Core-polarization effects in pion elastic scattering from polarized spin-$\frac{1}{2}$ nuclei

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We have studied the core-polarization effects in pion elastic scattering from polarized ^{13}C and ^{15}N. The pion-nucleus spin-flip amplitude has been calculated within a framework of distorted-wave impulse approximation. To calculate the nuclear form factors we have used the wave function including the first-order core polarization with the intermediate states with excitation energies up to $12\hbar\omega$. It is shown that the core-polarization effects reduce the absolute values of the asymmetry.

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I. INTRODUCTION

Recently, experiments of pion scattering from polarized nuclear targets have been carried out for $0p$-shell nuclei $^{13}\vec{\text{C}}$, $^{15}\vec{\text{N}}$, $^{6}\vec{\text{Li}}$, and $^{7}\vec{\text{Li}}$ [1–5]. In the pion scattering from polarized spin-$\frac{1}{2}$ nuclei, the right-left asymmetry comes from an interference between pion-nucleus spin-flip and spin-nonflip amplitudes and is sensitive to the spin-flip amplitude. The experiments of pion elastic scattering from polarized spin-$\frac{1}{2}$ nuclei have been carried out on ^{13}C [3] and ^{15}N [1]. In these experiments, the observed asymmetry takes small absolute value due to dominant pion-nucleus spin-nonflip amplitude except for the angular range where spin-nonflip amplitude has a dip structure. Theoretical calculations have been carried out for ^{13}C within a framework of distorted-wave impulse approximation (DWIA) using the Cohen-Kurath wave function [6,7]. The theoretical value of the asymmetry is consistent with the experimental data at forward direction but disagrees with the data around the angle $\theta \sim 90°$. The wave function by Tiator [8,9] was also examined and was shown to give the sign of the asymmetry opposite to that calculated with the Cohen-Kurath wave function around the second dip. This shows the sensitivity of the asymmetry to the choice of nuclear wave function. The calculation of the elastic scattering π^+-$^{15}\vec{\text{N}}$ has been done also with an optical model based on the momentum-space coupled-channel formalism [10]. Although the calculated differential cross section was consistent with the experimental data, large discrepancies were found for the asymmetry; the measured asymmetry is nearly zero, while the theoretical results exhibit a sharply diffractive structure. Generally speaking, theoretical values of the asymmetry are larger than the data and do not always have the correct sign.

For spin-$\frac{1}{2}$ nuclei, two kinds of nuclear form factors $[Y_0 \times \sigma]^1$ and $[Y_2 \times \sigma]^1$ contribute to the pion-nucleus spin-flip amplitude. Siegel and Gibbs used the Cohen-Kurath wave function and have shown that the $[Y_2 \times \sigma]^1$ term dominates the spin-flip amplitude for ^{13}C [6]. Their theoretical value of the asymmetry does not agree with the experiment around the angle of the second dip of the elastic scattering cross section.

It is well known, however, that the $M1$ form factors calculated by the Cohen-Kurath wave function disagree with the experiment for ^{12}C and ^{13}C. The theoretical value is too large for ^{13}C and is too small for ^{12}C around the second peak of the $M1$ form factors. Several authors pointed out the importance of core-polarization effects on the $M1$ form factors for these nuclei [11–17] and it was shown that the above discrepancy can be explained by taking into account the first-order core polarization and the exchange-current contributions. For the case of ^{13}C, Suzuki *et al.* [17] showed that the core polarization largely enhances the isovector $[Y_0 \times \sigma]^1$ component and changes the sign of its contribution at $q \geq 1.4$ fm^{-1} while it little affects the $[Y_2 \times \sigma]^1$ component. The $M1$ form factors of the ^{12}C and ^{13}C were consistently explained mainly by the effect of the first-order core polarization [17]. It was pointed out that the proper treatment of the open-shell nature of the nucleus is important for the evaluation of the core polarization. If we adopt the simplified particle-hole method, core-polarization effects are considerably underestimated. They also pointed out that the proper subtraction of the Hartree-Fock one-body term is important. The relevant matrix elements in the spin-flip pion-nucleus amplitude are quite similar to those of the magnetization contribution in the $M1$ form factor, and hence we expect that the core-polarization effects play an important role for the pion asymmetry. There have been several theoretical calculations for the asymmetry of pion elastic scattering, but the core-polarization effects have never been examined. In order to make detailed comparison with the experimental data, it should be important to use the nuclear wave function which is consistent with the experimental $M1$ form factor. The nuclear wave function with the admixture of $2\hbar\omega$ component is used for the study of various reactions on ^{15}N [18] and for the single-charge exchange reaction on ^{13}C [19,20]. Oset *et al.* studied the core-polarization effects in pion single- and double-charge exchange reactions above the delta-resonance region [21].

The purpose of the present work is to investigate the effects of the first-order core polarization for the asymmetry in the pion elastic scattering from the polarized ^{13}C and ^{15}N. Our aim is not to make a detailed comparison with the experimental data, but to evaluate the correction coming from the core polarization. Our method of calculation for the core polarization is the same as that of Suzuki *et al.* [17]. In Sec. II, we briefly describe the calculation of pion-nucleus scattering amplitude including the core-polarization effects. The spin-nonflip amplitude is calculated with an optical potential of Stricker *et al.* [22–24] and the spin-flip amplitude is calculated under DWIA. In Sec. III, we show the results of our calculation. The core-polarization effects are shown to reduce the absolute values of the asymmetry for both ^{13}C and ^{15}N. We summarize the results in Sec. IV.

II. ASYMMETRY FOR PION ELASTIC SCATTERING

The scattering amplitude in the pion elastic scattering from spin-$\frac{1}{2}$ nucleus can be written as

$$\mathcal{F}(\theta) = f(\theta) + ig(\theta)\boldsymbol{\sigma}\cdot(\hat{\mathbf{k}}_i \times \hat{\mathbf{k}}_f), \tag{1}$$

where $f(\theta)$ and $g(\theta)$ are spin-nonflip and spin-flip amplitudes, respectively. $\hat{\mathbf{k}}_i(\hat{\mathbf{k}}_f)$ is a momentum of incident (outgoing) pion. Then we can express the differential cross section as

$$\left(\frac{d\sigma}{d\Omega}\right) = |f(\theta)|^2 + |g(\theta)|^2 \sin^2\theta. \tag{2}$$

The asymmetry $Ay(\theta)$ can be given as an interference between $f(\theta)$ and $g(\theta)$,

$$Ay(\theta) = \frac{2\mathrm{Im}(fg^*)\sin\theta}{|f|^2 + |g|^2\sin^2\theta}. \tag{3}$$

The spin-nonflip amplitude $f(\theta)$ can be decomposed into Coulomb and nuclear parts as

$$f(\theta) = f_C(\theta) + f_N(\theta), \tag{4}$$

where $f_C(\theta)$ and $f_N(\theta)$ are the Coulomb and the nuclear scattering amplitudes. In order to calculate the spin-nonflip amplitude $f(\theta)$, we use the pion-nucleus optical potential by Stricker *et al.* (MSU potential) [22–24] with the absorption parameters B_0 and C_0 determined phenomenologically by Gmitro *et al.* [25]. On the other hand, the spin-flip amplitude $g(\theta)$ is calculated under DWIA. For the pion scattering from spin-$\frac{1}{2}$ nuclei, the spin-flip DWIA amplitude is expressed as [26]

$$g(\theta) = \sqrt{\frac{2\pi}{3}}\sum_l (2l+1)P_l'(\cos\theta)\int_0^\infty dr\, r^2 \Bigg\{ \frac{2}{r}\left(-\frac{d}{dr} - \frac{1}{r}\right)(u_l^{(f)}u_l^{(i)})\left(\frac{c_0}{\sqrt{2}}F_{011}^{(0)}(r) - \alpha\frac{c_1}{\sqrt{6}}F_{011}^{(1)}(r)\right)$$
$$+\frac{1}{r}\left(-\frac{d}{dr} + \frac{2}{r}\right)(u_l^{(f)}u_l^{(i)})\left(\frac{c_0}{\sqrt{2}}F_{211}^{(0)}(r) - \alpha\frac{c_1}{\sqrt{6}}F_{211}^{(1)}(r)\right)\Bigg\}, \tag{5}$$

where $u_l^{(i)}(u_l^{(f)})$ is incident (outgoing) pion wave function and $\alpha = \pm 1$ for $\pi^\pm$. $c_k(k = 0, 1)$ are the coefficients of the isoscalar and the isovector pion-nucleon spin-flip amplitudes and we have used the pion-nucleon phase shifts by Rowe *et al.* [27]. Nuclear multipole densities $F_{LSJ}^{(k)}(r)$ are defined as

$$F_{LSJ}^{(k)}(r) = (-)^k\sqrt{2(2k+1)}\langle\Phi_i||\sum_j \frac{\delta(r-r_j)}{r^2}$$
$$\times[Y_L(\hat{\mathbf{r}}_j)\otimes\sigma_j^{(S)}]^J\tau_j^{(k)}||\Phi_i\rangle, \tag{6}$$

where Φ_i is the initial nuclear wave function. If we adopt the Cohen-Kurath wave function, the calculated $M1$ form factor is larger than the data for ^{13}C while smaller about an order of magnitude for ^{12}C around the second peak. These discrepancies are explained mainly by taking into account the first-order core-polarization effects. As was shown by Suzuki *et al.* [17], the core-polarization effects largely enhance the isovector $[Y_0\times\sigma]^1$ component. For ^{13}C, two terms $[Y_0\times\sigma]^1$ and $[Y_2\times\sigma]^1$ have the opposite signs and thus the core polarization reduces the $M1$ form factor. For the case of ^{12}C, these two terms have the same signs and the $M1$ form factor is largely enhanced in conformity with the experiment.

If we take into account the first-order core polarization, the nuclear density can be expressed as

$$F_{LSJ}^{(k)}(r) = (-)^k\sqrt{2(2k+1)}\Bigg[\langle\Phi_i||\sum_j \mathcal{O}_j||\Phi_i\rangle$$
$$+\sum_m\langle\Phi_i||\sum_j\mathcal{O}_j||\Phi_m\rangle\frac{1}{e}\langle\Phi_m|V_{\mathrm{res}} - U|\Phi_i\rangle$$
$$+\sum_m\langle\Phi_i|V_{\mathrm{res}} - U|\Phi_m\rangle\frac{1}{e}\langle\Phi_m||\sum_j\mathcal{O}_j||\Phi_i\rangle\Bigg]. \tag{7}$$

Since we are concerned with the one-body operator $\mathcal{O}_j$, the intermediate nuclear states $|\Phi_m\rangle$ have $1p1h$ configuration with respect to the ground state. V_{res} is a two-body residual interaction and U is the one-body potential for the Hartree-Fock bubble contribution.

III. RESULTS AND DISCUSSION

As was pointed out in Ref. [17], we should properly take into account the open-shell nature of the ^{13}C for the core-polarization calculation. If we assume the simplified particle-hole method, core-polarization effects are considerably underestimated for ^{13}C. Furthermore, the subtraction of the Hartree-Fock one-body term was shown to be important. Our method of calculation of core polarization is the same as that of Ref. [17]. We adopted the Cohen-Kurath wave function [7] with (8–16)POT two-body matrix elements as the unperturbed ground state. We did not examine the purely phenomenological wave function by Tiator and Wright for ^{13}C [8,9]. They determined the $0p$-shell reduced matrix element with the constraints coming from the experimental data on $M1$ form factor, magnetic moment, and β-decay rate and they could reproduce the (γ, π^-) reaction cross section. But it was shown that, for the (γ, π^-) reaction, rescattering correction plays a non-negligible role, especially for the case of $0p_{1/2} \to 0p_{1/2}$ transition [28]. As the one-body potential U, we assumed the harmonic-oscillator potential with size parameter $b = 1.543$ fm for ^{13}C, which was determined to fit the elastic charge form factor of ^{13}C. For ^{15}N, we used the oscillator parameter $b = 1.67$ fm which is taken from Ref. [18]. As the two body residual interaction $V_{\rm res}$, we used the two-body interaction which was successfully applied to calculate the $M1$ form factors in the electron scattering for ^{12}C and ^{13}C. The central part is the Gaussian form with the Rosenfeld-type exchange mixture. The force range is assumed to be $r_c = 1.6$ fm and the potential depth in the triplet-even state is taken to be $V_c = -60$ MeV. The tensor part is taken from the nucleon-nucleon interaction by Hamada and Johnstone [29] with a radial cutoff at 0.7 fm. We used the value $\hbar\omega = 15.4$ MeV to calculate the energy denominator. We have taken into account the intermediate states with excitation energies up to $12\hbar\omega$. For the $M1$ form factor of ^{13}C, two terms $[Y_0 \times \sigma]^1$ and $[Y_2 \times \sigma]^1$ contribute and the $[Y_2 \times \sigma]^1$ term is dominant around the second peak since the $p_{1/2} \to p_{1/2}$ matrix element is the main component. Similarly, the $[Y_2 \times \sigma]^1$ component gives dominant contribution for the case of pion-nucleus spin-flip amplitude for ^{13}C. The first-order core polarization is known to enhance the isovector $[Y_0 \times \sigma]^1$ term largely around $q \geq 1.4$ fm^{-1} and this effect is crucial for the explanation of the $M1$ form factor. Before making detailed comparison of the pion asymmetry, it should be necessary to take into account the core-polarization effects and to evaluate the correction coming from core polarization. The effects of the higher configurations are also studied by Bydzovsky *et al.* for single-charge exchange reactions [20]. They used the $0p$ shell-model wave function supplemented with the admixture of the $2\hbar\omega$ component. The wave function, however, fails to reproduce the $M1$ form factor around the second peak. It would be important to use the nuclear wave function which is, at least, consistent with the $M1$ form factor.

The pion-nucleus spin-nonflip amplitude $f(\theta)$ is calculated with the pion-nucleus optical potential by Stricker *et al.* (MSU potential) [22–24] which was constructed to describe the low-energy pion-nucleus elastic scattering. Since we are concerned with the energy region $T_\pi = 130$–160 MeV, we have to extrapolate the absorption parameters B_0 and C_0 to the higher-energy region. We adopted the values determined phenomenologically by Gmitro *et al.* [25]. Their approach is different from that of Stricker *et al.* The first-order optical potential is supplemented by the phenomenological ρ^2 terms which simulate the pion absorption and the higher-order effects. The coefficients B_0 and C_0 are determined from a fit to the experimental elastic scattering cross section. They have obtained the energy-dependent parameters and these parameters are close to the value of the MSU potential at the low-energy region. The pion-nucleus spin-flip amplitude $g(\theta)$ can be easily calculated according to Eq. (5) by using the pion distorted waves generated by the MSU potential. Since the form of pion-nucleus optical potential used by Gmitro *et al.* is different from that of Stricker *et al.*, the use of Gmitro's absorption parameters is not a fully consistent procedure and also the use of the impulse values for the potential parameters might not be justified. Hence, we have varied the isoscalar s- and p-wave potential parameters $\bar{b}_0$ and c_0, and searched for the best-fit values for the elastic scattering cross section. The parameters thus determined for 130 MeV are $\bar{b}_0 = -0.065 + 0.107i$ fm and $c_0 = 0.587 + 0.479i$ fm^3 for π^+ and $\bar{b}_0 = -0.110 + 0.068i$ fm and $c_0 = 0.484 + 0.398i$ fm^3 for π^-, respectively. For the p-wave parameter c_0, the best-fit values are reduced about 60–70 % from the impulse values $c_0(\text{best fit})/c_0(\text{impulse}) \sim 0.6$–0.7 for both the $\pi^\pm$. Figure 1 shows the results for π-^{13}C elastic scattering with the potential parameters calculated by pion-nucleon phase shifts. The contribution from the spin-flip amplitude is quite small and consequently the asymmetry is fairly small except for the angular range where spin-nonflip contribution has a dip structure. The core-polarization effect is not very large but works to reduce

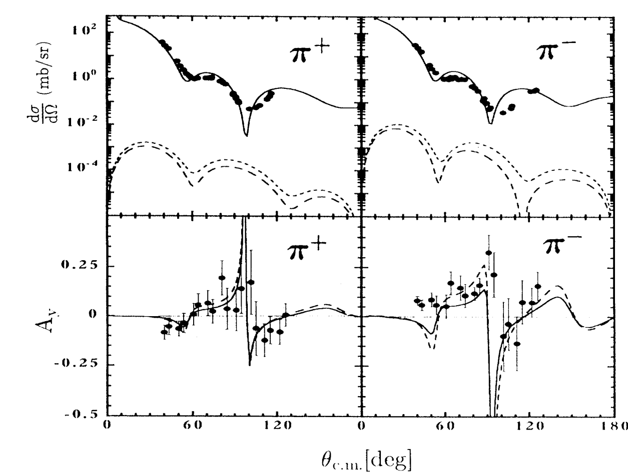

FIG. 1. Core-polarization effects for the elastic scattering $\pi^\pm$-^{13}C at $T_\pi = 130$ MeV. For the cross section, the long-dashed and the dashed lines correspond to the spin-flip contribution with and without the core polarization, respectively. For the asymmetry, the solid and the dashed lines present the results with and without the core-polarization effects, respectively. The experimental data are taken from Ref. [3].

the absolute value of the asymmetry. Figure 2 shows the results with the best-fit potential parameters. Around the second dip of the cross section, it gives shallower dip structure and the asymmetry is moderately affected. In both of these calculations, the core-polarization effects reduce the absolute value of the asymmetry. Siegel and Gibbs [6] examined the different treatment of the spin-flip interactions: spin-dependent optical potential versus DWIA. They showed that the results are almost the same for both of these treatments because of the rather weak spin-dependent interaction. Then our DWIA treatment would be sufficient for the calculation of the spin-dependent interaction. Next, in order to see the q dependence of the core-polarization effects, we show the asymmetry for the elastic scattering for various incident energies in Fig. 3. We used the best-fit potential parameters for 130 MeV. For the other energies, the number of the experimental data are quite few and it is impossible to search for the potential parameters, and then we used the impulse values. Obviously, the core-polarization effects become appreciable at the high-q region.

Figure 4 shows the results of the ^{15}N at 164 MeV. We used the best-fit potential parameters: $\bar{b}_0 = 0.039 + 0.452i$ fm and $c_0 = 0.387 + 0.322i$ fm^3 for π^+ and $\bar{b}_0 = 0.060 + 0.345i$ fm and $c_0 = 0.446 + 0.439i$ fm^3 for π^-. In this case, the imaginary part of the p-wave parameter c_0 is considerably reduced from the impulse values. If we use the impulse values, the results are highly diffractive and the resulting asymmetry also exhibits sharp positive-negative patterns. Best-fit potential gives shallower structure for the cross section and then the absolute value of the asymmetry becomes smaller.

Our results show that the core polarization is more appreciable for π^--^{13}C and π^+-^{15}N. The spin-flip amplitude itself is an order of magnitude larger for these cases than for π^+-^{13}C and π^--^{15}N and this is simply because the dominant configuration is $p_{1/2}$-neutron for ^{13}C and $p_{1/2}$-proton hole for ^{15}N. Though our primary interest is to evaluate the correction coming from the core-polarization effects and not to make a detailed comparison with the experimental data, we obtained overall agreement with the experimental data.

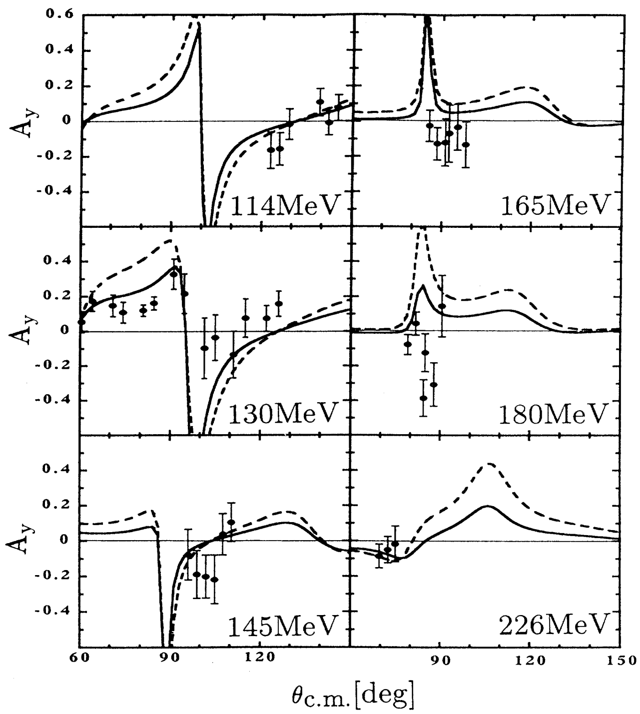

FIG. 3. Core-polarization effects for the asymmetry in the case of π^--^{13}C elastic scattering for various incident energies. The solid and the dashed lines are the results with and without the core-polarization effects. The experimental data are taken from Ref. [3].

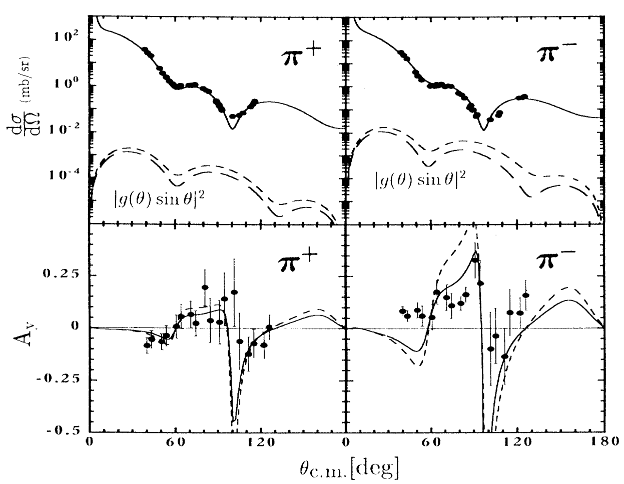

FIG. 2. The same as in Fig. 1. We used the best-fit potential parameters $\bar{b}_0$ and c_0 to calculate the elastic scattering cross section and distorted waves. The experimental data are taken from Ref. [3].

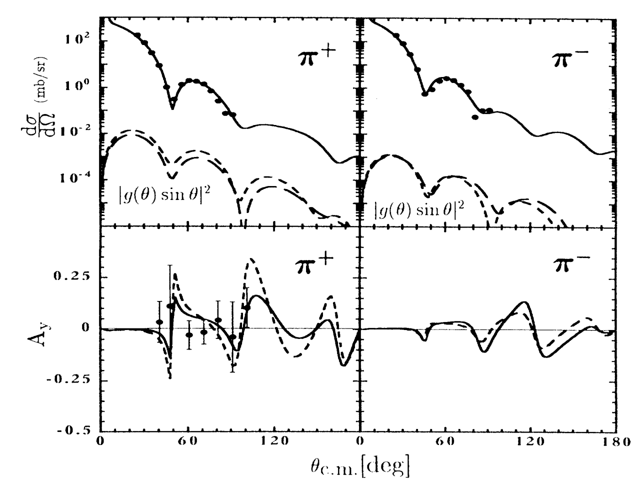

FIG. 4. Core-polarization effects for the elastic scattering $\pi^{\pm}$-^{15}N at $T_\pi = 164$ MeV. The lines have the same meaning as those in Fig. 1. The experimental data are taken from Refs. [1,30].

IV. SUMMARY

We have studied the first-order core-polarization effects in the pion elastic scattering from the polarized spin-$\frac{1}{2}$ nuclei ^{13}C and ^{15}N. The pion-nucleus spin-flip amplitude has been calculated under DWIA and is expressed in terms of the nuclear form factors $[Y_0 \times \sigma]^1$ and $[Y_2 \times \sigma]^1$. These nuclear form factors are calculated by using the nuclear wave function including the first-order core polarization with intermediate states with excitation energies up to $12\hbar\omega$. The core-polarization effects are not very large but work to reduce the absolute values of the pion-nucleus spin-flip amplitude at almost all the angles and hence reduce the absolute value of the asymmetry somewhat. At the delta-resonance region, the valence neutron couples more strongly to π^- than π^+. Accordingly the effects of the core polarization are larger for π^--^{13}C and π^+-^{15}N than for π^+-^{13}C and π^--^{15}N.

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