Consequences of a covariant description of heavy-ion reactions at intermediate energies

E. Lehmann, Rajeev K. Puri, Amand Faessler, G. Batko, and S.W. Huang

Institut für Theoretische Physik, Universität Tübingen, Auf der Morgenstelle 14, D-72706 Tübingen, Germany

Heavy-ion collisions at intermediate energies are studied by using a new relativistic quantum molecular dynamics (RQMD) code, which is a covariant generalization of the quantum molecular dymanimcs (QMD) approach. We show that this new implementation is able to produce the same results in the nonrelativistic limit (i.e., 50 MeV/nucleon) as the noncovariant QMD. Such a comparison is not available in the literature. At higher energies (i.e., 1.5 GeV/nucleon and 2 GeV/nucleon) RQMD and QMD give different results in respect to the time evolution of the phase space, for example for the directed transverse flow. These differences show that consequences of a covariant description of heavy-ion reactions within the framework of RQMD exist even at intermediate energies.

PACS number(s): 25.70.−z, 24.10.Jv, 02.70.Ns

I. INTRODUCTION

The only way to probe nuclear matter under extreme conditions of density and thermal excitations is to study heavy-ion reactions. The main aim of these experiments at intermediate energies, i.e., between 100 MeV/nucleon to about a few GeV/nucleon, is to get some information on the nuclear equation of state (EOS), which is characteristic of the physical structure of the considered system. Knowledge of the EOS is of immense theoretical and practical importance for nuclear physics as well as astrophysics.

In the last decade strong efforts were directed to develop microscopic models to describe the dynamics of heavy-ion collisions at intermediate energies. Today mainly two different semiclassical theoretical approaches are used with great success: One of them, the so-called Boltzmann-Uehling-Uhlenbeck (BUU) type models [1–6] simulate, with the help of the test particle method, kinetic equations like the Boltzmann-Uehling-Uhlenbeck equation (BUU equation) which describes the time evolution of the one-body distribution function in phase space. The other one, the so-called "quantum" molecular dynamics (QMD) [7, 8] is based on the classical molecular dynamics and hence propagates the particles under mutual interactions. In addition some important quantum theoretical characteristics, which are also contained in the BUU-type models, like Fermi motion of the nucleons, stochastic scattering, and Pauli blocking, are also included.

In QMD and BUU-type approaches the nucleon-nucleon interaction is split into a long-range and a short-range part. While the long-range part influences the particle trajectories in a steady way the short-range part is responsible for the so-called "hard collisions," in which strong changes of the momenta of the particles can appear.

With the help of such phase-space simulations a clearer picture of the dynamics of heavy-ion reactions has been created. But the main aim of studying heavy-ion collisions at intermediate energies, namely to extract some information on the EOS, could not be reached due to the sensitivity of the observables, like the collective flow, to different ingredients.

One of these questions that still remains is how strong do relativistic effects influence the dynamical evolution of a heavy-ion reaction at intermediate energies? Since even at this energy range the nucleons are moving with a velocity which is not negligible with respect to the speed of light, one has to work in a covariant framework to get a reasonable description of heavy-ion collisions at intermediate energies. Therefore, it was worth developing covariant generalizations of these non-covariant microscopic models listed above.

Unlike the covariant generalizations of the BUU-type models [9–11], which are closely connected to field theoretical features, the covariant generalization of the QMD approach is not straightforward. The reason for these difficulties is that the mutual nucleon-nucleon interaction in QMD is dealt as a simultaneous action at a distance. Hence one enters the problems of a Poincaré invariant action at a distance if one wants to generalize the QMD approach in a manifestly covariant way.

But these problems can be treated in the framework of constrained Hamilton dynamics. This formalism was introduced by Dirac [12] to express a theory based on a singular Lagrangian in a generalized phase-space approach with the help of constraints and later on extended by others [13–17] to a form which is suitable for our purpose. These authors picked up the idea of Dirac [12] who has realized for the first time that constraints are not only reducing the degrees of freedom but can also determine the dynamics.

Of course, all these covariant models are based on analytical expressions with a well-defined nonrelativistic limit. However, it was not shown for any of them that the numerical procedures used in the implementations give in the nonrelativistic limit really the same results as the corresponding noncovariant model. But one should make sure that the covariant codes are working well at

<u>51</u> 2113

nonrelativistic energies by checking sensitive quantities like the transverse flow before one is able to discuss possible relativistic effects at higher energies. This was also not done in [18] because these authors studied only insensitive quantities at nonrelativistic energies.

Therefore we present here a detailed analysis of phase space simulations at 50 MeV/nucleon. These results show that our implementation of RQMD gives the same results as the noncovariant QMD in the nonrelativistic limit. Another aim of this paper is to study whether this covariant description of heavy-ion reactions in the framework of RQMD has consequences for the time evolution of the phase space in the intermediate-energy range. This is done by comparing results of sensitive quantities like the directed transverse flow extracted from QMD and RQMD calculations.

The paper is organized as follows: In the following section we will briefly discuss the formalisms used in QMD and RQMD and will point out their main differences. The third section contains a few details of the implementation of these methods. In the fourth section results of simulations of semicentral Ca+Ca collisions at different energies are presented. These results are analyzed with respect to discussing the differences of the approaches used, e.g., QMD and RQMD. Finally, in the last section, we summarize and also give an outlook.

II. THE FORMALISM USED IN QMD AND ITS COVARIANT GENERALIZATION

In this section we give a brief description of both the QMD approach and its covariant generalization, the RQMD approach. For more details, especially for the foundations of these formalisms, we refer to [7] in the case of QMD and to [19] in the case of RQMD. First we concentrate on the QMD.

A. Quantum molecular dynamics (QMD)

The dynamics of heavy-ion reactions can be studied in a dynamical many-body approach on an event-by-event base using QMD. This model gives a microscopic description of heavy-ion collisions at the nucleon level and was developed by Aichelin and Stöcker [8].

Each nucleon is represented by its Wigner density in phase space in a Gaussian parametrization

$$f_i(\vec{r},\vec{p},t) = \frac{1}{\pi^3}\exp\{-[\vec{p}-\vec{p}_i(t)]^2 2L - [\vec{r}-\vec{r}_i(t)]^2/2L\}$$

(1)

with a fixed width. The initial distribution of the two nuclei in orbital space and momentum space are generated by a standard Monte Carlo procedure by taking care of the right radii of the nuclei, the Fermi energy, and an acceptable binding energy of each nucleus. In order to simulate heavy-ion collisions these two well-prepared nuclei are boosted towards each other.

Since the nucleon-nucleon interaction in QMD is split into a long-range part and a short-range part, we calculate the time evolution of the system of the two colliding nuclei in two parts.

(1) As the width of the Gaussians is fixed the centroids of each nucleon $\vec{r}_i(t)$, $\vec{p}_i(t)$ are propagated by the classical equations of motion

$$\frac{d\vec{r}_i}{dt} = \frac{\partial H}{\partial \vec{p}_i},$$

(2)

$$\frac{d\vec{p}_i}{dt} = -\frac{\partial H}{\partial \vec{r}_i},$$

(3)

where the Hamiltonian is given by the classical N-body Hamiltonian

$$H = \sum_i \frac{\vec{p}_i^2}{2m_i} + \frac{1}{2}\sum_{i,j} U_{ij}^{(2)} + \frac{1}{3!}\sum_{i,j,k} U_{ijk}^{(3)}.$$

(4)

The potentials in Eq. (4) are calculated as classsical expectation values by folding the two- and three-body parts of the interaction with the Wigner densities of the interacting nucleons:

$$U_{ij}^{(2)} = \int f_i(\vec{p}_i,\vec{r}_i,t)\, f_j(\vec{p}_j,\vec{r}_j,t)\, V_I^{(2)}(\vec{r}_i,\vec{r}_j,\vec{p}_i,\vec{p}_j)$$

$$\times d^3\vec{r}_i\, d^3\vec{r}_j\, d^3\vec{p}_i\, d^3\vec{p}_j,$$

(5)

$$U_{ijk}^{(3)} = \int f_i(\vec{p}_i,\vec{r}_i,t)\, f_j(\vec{p}_j,\vec{r}_j,t)\, f_k(\vec{p}_k,\vec{r}_k,t)\, V_I^{(3)}$$

$$\times (\vec{r}_i,\vec{r}_j,\vec{r}_k)\, d^3\vec{r}_i\, d^3\vec{r}_j\, d^3\vec{r}_k\, d^3\vec{p}_i\, d^3\vec{p}_j\, d^3\vec{p}_k.$$

(6)

Shrinking on a local Skyrme force only one gets for the total potential energy by using (5) and (6)

$$V_{\mathrm{Skyrme}} = \sum_{i=1}^{N}\left[\frac{\alpha}{2}\left(\sum_{j\neq i}\frac{\tilde{\rho}_{ij}}{\rho_0}\right) + \frac{\beta}{\gamma+1}\left(\sum_{j\neq i}\frac{\tilde{\rho}_{ij}}{\rho_0}\right)^{\gamma}\right],$$

(7)

with the so-called interaction density

$$\tilde{\rho}_{ij} = \frac{1}{(4\pi L)^{3/2}}\exp(-\vec{r}_{ij}^2/4L),$$

(8)

where $\vec{r}_{ij}$ is the distance of the centers of two Gaussians.

(2) If the centroids of two Gaussians come closer than a certain distance $d_{\min} = \sqrt{\sigma_{\mathrm{tot}}(\sqrt{s})/\pi}$ during their propagation, a stochastic collision of the two corresponding nucleons is calculated by a Monte Carlo procedure. In order to respect the Pauli principle the collision will be blocked if the phase-space elements of the final states are already occupied by other nucleons. The collisions determined in this way can be elastic or inelastic. The inelastic channels included in the calculations presented in this paper are creation and reabsorption processes of the $\Delta(1232)$ resonance. The whole collision part is dealt by using a full relativistic kinematic as explained in [2].

In the propagation part one can also include some relativistic kinematics by replacing the kinetic energies in the Hamiltonian (4) by $\sqrt{\vec{p}_i^2 + m_i^2}$. This small modification is always used in QMD calculations at relativistic energies. But a manifestly covariant description also requires a covariant formulation of the interaction. Only in such a full covariant model one can make sure that Lorentz scalar observables are independent of the observer frame. How one can generalize QMD to a manifestly covariant model will be described in the next subsection.

B. Relativistic quantum molecular dynamics (RQMD)

The QMD approach contains as an essential part the classical propagation of particles under instantaneous mutual interactions "at a distance." The generalization of this nonrelativistic particle dynamics to a manifestly covariant particle dynamics is not trivial because one has to know how to deal action at a distance in a covariant manner. The conceptual problems in this field are formulated in the famous no interaction theorem [20], which states that a Hamiltonian description of a multibody system with a canonical representation of the Poincaré group, where world line invariance is demanded and the physical coordinates are identified with the canonical coordinates is incompatible with interaction. This negative implication of the no interaction theorem can be avoided by dealing not with the whole phase space but with a submanifold of it. One possibility in this fashion is given in the framework of constrained Hamilton dynamics.

In a covariant theory one has to respect Poincaré invariance and hence every particle has to be described by its four-momentum p_i^μ and its four position vector q_i^μ means every particle carries its own time coordinate. Therefore, an N-particle system is connected with an $8N$-dimensional phase space. In the formalism of constrained Hamilton dynamics this phase space is reduced to a $6N$-dimensional phase space with the help of $2N$ constraints fixing the energies and the relative times of the particles. In addition a global evolution parameter has to be introduced by these constraints in order to gauge the evolution of the system. Doing this one defines a $6N$-dimensional hypersurface in the original $8N$-dimensional phase space on which the system is allowed to move during its evolution. This formalism is not changing the notion of simultaneity by a change of the frame of reference, which means one gets an invariant notion of simultaneity as well as invariant world lines in this fashion.

The QMD model was extended to its covariant version, the RQMD model, first by Sorge *et al.* [19] to study heavy-ion collisions at ultrarelativistic energies. We use a similar method in order to study heavy-ion collisions at the intermediate-energy range. In the following we will discuss some details of this method.

We use here similar constraints as introduced in [19] and hence the first N constraints are chosen as on-shell constraints[1]

$$K_i = p_i^\mu p_{i\mu} - m_i^2 - \tilde{V}_i = 0, \quad i = 1, ..., N, \tag{9}$$

which request that the particles move between collisions on energy shell. This choice of the first N constraints requires that the potential part $\tilde{V}_i$ should be a Lorentz scalar and therefore a function of Lorentz scalars. Since we want to define a system with mutual two-body interactions as in QMD, $\tilde{V}_i$ should be given by a sum of these

---

[1] We use the Einstein convention for the tensor indices, but no summation over repeated particle indices, except if explicitly specified.

two-body interactions. Following [19] we therefore use

$$\tilde{V}_i = \sum_{j \neq i} \tilde{V}_{ij}(q_{Tij}^2), \tag{10}$$

which means that the two-body interactions depend only on the Lorentz invariant squared transverse distance

$$q_{Tij}^2 = q_{ij}^2 - \frac{(q_{ij}^\mu p_{ij\mu})^2}{p_{ij}^2}, \tag{11}$$

with $q_{ij}^\mu = q_i^\mu - q_j^\mu$ being the four-dimensional distance and $p_{ij}^\mu = p_i^\mu + p_j^\mu$ the sum of the momenta of the two interacting particles i and j.

Motivated by a comparison in the nonrelativistic limit we use for the potential part finally

$$\tilde{V}_i = 2m_i \left[\frac{\alpha}{2} \left(\sum_{j \neq i} \frac{\exp[q_{Tij}^2/4L]}{\rho_0(4\pi L)^{3/2}} \right) + \frac{\beta}{(\gamma+1)} \left(\sum_{j \neq i} \frac{\exp[q_{Tij}^2/4L]}{\rho_0(4\pi L)^{3/2}} \right)^\gamma \right]. \tag{12}$$

In this way the local Skyrme interaction used in QMD is generalized by replacing the squared distance $-\vec{r}_{ij}^2$ in the interaction densities (8) by the Lorentz invariant squared transverse distance q_{Tij}^2. One should notice that due to the second term on the right-hand side of Eq. (11) this generalized interaction used in RQMD is slightly implicit momentum dependent. Because of this term, which gives the longitudinal squared distance, the interaction used in RQMD depends not only on the distance of the two interacting particles, as in QMD, but also on the direction of their center-of-mass motion in the rest frame of the two nuclei.

Since the on-shell constraints alone do not specify the world lines one needs additional N constraints which are fixing the relative times of the particles. In order to respect world line invariance, causality, and cluster separability this N time fixations are defined as

$$\chi_i = \sum_{j(\neq i)} g_{ij} p_{ij}^\mu q_{ij\mu} = 0, \quad i = 1, ..., N-1, \tag{13}$$

$$\chi_N = P^\mu Q_\mu - \tau = 0, \tag{14}$$

with $P^\mu = p^\mu/\sqrt{p^2}$, $p^\mu = \sum_i p_i^\mu$, $Q^\mu = \frac{1}{N} \sum_i q_i^\mu$, and the dimensionless scalar weighting function

$$g_{ij} = \frac{1}{q_{ij}^2/L_C} \exp(q_{ij}^2/L_C) \tag{15}$$

with $L_C = 8.66$ fm^2. Conditions (13) are motivated by studies in the framework of singular Lagrangians [21]. Using this method one gets up to the weighting functions g_{ij} the same conditions as secondary constraints and the on-shell conditions (9) as primary constraints in a natural way. The important fact for using the expression (15) as weighting function is that this scalar function respects the principle of causality, while the ones used in the singular Lagrangian theories and in the model of Samuel

[17] can violate this important physical restriction.

Constraints (13) take care that the times of interacting particles are not dispersed too much in their common center-of-mass system. Furthermore, they specify the dynamics by fixing the times at which the forces have to be calculated but they do not specify the global evolution parameter τ. This evolution parameter must be defined to be determined dynamically since the no-interaction theorem can only be avoided in this way as pointed out in [16]. The evolution parameter τ is defined by the gauging condition (14) in a way that the individual times are increasing with increasing τ.

This set of $2N$ constraints given by (9), (13) and (14) determines covariant world lines parametrized by the initial data at equal τ, which means the starting conditions are given by the values of p_i^μ and q_i^μ at a given starting value of τ.

With the help of this set of constraints the reduction of the phase space can be done in a well-defined way and the dynamics of the system can be determined by the Hamiltonian

$$H = \sum_{i=1}^{N} \lambda_i K_i + \sum_{i=1}^{N-1} \lambda_{N+i}\chi_i, \tag{16}$$

given by a linear combination of the τ independent $2N-1$ constraints and hence, in the sense of Dirac [12], these constraints determine the dynamics. The Hamiltonian (16) generates equations of motion with the help of Poisson brackets as

$$\frac{dq_i^\mu}{d\tau} = [H, q_i^\mu], \tag{17}$$

$$\frac{dp_i^\mu}{d\tau} = [H, p_i^\mu]. \tag{18}$$

The unknown Lagrange multipliers λ_i can be determined using the fact that the complete set of $2N$ constraints must be fulfilled during the whole time evolution.

This formalism has a well-defined nonrelativistic limit as shown in [19]. This fact is a prior condition that RQMD and QMD calculations should give the same results at nonrelativistic energies if one uses in RQMD the same type of interaction as in QMD but generalized in the way as described above for the Skyrme interaction.

The binary collisions are dealt in RQMD in the same way as in the noncovariant QMD by using Monte Carlo methods. But in RQMD we use a full covariant kinematic to determine these collisions whereas in QMD the kinematic is relativistic, but not manifestly covariant.

III. DETAILS OF THE IMPLEMENTATION

Before we present some results of calculations in the next section we will discuss in this section some details of the numerical realization of QMD and RQMD.

First of all we would like to stress that in contradiction to earlier RQMD codes [19, 18] our new code is fully integrated in a simulation package which contains the noncovariant QMD approach with its different options and the covariant RQMD approach as well. The whole

package is called UNISCO standing for unified simulation code.[2] In this development special care is given to keep exactly the same initial conditions in both approaches, QMD and RQMD. In addition, this way of implementation assures us to use the same parameters in QMD and RQMD calculations. This is extremely important when one wants to look for relativistic effects at intermediate energies and guarantees that disagreements of results extracted from these two approaches have a physical origin.

In QMD RQMD heavy-ion collisions are simulated with the help of independent runs. In each run a single event is calculated. In the full simulation the average of these independent events is taken in order to get representative results.

In each event two carefully prepared nuclei are boosted against each other. A single nucleus is built up with the help of a Monte Carlo sampling method. This procedure used in QMD is applied in RQMD as well. But in RQMD one has additionally to take care of the constraints. A violation of these constraints right from the start would be conserved during the whole time evolution and hence one would leave the base of the formalism in such a case. Therefore one can only accept distributions in RQMD which are able to fulfill the constraints and special care is taken in the code to fulfill this condition.

By this procedure one gets nuclei which are stable for a much longer time span than the usually considered reaction time. This was already carefully proven for QMD in [7]. For RQMD we present here results in Fig. 1. As a measure of the stability of the nuclei we have plotted the time evolution of the root mean square radius $R_{\rm rms}$ for various single nuclei, which are prepared by the procedure discussed above. For all three nuclei (^{12}C, ^{28}Si, ^{40}Ca) the time evolution of the root mean square radii of 10 different initializations boosted to a certain energy is plotted. As can be observed from Fig. 1, very light nuclei like ^{12}C suffer strong vibrations whereas the root mean square radii of heavier nuclei, like ^{40}Ca, are more smooth. Finally, these results show clearly that these nuclei, which are not even heavy ones, are stable for a very long time span (at least 200 fm/c).

In the part of the code where the centers of the Gaussians are propagated we integrate the Hamilton equations (2) and (3) in the case of QMD and the equations of motion generated by formulas (17) and (18) in the case of RQMD. The numerical integrations are done by standard integration routines.

In order to decide if a baryon-baryon collision will occur or not we use a strictly geometrical interpretation of the cross section. Therefore two baryons will collide if their distance becomes closer than $d_{\rm min} = \sqrt{\sigma_{\rm tot}(\sqrt{s})/\pi}$ within a small time interval, whereas those passing each other at a larger distance will not suffer a collision during this time interval. $\sigma_{\rm tot}(\sqrt{s})$ is the total nucleon-nucleon cross section at a given c.m. energy of the colliding nu-

---
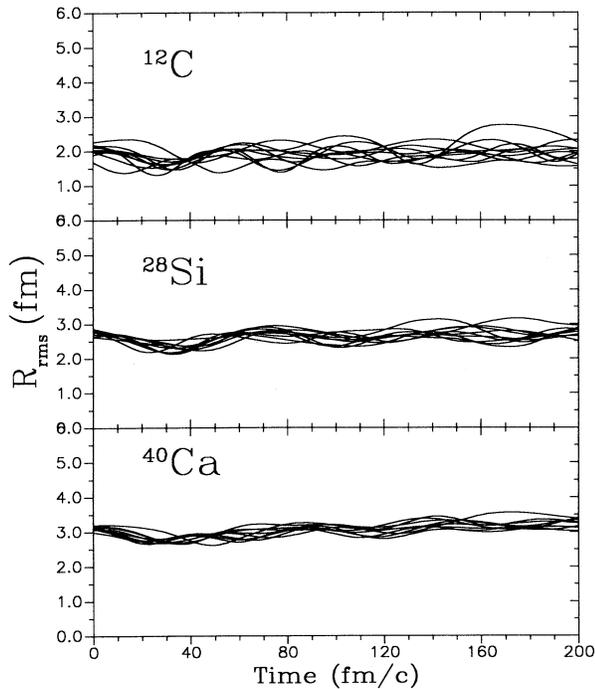
FIG. 1. Root mean square radii of various nuclei (C, Si, Ca) as a function of time.

cleon system. The time interval is given by the actual time step size of the numerical integration used in the propagation part.

Using this minimal distance concept in QMD one compares $|\vec{r}_i - \vec{r}_j|$ with $d_{\min}$ in order to decide if particles i and j became canditates for a collision. Since this collision criterion is not Lorentz invariant the collision sequence and the number of collisions can depend on the frame of reference as has been shown in [23]. In RQMD we use an invariant measure, namely the Lorentz invariant transverse distance by comparing $\sqrt{-q_{Tij}^2}$ with $d_{\min}$ and we are therefore avoiding these difficulties.

The scattering angles of a single baryon-baryon collision are determined randomly by a standard Monte Carlo procedure whereas the magnitudes of the final momenta are fixed by conservation of energy and momentum. For the cross sections used in this procedure we use the so-called Cugnon parametrization [24] of the free nucleon-nucleon scattering data, which includes excitation and reabsorption processes of the $\Delta(1232)$ resonance and takes care of the different isospin channels of the processes included.

Whenever the final state of a nucleon-nucleon collision is determined in this way we compute for each scattered nucleon the overlap in phase space with the surrounding nucleons. In this procedure we assume that each nucleon occupies a sphere in coordinate and momentum space. Therefore, in RQMD this overlap has to be determined in the rest frame of the nucleon of interest to justify the assumption of a spherical distribution of the nucleon. In QMD the overlap is calculated in the CMS of the two

nuclei and hence a spherical distribution of the nucleon is not ensured in the noncovariant approach. From this overlap in phase space one can then determine the probability with which the collision is blocked. Using this procedure one is able to respect the Pauli principle on a semiclassical level in a satisfying way as proven in [7].

IV. RESULTS AND THEIR DISCUSSION

A. Nonrelativistic limit

As we have already stressed in this paper, it makes absolutely no sense to discuss possible relativistic effects without making sure that the methods used for these studies, in our case QMD and RQMD, do not give the same results in the nonrelativistic limit. Therefore we first present here a comparison of QMD and RQMD at 50 MeV/nucleon, which is an energy where no relativistic effect should occur.

As a measure of the evolution of the phase space we have extracted three different quantities from QMD-RQMD calculations: the nucleon density in the central zone of a heavy-ion reaction as a measure of how much the nuclear matter was compressed during the reaction, the rapidity distribution as a measure of the stopping and the directed transverse momentum as a measure of the bounce off. The last quantity is defined as

$$p_x^{\text{dir}} = \frac{1}{N} \sum_{i=1}^{N} \text{sign}[Y_i^{(\text{CM})}] p_{ix} \qquad (19)$$

with the rapidity of the ith particle evaluated in CMS from

$$Y_i = \frac{1}{2} \ln \left(\frac{E_i + p_{iz}}{E_i - p_{iz}} \right). \qquad (20)$$

Another measure of the bounce off, namely the transverse momentum distribution p_x/A as a function of the rapidity, is often used instead of p_x^{dir}, especially to compare calculations with experimental data. In this paper we are interested in a relative comparison of two theories and hence we are using here p_x^{dir}, which has the following advantages for our purpose.

In contradiction to p_x/A the quantity p_x^{dir} integrates over all rapidity bins and provides therefore one value at a given time and allows one to follow this quantity easily as a function of time. Due to this fact p_x^{dir} as a function of time also reflects the creation of the bounce off during the reaction.

Figure 2 presents the results for the nucleon density as a function of time, calculated in a sphere with 2 fm radius around the origin of the CMS, the rapidity distribution of the final state (after 120 fm/c), and the directed transverse momentum as a function of time as extracted from QMD and RQMD calculations of semicentral C+C collisions at 50 MeV/nucleon. The impact parameter (b=1.3 fm) is half the radius of the two colliding nuclei.

As can be recognized from these plots, all three quantities reflect their typical behavior at low energies, e.g., the nucleon density is in all cases far less than $2\rho_0$ because of the small compression at this low energy, the rapidity

distribution shows one maximum due to the small bombarding energy and the directed flow is negative.

All results of these three quantities for semicentral C+C collisions presented in Fig. 2 show an excellent agreement of QMD and RQMD. Even the directed transverse flow, which is a highly sensitive quantity with respect to all ingredients (physics and numerics), shows this excellent agreement during the whole reaction time.

The analysis of semicentral collisions of a higher mass system, namely Ca+Ca, also gives good agreement of

QMD and RQMD as shown in Fig. 3. In this figure the same three quantities as in Fig. 2 are displayed. (The rapidity distribution is analyzed at 100 fm/c in this reaction.) The small differences in the directed flow (few %) are understandable by taking into account that the constrained dynamics used in RQMD needs more numerical efforts than the simple Hamilton dynamics used in QMD.

Therefore, in conclusion, we can point out that our new RQMD gives in the nonrelativistic limit the same
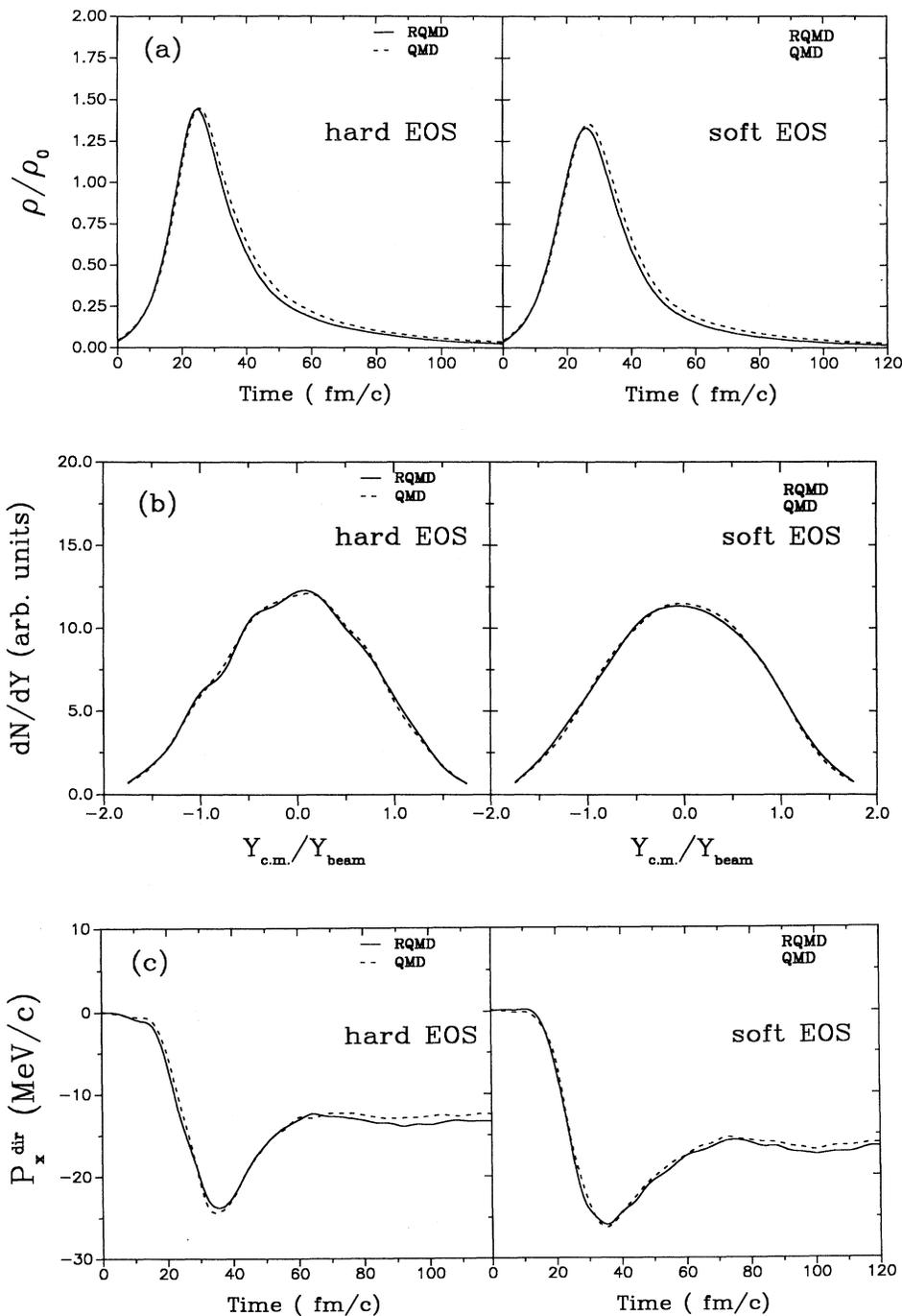
FIG. 2. Nucleon density (a) and directed transverse flow (c) as function of time and the rapidity distribution (b) at the final state (120 fm/c) of semicentral C+C collisions at 50 MeV/nucleon as extracted from QMD and RQMD calculations. The impact parameter was 1.3 fm and a static hard (left) as well as a static soft (right) Skyrme interaction was used.

results as the noncovariant QMD as it should be. The fulfilling of this critical benchmark convinces that one can study, with the help of this code, relativistic effects in heavy-ion collisions comparing RQMD and QMD at higher energies.

B. Relativistic energies

In order to test whether the covariant description of heavy-ion reactions as given in RQMD already shows consequences at intermediate energies we compare here QMD and RQMD calculations at 1.5 and 2 GeV/nucleon. At these energies the nucleons are moving with a velocity of about 85% and 92% of the speed of light, respectively, and hence possible relativistic effects should become vis-

ible at these energies.

We concentrate here our investigations on semicentral Ca+Ca collisions and analyze one of the most sensitive quantities, namely the directed transverse momentum as defined by Eq. (19). The impact parameter is 2 fm, which is about half the radius of the considered nuclei.

In Fig. 4 the directed transverse momentum is displayed as a function of time as extracted from QMD and RQMD calculations of semicentral Ca+Ca collisions at a bombarding energy of 1.5 GeV/nucleon. In Fig. 5 results of the same reaction, but at 2 GeV/nucleon are plotted.

Analyzing the time evolution of the directed flow created during the reaction one observes an appreciable difference among the different equations of state and, more important for our intention, among QMD and RQMD.

In the early time stage of the reaction the directed flow is negative since the interaction between the nuclei is attractive. After the two nuclei collide the flow becomes positive because a repulsion is induced by the high density in the region where the two nuclei overlap each other. From this region particles are bounced off due to their interactions with the other particles.

This repulsion is stronger in the case of the hard EOS and hence the transverse flow becomes larger in this case as for the soft EOS. In RQMD the interaction gets less attractive in the early time stage and hence starts earlier

FIG. 3.   Same as Fig. 2 but for semicentral Ca+Ca collisions. The impact parameter was 2 fm and a static hard Skyrme interaction was used. The rapidity distribution (b) is plotted at the final state (after 100 fm/c).
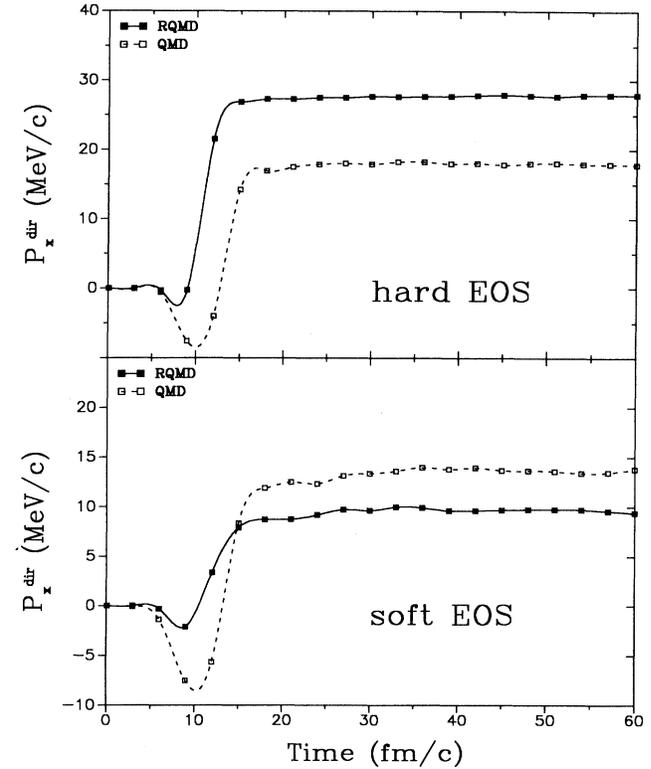
FIG. 4.   Comparison of QMD and RQMD calculations of semicentral Ca+Ca collisions at a bombarding energy of 1.5 GeV/nucleon and an impact parameter of 2 fm. The directed transverse flow is plotted as a function of time. Results obtained with a hard EOS (above) are shown as well as results obtained with a soft EOS (below).
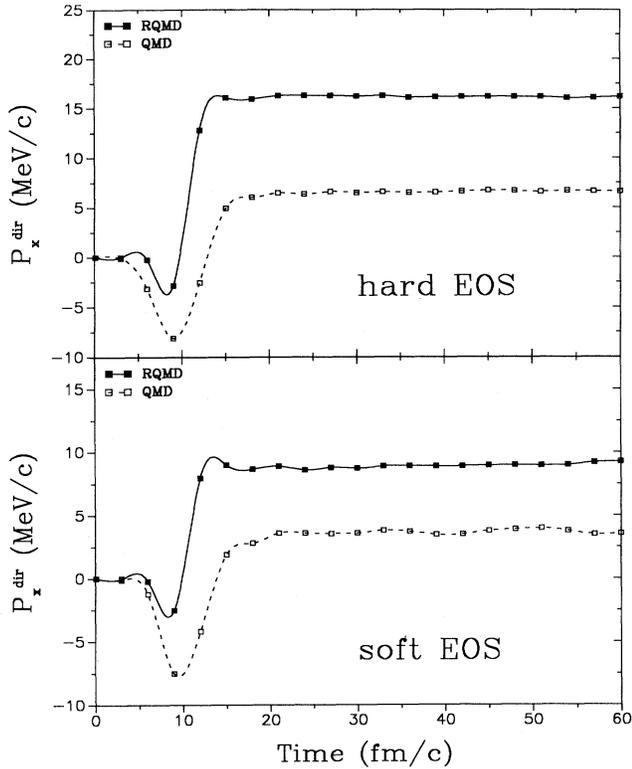
FIG. 5. Same as Fig. 4 but at a bombarding energy of 2 GeV/nucleon.

to become repulsive than in QMD as all results plotted in Figs. 4 and 5 are showing.

Due to the covariant treatment of the interaction in RQMD the strength of the attractive forces, dominating the early time stage of the reaction, is smaller in RQMD than in QMD. This can be understood by the fact that the forces will be modified in the covariant treatment by correcting factors which convert the nonrelativistic force to the spatial part of the corresponding Minkowski force.

Furthermore, the final values of the directed flow in RQMD and QMD are very sensitive to the energies considered.

At 2 GeV/nucleon as shown in Fig. 5, the flow is enhanced in RQMD in comparison to QMD in case of the hard and the soft EOS as well. At 1.5 GeV/nucleon (see Fig. 4) the flow is decreased in RQMD in comparison to QMD for the soft EOS but increased for the hard EOS. Therefore, the comparison of the results at 1.5 and 2 GeV/nucleon plotted in Figs. 4 and 5, respectively, shows that the covariant description affects the flow strongly but not in a unique way.

Information about the causes of the differences in the flow can be achieved by comparing QMD and RQMD calculations in a Vlasov mode (no collisions) as well as in a cascade mode (collisions only). Whereas calculations in the Vlasov mode give a measure of the influence of the mutual potential interactions on the flow, calculations in the cascade mode measure the influence of two-body col-

lisions on the flow. Results obtained by these calculations for semicentral Ca+Ca collisions at 2 GeV/nucleon are displayed in Fig. 6. These results show an increase of the flow in RQMD calculations in the Vlasov limit compared to equivalent QMD calculations. RQMD creates due to the Lorentz contraction a higher density and therefore already a positive flow in the final state of the considered reaction whereas in QMD the flow is negative. This fact reflects that in QMD due to the lower density mainly attractive interactions are created.

In the cascade limit the flow is decreasing in RQMD compared with QMD. This difference has its reason in the different treatment of the two-body collisions in the two considered approaches: Although QMD contains a relativistic kinematic RQMD uses a full manifest covariant kinematic, which avoids problems of reference frame dependent collision sequences. Furthermore, in RQMD due to relativistic contractions more collisions are Pauli blocked than in QMD, which also reflects the different treatment of the two-body collisions in the two approaches.

The comparison of the Vlasov limit and the cascade limit makes clear that the mutual potential interactions and the two-body collisions affect the flow in opposite directions in RQMD and in QMD. Therefore, it becomes understandable that one can get an increased as well as a decreased flow when using RQMD instead of QMD,
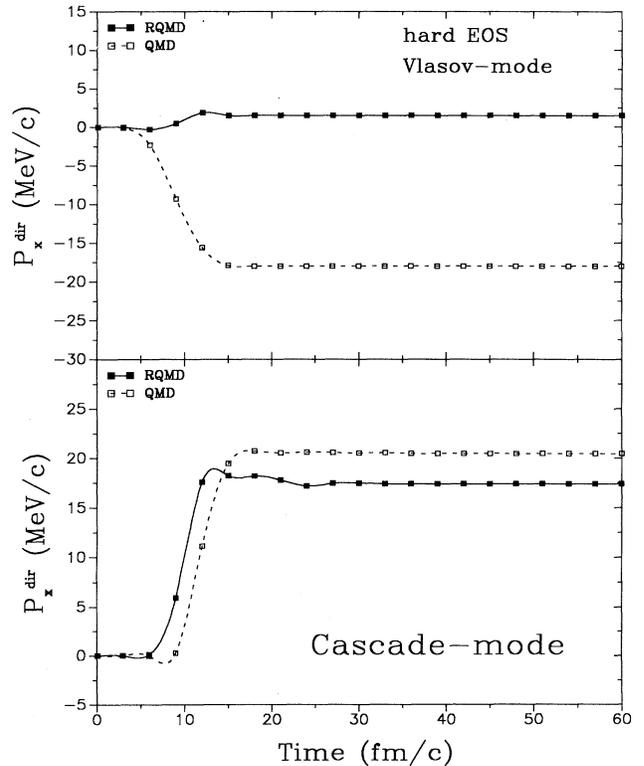
FIG. 6. Directed transverse flow as a function of time as obtained from QMD and RQMD calculations in a Vlasov mode (above) and a cascade mode (below). Ca+Ca collisions at 2 GeV/nucleon and an impact parameter of 2 fm were calculated by using a hard EOS.

as it was shown for semicentral Ca+Ca collisions at 1.5 GeV/nucleon in Fig. 4.

The interplay of these two contrary effects is studied again in Fig. 7, which shows a decomposition of the flow for the same reaction as considered in Fig. 5. In full QMD-RQMD calculations both effects are acting together because they are both present, whereas in the Vlasov limit or in the cascade limit either the collisions or the mutual potential interactions are switched off, respectively. Here, the contribution created by the mutual potential interactions contained in the Hamiltonian as well as the contribution created by two-body collisions is calculated separately as a function of time. This decomposition is extracted from full QMD-RQMD calculations and is done in the following way.

In each time step the momentum transfer created by the mutual potential interactions and the momentum transfer produced by two-body collisions is analyzed separately by using Eq. (19). Therefore, this analysis gives two values in each time step and both contributions can be easily followed during the whole reaction. If one adds both contributions one gets the total directed transverse flow. From this analysis one realizes that in QMD the flow is mainly created by two-body collisions in the compression stage in the considered reaction whereas the mutual potential interactions even decrease the flow because
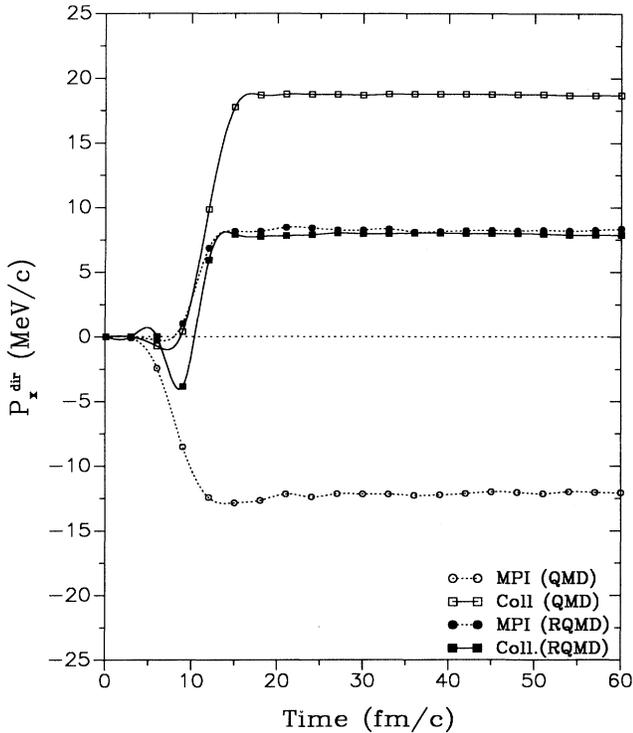
they create a negative contribution. In RQMD the collisions as well as the mutual potential interactions influence the flow in the same direction and nearly with the same amount. Therefore, one finally gets a higher sidewards flow in RQMD in the considered reaction. The contribution of the flow created by mutual potential interactions reflects the same effect as was already discussed in the Vlasov limit: The mutual potential interactions alone create no positive flow in QMD whereas in RQMD one already gets a positive flow in the Vlasov limit in the considered reaction (see Fig. 6). The enhancement of the repulsion in RQMD in comparison to QMD, which is responsible for this effect, is mainly created by the Lorentz contraction of the nuclei in the CMS (compare also Fig. 8).

Figure 6 as well as Fig. 7 show that the mutual potential interactions create more positive flow in a covariant treatment whereas binary collisions create less flow in RQMD than in QMD. Comparing covariant approaches with corresponding non-covariant approaches Ko et al. [9] as well as Schmidt et al.[3] [25] found that the flow is strongly increased if binary collisions are disregarded. The flow might be reduced also in their approaches if binary collisions are taken into account.

In order to gain more information about the differences in the flow we have done two additional calculations at 2 GeV/nucleon for the same reaction, which are shown for the hard EOS in Fig. 8, together with the results already plotted in Fig. 5.

In one calculation, assigned with QMD*, we have changed the starting conditions in QMD by using Lorentz contracted distributions of the nuclei in orbital space and Lorentz elongated distributions in momentum space in the CMS of the two nuclei. The phase space is always initialized in this way in RQMD but not in the usual QMD calculations. Since we use only static Skyrme forces in our studies presented in this paper the nuclei are stable for a longer time span than the reaction time also when this special initialization of the nuclei is applied in QMD.

Comparing the results plotted in Fig. 8 one realizes that QMD calculations with modified initial conditions result in even larger values for the transverse flow in the expansion stage than with RQMD. Due to the Lorentz contraction of the nuclei the density is increased. Therefore, the repulsive interaction gets stronger and the flow is increased. This effect is partly counterbalanced in RQMD by the covariant treatment of the interaction.[4] But in comparison to normal QMD calculations RQMD increases the transverse flow for the reaction shown in Fig. 8.

In another calculation, assigned with RQMD* in Fig. 8, we have modified RQMD in order to study the importance of the multitime description in this approach. Our intention in this modification was to force the particles

FIG. 7. Different contributions to the directed transverse flow as extracted from QMD and RQMD calculations for the same reaction (Ca+Ca at 2 GeV/nucleon, $b=2$ fm) as in Fig. 5. Here the contribution of the directed transverse flow created by mutual potential interactions (MPI) contained in the Hamiltonian and the contribution created by two-body collisions (Coll) are displayed separately.

[3]This approach based on classical molecular dynamics was only Lorentz invariant up to the order of $(v/c)^2$.

[4]One should note that this result is an improvement over earlier RQMD calculations [26].
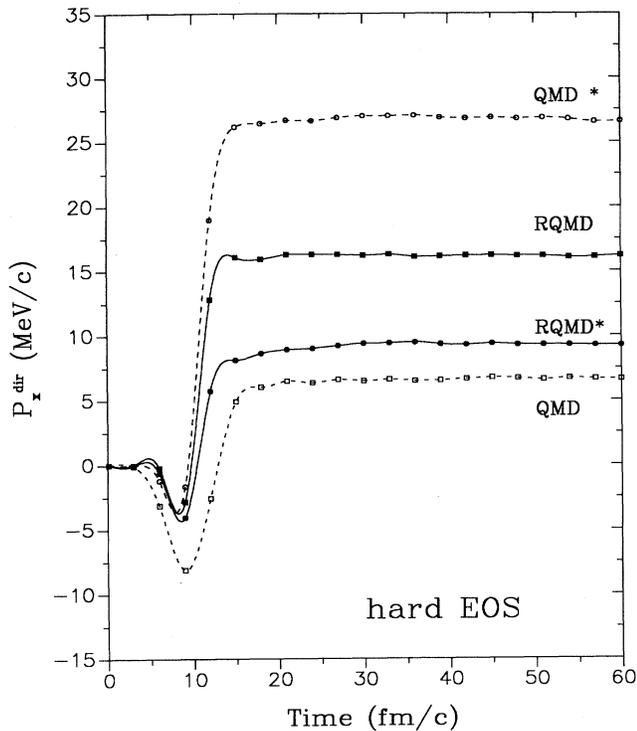
FIG. 8. Directed transverse flow as a function of time as extracted from various QMD and RQMD calculations for semicentral Ca+Ca collisions at a bombarding energy of 2 GeV/nucleon. Whereas usual QMD and RQMD calculations are assigned with QMD and RQMD, respectively, QMD calculations with Lorentz contracted starting conditions (see text) are assigned with QMD*. RQMD calculations with equal time constraints [see Eq. (23) and text] are assigned with RQMD*. The impact parameter was 2 fm and a hard EOS was used.

to have the same time coordinate in a defined frame of reference during the whole reaction. This cannot be done by defining simple nonrelativistic time fixations like

$$\chi_i = q_i^0 - q_N^0 = 0, \quad i = 1, ..., N-1, \tag{21}$$

$$\chi_N = q_N^0 - \tau = 0, \tag{22}$$

because these constraints are not Poincaré invariant and hence would strongly violate the foundations of the underlying formalism of constrained Hamilton dynamics. Therefore, in order to violate the restrictions of the formalism as less as possible, we have used more refined constraints even in this modification of RQMD and have replaced the τ-independent $N-1$ time fixations (13) by

$$\chi_i = P^\mu \left(q_{i\mu} - Q_\mu \right) = 0, \quad i = 1, ..., N-1, \tag{23}$$

whereas the gauging condition (14) is kept the same. This set of time constraints given by Eqs. (23) and (14) stresses the particles to have equal time coordinates in the CMS. Therefore, one can use this modification only for calculations in the CMS if one wants to have equal time coordinates of all particles. In addition, one should remark that these time constraints are not really an alter-

native to the constraints defined in Eqs. (13) and (14), because they violate cluster separability and causality (compare [27]). But condition (23) fixes that all particles have the same time coordinate in the CMS. This allows one to study the importance of the multitime description in RQMD requested by Lorentz invariance.

The results displayed in Fig. 8 clearly show that the difference among RQMD and QMD is drastically reduced if one disregards the multitime description and requests equal times according to Eq. (23) in the RQMD approach. However, this multitime description presents the price which one has to pay to have a full covariant formalism. Therefore, it is not surprising that individual times of the different baryons in RQMD play an important role for relativistic effects in the transverse flow. The covariant treatment of the interaction in this multitime description enhances the flow in comparison to noncovariant QMD calculations in the considered reaction as can be seen from Fig. 8.

In order to gain a better overview it is helpful to summarize here the different influences on the flow connected with the covariant description in the framework of RQMD. (1) Different starting conditions affect the flow to a certain amount. A Lorentz contracted distribution in coordinate space enhances the flow drastically. This effect is partly counterbalanced by the covariant treatment of the interaction in RQMD. This covariant treatment of the interaction can only be done in a multitime description, which takes care of individual time coordinates of all particles. Disregarding this fact, which violates the foundations of the underlying formalism, produces a flow which is not much different to the one obtained by noncovariant QMD calculations.

(2) The "long-range part" of the nucleon-nucleon interaction, contained as mutual potential interactions in the Hamiltonian in the approaches discussed, increases the flow in RQMD in comparison to QMD, mainly due to the Lorentz contraction of the nuclei. In addition, one should note that the implemented forces are slightly different although they are based on the same static Skyrme force in both approaches. This difference in the forces used is also responsible for some differences of the flow between QMD and RQMD.

(3) The directed sidewards flow is also affected by the covariant treatment of the two-body collisions, which represent mainly the "short-range part" of the nucleon-nucleon interaction. These effects reduce the flow in RQMD compared with QMD, where the treatment of the two-body collisions is only relativistic but not manifestly covariant.

Finally, all these effects produce the difference in flow between RQMD and QMD at relativistic energies reflecting kinematical and dynamical relativistic effects. The results for the directed transverse flow show clearly that a Lorentz invariant treatment is necessary to study heavy-ion reactions in the GeV energy region.

V. SUMMARY AND OUTLOOK

Heavy ion collisions were studied in the framework of QMD and RQMD at relativistic energies as well as in

the nonrelativistic limit. The formalisms used in QMD and RQMD were discussed by working out their main differences.

Although the nucleon-nucleon interaction is described in both approaches by static Skyrme forces it turned out that the correct covariant treatment of this interaction, as done in RQMD, results in some differences in comparison to the noncovariant approach used in QMD. Further differences can be produced by a manifest covariant treatment of the two-body collisions which assures reference frame independent collision sequences.

It was proven explicitly that our new implementation of the RQMD approach produces the same results as QMD in the nonrelativistic limit by comparing RQMD and QMD calculations of semicentral C+C and Ca+Ca collisions at 50 MeV/nucleon. This critical benchmark is even fulfilled for highly sensitive quantities like the directed transverse flow during the whole reaction.

At relativistic energies the covariant description of heavy-ion reactions as given in the framework of RQMD shows consequences on the dynamics of heavy-ion reactions. These consequences were studied by comparing the directed flow produced during semicentral Ca+Ca collisions at 1.5 and 2 GeV/nucleon as extracted from QMD and RQMD calculations.

The careful analysis of the observed differences turned out that the flow is affected by several reasons, based on various differences among QMD and RQMD.

A simple modification of the initial conditions in QMD towards relativity, e.g., Lorentz contracted distributions in coordinate space and Lorentz elongated distributions in momentum space, but treating the interaction as not covariant leads to a drastic overestimation of the flow. Disregarding the multitime description, which presents the price to pay for a manifest covariant treatment, leads to a flow which is not much different in comparison to the flow obtained by QMD calculations. But the correct treatment of the dynamics in a multitime formalism produces quite different results for the flow in RQMD compared to the flow in QMD.

Whereas the difference in the treatment of the mutual potential interaction contained in the Hamiltonian increases the flow in RQMD the manifest covariant treatment of the two-body collisions decreases the flow in RQMD in comparison to QMD. Finally, the interplay of these contrary effects is responsible for the differences of the directed flow detected from comparisons of QMD and RQMD calculations at relativistic energies. Due to the fact that these two effects act in opposite directions, one can get an increased but also a decreased flow in RQMD in comparison to QMD.

All RQMD calculations presented were obtained by describing the mutual two-body potential interactions with help of generalized static Skyrme forces, treated as scalar potentials in constrained Hamilton dynamics. However, a more reasonable and realistic description of heavy-ion collisions should take care of the complete Lorentz structure of the nucleon-nucleon interaction and has to include large scalar and vector potentials as well.

Therefore, in future, on should work with modified on-shell constraints given by

$$\tilde{K}_i = \Pi_i^\mu \Pi_{i\mu} - m_i^{*2} = 0, \tag{24}$$

where the effective momenta

$$\Pi_i^\mu = p_i^\mu - g_v A_i^\mu \tag{25}$$

contain all mutual two-body vector interactions, e.g., $A_i^\mu = \sum_j A_{ij}^\mu$ and the effective masses

$$m_i^* = m_i + g_s \Phi_i \tag{26}$$

contain all mutual two-body scalar interactions, e.g. $\Phi_i = \sum_j \Phi_{ij}$.

In order to take care of the nuclear medium and the correct energy and momentum dependence of the nucleon-nucleon interaction one should work with realistic forces and hence, these interactions should be extracted from self-consistent Dirac-Brückner calculations.

However, such a treatment is not at all trivial, but nevertheless, work on this line is in progress.

ACKNOWLEDGMENTS

[1] A. Bonasera, G. Burgio, M. Di Toro, and H. H. Wolter, Phys. Rev. C **39**, 2385 (1989).

[2] G. F. Bertsch and S. Das Gupta, Phys. Rep. **160**, 189 (1988).

[3] C. Grégoire, B. Remaud, F. Sébille, L. Vinet, and Y. Raffray, Nucl. Phys. **A465**, 317 (1987).

[4] W. Bauer, G. F. Bertsch, W. Cassing, and U. Mosel, Phys. Rev. C **34**, 2127 (1986).

[5] H. Kruse, B. Jacak, and H. Stöcker, Phys. Rev. Lett. **54**, 289 (1985).

[6] G. F. Bertsch, H. Kruse, and S. Das Gupta, Phys. Rev. C **29**, 673 (1984).

[7] J. Aichelin, Phys. Rep. **202**, 233 (1991).

[8] J. Aichelin and H. Stöcker, Phys. Lett. B **176**, 14 (1986).

[9] C. M. Ko, Q. Li, and R. Wang, Phys. Rev. Lett. **59**, 1084 (1987).

[10] W. Cassing and U. Mosel, Prog. Part. Nucl. Phys. **25**, 235 (1990).

[11] C. Fuchs, L. Sehn, and H. H. Wolter, Nucl. Phys. **A545**, 151c (1992).

[12] P. A. M. Dirac, Rev. Mod. Phys. **21**, 392 (1949).

[13] I. T. Todorov, Phys. Rev. D **3**, 2351 (1971).

[14] F. Rohrlich, Ann. Phys. **117**, 292 (1979).

[15] A. Komar, Phys. Rev. D **18**, 1881, (1978); **18**, 1887

(1978); **18**, 3617 (1978).

[16] E. C. G. Sudarshan, N. Mukanda, and J. N. Goldberg, Phys. Rev. D **23**, 2218 (1981).

[17] J. Samuel, Phys. Rev. D **26**, 3475 (1982); **26**, 3482 (1982).

[18] T. Maruyama, S. W. Huang, N. Ohtsuka, G. Q. Li, A. Faessler, and J. Aichelin, Nucl. Phys. **A534**, 720 (1991).

[19] H. Sorge, H. Stöcker, and W. Greiner, Ann. Phys. **192**, 266 (1989).

[20] D. G. Currie, T. F. Jordan, and E. C. G. Sudarshan, Rev. Mod. Phys. **35**, 350 (1963).

[21] D. Dominici, J. Gomis, and G. Longhi, Nuovo Cimento A **48**, 257 (1978); B **48**, 152 (1978); A **56**, 263 (1980).

[22] E. Lehmann, R. K. Puri, A. Faessler, T. Maruyama, G. Q. Li, N. Ohtsuka, S. W. Huang, D. T. Khoa, and M. A. Matin, Prog. Part. Nucl. Phys. **30**, 210 (1993).

[23] T. Kodama, S. B. Duarte, K. C. Chung, R. Donangelo, and R. A. M. S. Nazareth, Phys. Rev. C **29**, 2146 (1984).

[24] J. Cugnon and M.-C. Lemaire, Nucl. Phys. **A489**, 781 (1988).

[25] R. Schmidt, B. Kaempfer, and H. Feldmeier, Phys. Lett. B **229**, 197 (1989).

[26] T. Maruyama, G. Q. Li and A. Faessler, Phys. Lett. B **268**, 160 (1991).

[27] A. P. Balachandran *et al.*, Phys. Rev. D **26**, 3482 (1982).