

TABLE I. Coupling constants C_σ^2 and C_ω^2; binding energy E_b (MeV) at the density $\rho_0(\text{fm}^{-3})$, $\rho_s(\text{fm}^{-3})$, and m^* for the indicated models.

Models	C_σ^2	C_ω^2	E_b	ρ_0	ρ_s	m^*
Walecka	357.4	273.8	-15.75	0.148	0.138	0.54
ZM	169.2	59.1	-15.9	0.160	0.155	0.85
ZM2	219.3	100.5	-15.77	0.152	0.147	0.82
ZM3	443.3	305.5	-15.76	0.149	0.143	0.72

soft the equation of state is for each model we present in Fig. 3 the pressure (p) versus ρ. Since the incompressibility is related directly to $\partial p/\partial \rho$ we see directly that the ZM3 model gives the softest equation of state. Simultaneously from Fig. 2 (or Table I) we also see that it does not correspond to the largest value of $M^*(\rho_0)$. The first conclusion is that when the scalar field couples to the vector field [see Eq. 6 with $\alpha \neq 0$] , the statement that the stiffest equation of state is obtained for the lowest effective ground-state mass $M^*(\rho_0)$ does not apply. In fact, there was no established statement for it, but many results obtained by nonlinear scalar models have implicitly suggested it [5,12]. Nevertheless it means that the ZM2 and ZM3 models are bringing a new qualitative behavior regarding $M^*(\rho_0)$ and K. Notice that we have gotten a complete agreement in calculating K by using its direct definition given by Eq. (25) as well as the analytical form presented in Eq. (26). Table II presents the results for S, V, $S+V$, $V-S$, and K at the saturation point of the nuclear matter for the models. To calculate S we have used our definition given by Eq. (15) which as we have pointed out in the previous section differs by a factor $1/(1+S_0/M)$ from that of Eq. (16). In order to see the impact of neglecting higher orders terms in these models we give the values of S/S_0. They are 0.85 for ZM, 0.82 for ZM2, and 0.71 for ZM3. There is a simple check for our definitions of S and V given by Eqs. (15) and (17). At $T=0$ and at $\rho=\rho_0$, Eq. (23) gives $\mathcal{E}(\rho_0)/\rho_0 = \mu = \nu + V$, where ν is the Fermi energy (E_F^*). Since $M^* = S + M$, the following relation has to be satisfied:

$$\frac{\mathcal{E}}{\rho_0} = V + [k_F^2 + (M+S)^2]^{\frac{1}{2}}. \quad (34)$$

Our results for S and V are consistent with the above requirement.

We have made a detailed analysis of each model, by varying the coupling constants and keeping E_b and ρ_0 approximately fixed (ρ_0 between 0.14 fm^{-3} and 0.17 fm^{-3} while E_b was kept between -15 MeV and -16 MeV). The values of S, V, and K have only a very weak dependence on C_σ^2 and C_ω^2. Figure 4 exhibits how M^* behaves as a function of T at zero density. In this regime (ρ=0, $T \neq$0), after a careful numerical investigation, we have concluded that none of the proposed ZM models (unlike the Walecka model [11]) is able to present a first-order phase transition.

A settled point coming from our calculations is that all ZM models give soft equations of state compared to the Walecka model. Among ZM models themselves, ZM3 model is the softest, giving $K = 155.43$ MeV but also giving the value of $V-S$ which most approaches that

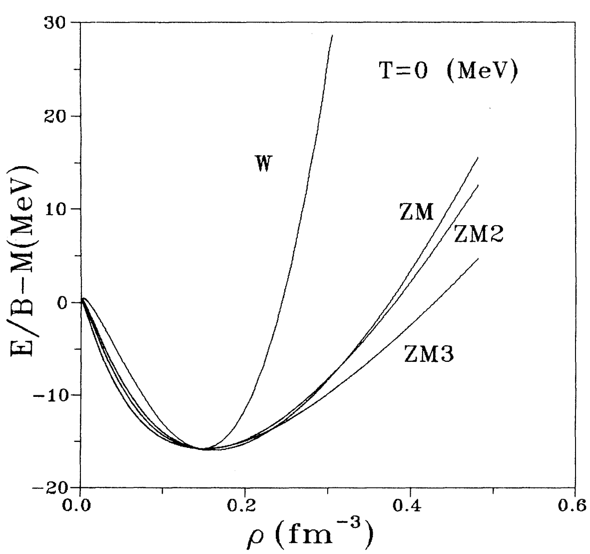

FIG. 1. Proper energy/baryon as a function of baryon density for the Walecka model (W) and Zimanyi-Moszkowski models (ZM, ZM2, ZM3).

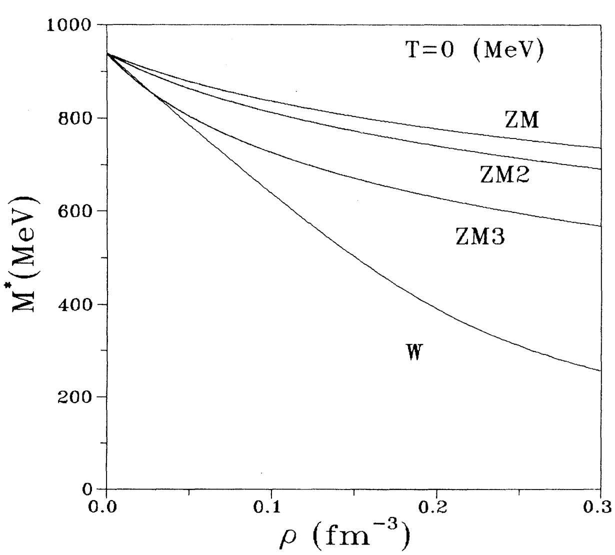

FIG. 2. Baryon effective mass in nuclear matter as a function of the proper baryon density for the models.

TABLE II. Values in MeV for $S, V, V+S, V-S$ and K at ρ_0 given in Table I for the indicated models.

Models	S	V	$V+S$	$V-S$	K
Walecka	-431.02	354.15	-76.87	785.18	550.82
ZM	-140.64	82.50	-58.13	223.13	224.71
ZM2	-167.83	109.73	-58.09	277.56	198.32
ZM3	-267.00	203.71	-63.28	470.71	155.74

one obtained from Walecka model. The expected value for $V-S$ can be related to the spin-orbit splitting of finite nuclei and if one accepts the value of the Walecka model as reasonable, this quantity is worsening to the ZM and ZM2 models while the incompressibility is improving. Apart from this behavior, the ZM3 model is which most approaches the Walecka model in other features. See, for instance, Fig. 2. The relativistic contents of each model can be estimated. It is given by the ratio $R = \rho_s/\rho_B$ at the nuclear matter saturation point. Particularly, if one neglects the contribuition of the small component of the Dirac nucleon spinor, $R = 1$. When this component is present, R estimates the nonrelativistic limit versus the relativistic one. From Table I the R values are 0.932, 0.975, 0.967, and 0.960 for the Walecka, ZM, ZM2, and ZM3 models, respectively. It indicates that among the ZM′*s* models, the ZM3 model is the one which more approaches the Walecka model in terms of relativistic effects.

From the nuclear matter mean-field approach we have used, it is possible to extract a momentum dependence of the averaged interaction of a nucleon in the nuclear medium and from this an energy dependent real optical potential as we have presented in the last section. This quantity can now be compared with the optical model fits obtained from measured nucleon-nucleus elastic cross sections [13, 14] in the limits of mass number going to infinity and radius going to zero. It has been implemented by Feldmeier and Lindner [12] in a simple empirical formula covering the kinetic energies from -50 to 1000 MeV. We plot in Fig. 5 the experimental values of the real optical potential [12–14] and the theoretical predictions of the models using Eq. (31). In order to see how U_{OPT} given by Eq. (31) deviates from U_{LIN} given by Eq. (32) we present in Fig. 6 the results for the ZM3 model using both approaches. Figure 5 singles out the ZM3 model. This model as we have already pointed out is the one which keeping some relativistic features of the Walecka model, gives a soft equation of state and a not so small value for $V-S$. All this together with the nice fitting this model gives for the experimental real optical potential suggest the importance of the nonlinear scalar coupling with the vector field. This "mixed coupling" can be seen directly from the rescaled Lagrangian, Eq. (6), for cases $\alpha = 1$ and $\alpha = 2$.

The discussions we have presented deserve some comments. The Walecka model is renormalizable while ZM models are not. However, nucleon loops correction for the Walecka model does not seem to converge [3,4] . We are aware that the different kinds of derivative scalar couplings are somewhat arbitrary, but we can look at them as effective models which give reasonable results for nuclear

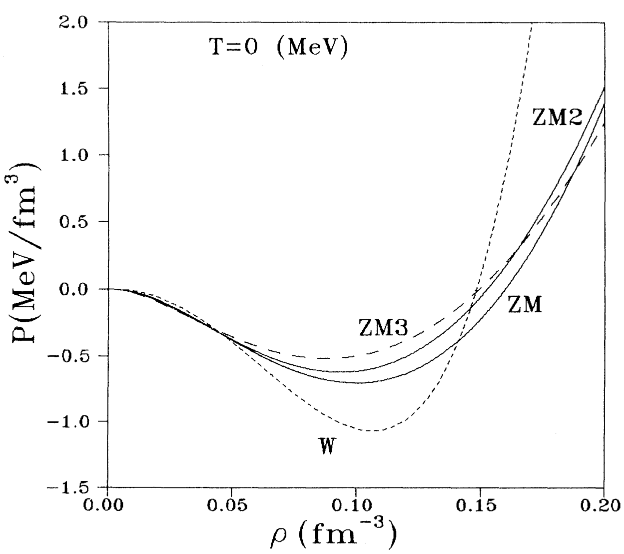

FIG. 3. Pressure as a function of proper baryon density for the models.

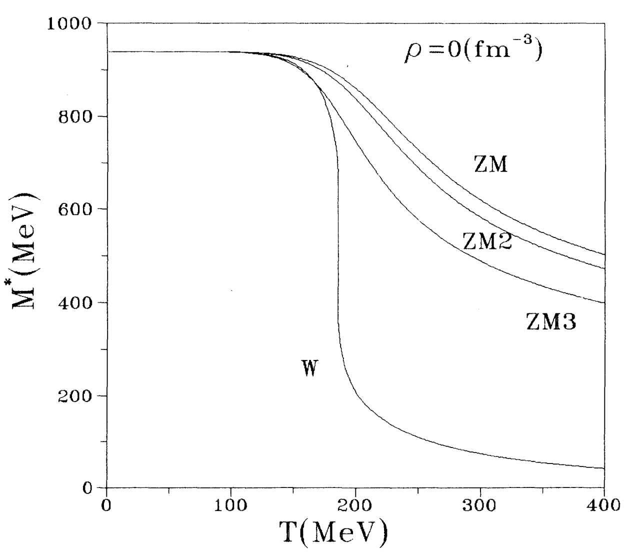

FIG. 4. Baryon effective mass in nuclear matter as a function of the temperature at $\rho = 0$.

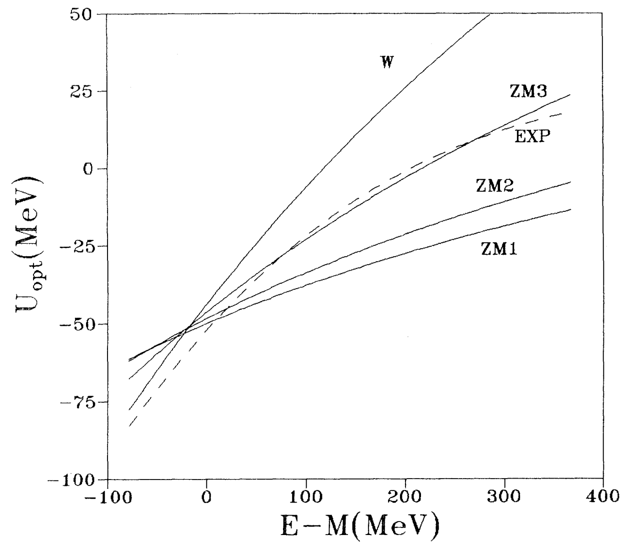

FIG. 5. The real part of the optical potential [Eq. (31)] for the models and the experimental curve extracted from Refs. [12–14]; in function of energy.

matter. It is in this sense that our calculations must be understood. However it is important to call attention to the recent work of Miyazaki [15] about the foundation of the derivative scalar coupling (DSC). He claims that such DSC can be obtained by utilizing the relativistic SU(6) model of the meson-baryon couplings. In summary, we have presented in a unified form the Walecka model and different derivative couplings suggested by Zimanyi and Moszkowski [6]. In particular, for the first time the ZM2 and ZM3 models were implemented. An analytical expression for the incompressibility for these models was derived. We have also calculated the real part of the optical potential for the models and compared them to the experiments. Unlike the Walecka model the ZM models do not exhibit phase transition for finite temperature at zero density.

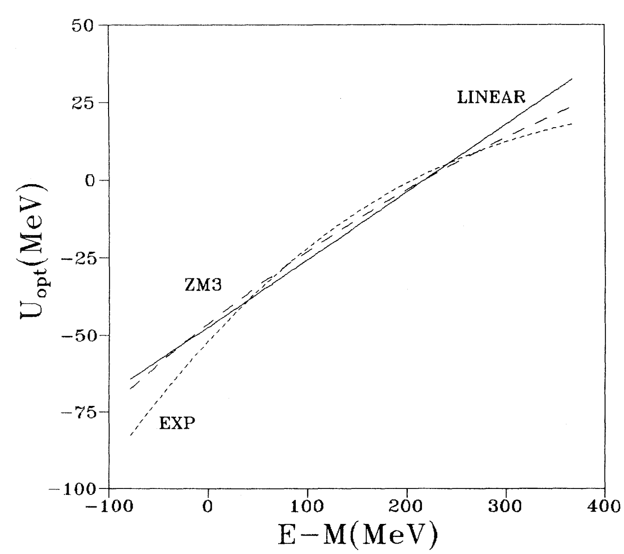

FIG. 6. The same as Fig. 5 for the ZM3 model (only) and $U_{\rm LIN}$ given by Eq. (32).

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APPENDIX: ANALYTICAL INCOMPRESSIBILITY FOR WALECKA AND ZM MODELS

Let us start this section by rewriting Eq. (25) in terms of the pressure. In order to do it we first use the definition of p in terms of the energy density (at $T = 0$),

$$p = \rho^2 \frac{\partial}{\partial\rho}\left(\frac{\mathcal{E}}{\rho}\right) . \tag{A1}$$

At the saturation point, $\rho = \rho_0$, the energy per particle has a minimum leading from the above expression to $p = 0$, meaning the hydrostatic equilibrium of nuclear matter. From this statment, the incompressibility given by our equation (25) can be rewritten as

$$K = 9\left.\frac{\partial p}{\partial \rho}\right|_{\rho=\rho_0} . \tag{A2}$$

From Eq. (A1) one has

$$\frac{\partial \mathcal{E}}{\partial \rho} = \frac{P+\mathcal{E}}{\rho} . \tag{A3}$$

By substituting this expression in our equation (25), the incompressibility can be rewritten as

$$K = 9\rho_0 \frac{\partial}{\partial\rho}\left(\frac{P+\mathcal{E}}{\rho}\right)\Bigg|_{\rho=\rho_0}$$
$$= 9\left[\frac{\partial}{\partial\rho}(P+\mathcal{E}) - \left(\frac{P+\mathcal{E}}{\rho}\right)\right]\Bigg|_{\rho=\rho_0} . \tag{A4}$$

To have K in this form is very convenient since Eqs. (18) and (19) show that by adding p and $\mathcal{E}$ the second right-hand side term of each of them cancel one other. At zero temperature the integrals appearing in those equations can be evaluated analytically. We have used these results to obtain

$$\mathcal{E} + p = \frac{C_\omega^2}{M^2} m^{*^\alpha} \rho_B^2 + E^*(k)\rho_B . \tag{A5}$$

Notice that the derivation in ρ_B contained in Eq. (A4) has to be done following the chain rule

$$\frac{\partial}{\partial\rho_B} = \frac{2\pi^2}{\gamma k_F^2}\frac{\partial}{\partial k_F} + \frac{\partial m^*}{\partial\rho_B}\frac{\partial}{\partial m^*} \ . \tag{A6}$$

The expressions we need to obtain K are

$$\frac{\partial}{\partial\rho_B}[E^*(k)\rho_B]\Big|_{\rho=\rho_0}$$

$$= \left(E^*(k) + \frac{K_F^2}{3E^*(k)} + M^2\frac{m^*\rho_B}{E^*(k)}\frac{\partial m^*}{\partial\rho_B}\right)\Bigg|_{\rho=\rho_0} \ , \tag{A7}$$

$$\frac{\partial}{\partial\rho_B}\left(\frac{C_\omega^2}{M^2}m^{*^\alpha}\rho_B^2\right)\Big|_{\rho=\rho_0}$$

$$= \left(\frac{2C_\omega^2}{M^2}m^{*^\alpha}\rho_B + \alpha\frac{C_\omega^2}{M^2}m^{*^{\alpha-1}}\rho_B^2\frac{\partial m^*}{\partial\rho_B}\right)\Bigg|_{\rho=\rho_0} \ , \tag{A8}$$

With these auxiliary derivations we end up with the unified incompressibility expression for the hadronic models we are dealing with given by Eq. (26). Only $\frac{\partial m^*}{\partial\rho}$ remains to be detailed. The explicit expression for $\rho_s = \langle\bar\psi\psi\rangle$ is obtained from Eq. (13) and given by Eq. (28). An intermediary step is the obtaining of

$$\frac{\partial\rho_s}{\partial\rho_B} = \frac{m_\sigma^2}{g_\sigma}\Bigg[\frac{1}{m^{*2\beta}}\left(\frac{-M[m^* + \beta(1-m^*)]}{g_\sigma m^{*\beta+1}} - \alpha\frac{(\alpha+1)}{2}\frac{C_\sigma^2 C_\omega^2}{g_\sigma M^5}m^{*^\alpha}\rho_B^2\right)\frac{\partial m^*}{\partial\rho_B} - \alpha\frac{C_\sigma^2 C_\omega^2}{g_\sigma M^5}\frac{m^{*^{\alpha+1}}}{m^{*2\beta}}\rho_B$$
$$-\frac{2\beta}{m^{*2\beta+1}}\left(\frac{M(1-m^*)}{g_\sigma m^{*\beta}} - \frac{\alpha}{2}\frac{C_\sigma^2 C_\omega^2}{g_\sigma M^5}m^{*^{\alpha+1}}\rho_B^2\right)\Bigg] \ . \tag{A9}$$

But as we have explained before

$$\frac{\partial\rho_s}{\partial\rho_B} = \frac{\partial\rho_s}{\partial k_F}\frac{\partial k_F}{\partial\rho_B} + \frac{\partial\rho_s}{\partial m^*}\frac{\partial m^*}{\partial\rho_B}$$
$$= \frac{M^*}{E_F^*} + 3M\left(\frac{\rho_s}{M^*} - \frac{\rho_B}{E_F^*}\right)\frac{\partial m^*}{\partial\rho_B} \ . \tag{A10}$$

Now we are in the position to obtain $\frac{\partial m^*}{\partial\rho}$ by using Eqs. (A9) and (A10) which turns out to be that given by Eq. (27). Notice that all quantities involved in Eqs. (25)–(28) have to be calculated at the saturation point $\rho = \rho_0$.

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