# Extension of fractional parentage expansion to the nonrelativistic and relativistic $SU^f(3)$ dibaryon calculations

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The fractional parentage expansion method is extended from $SU^f(2)$ nonrelativistic to $SU^f(3)$ and relativistic dibaryon calculations. A transformation table between physical bases and symmetry bases for the $SU^f(3)$ dibaryon is provided. A program package is written for dibaryon calculation based on the fractional parentage expansion method.

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## I. INTRODUCTION

Quantum chromodynamics (QCD) is a very promising theory for fundamental strong interactions. However, because of the complexity of QCD, for the present time and for the foreseeable future, one must rely on QCD-inspired models to study hadron physics. The existing models (potential, bag, soliton, etc.) are quite successful for the meson and baryon sectors, but not so successful for hadronic interactions. Recently, some hope has developed to obtain the full $N$-$N$ interaction from QCD models [1,2].

Since the first prediction of the $H$ particle by Jaffe [3], there have been tremendous efforts both theoretically and experimentally [4] to find possible candidates for quasistable dibaryon states. Nevertheless, there remains an outstanding question. Theoretically, all the QCD models, including lattice QCD calculations, predict that there should be quasistable dibaryons or dibaryon resonances, but in contrast, experimentally, no quasistable dibaryon whatsoever has been observed (except the molecular deuteron state). One has to ask if some important QCD characteristics are missing in all these dibaryon calculations. For example, in the potential (or cluster) model approach, the six-quark Hamiltonian is usually a direct extension of the three-quark Hamiltonian. This extension is neither reasonable nor successful. The two-body confinement potential yields color van der Waals forces which are in contradiction with experimental observation. Lattice gauge calculations and nonperturbative QCD both yield a stringlike structure inside a hadron instead of two-body confinement. Two-body confinement may be a reasonable approximation inside a hadron, but not for the interaction between quarks in two color-singlet hadrons [5]. Another possible missed general feature is that the quark, originally confined in a single hadron, may tunnel (or percolate) to the other hadron when two hadrons are close together [6]. In the potential model approach, the internal motion of the interacting hadrons is assumed to be unchanged. The product ansatz of the Skyrmion model approach makes the same approximation. In the bag model approach, another extreme approximation is assumed; i.e., the six quarks are merged into a single confinement space. The real configuration may be in between these two extremes, which is well known in molecular physics.

Except for a few cluster model calculations, in which a phenomenological meson exchange is involved to fit the $N$-$N$ scattering, for all the other dibaryon calculations, the model parameters are only constrained by hadron spectroscopy. In fact, the six-quark system includes new color structures, for which a single hadron cannot give any information. A six-quark Hamiltonian should be constrained by the existing baryon-baryon interaction data, especially the $N$-$N$ data; then, the model dibaryon states may be really relevant to the experimental measurement.

A model, the quark delocalization color screening model (QDCSM), has been developed which includes the new QCD-inspired ingredients mentioned above and is constrained by $N$-$N$ scattering data [2]. This model has been applied to a systematic search of the dibaryon candidates in the $u$, $d$, and $s$ three-flavor world [7] to provide a better estimate of dibaryon states on the one hand and to test the model assumption further on the other hand.

As pointed out in [4], a more realistic systematic search of dibaryons would be a tremendous task, for which a systematic and powerful method is indispensable. The fractional-parentage (fp) expansion developed in atomic and nuclear physics is one of such methods. A major obstacle in applying the fp-expansion technique to quark models is the occurrence of many $SU(mn) \supset SU(m)\times SU(n)$ isoscalar factors (ISF's) with $m, n \geq 2$, e.g., in the two "orbits," two spins, $n_f$ flavors, and three-color quark world. We need the $SU(2\times 3\times n_f \times 2) \supset SU^x(2)\times SU(6n_f)$, $SU(6n_f) \supset SU^c(3)\times SU(2n_f)$,

 

$SU(2n_f) \supset SU^f(n_f) \times SU^\sigma(2)$ ISF's, where $x$, $c$, $f$, and $\sigma$ indicate the space or orbit, color, flavor, and spin, respectively. Before 1991, only the $SU(4)\supset SU(2)\times SU(2)$ ISF [and some scattered results for the $SU(6)\supset SU(3)\times SU(2)$ ISF] was available. A breakthrough in group representation theory is the recognition of the fact that the $SU(n_1 n_2) \supset SU(n_1)\times SU(n_2)$ $n_2$-particle coefficients of fp (cfp) are precisely the ISF for the permutation group chain $S(n_1+n_2) \supset S(n_1) \times S(n_2)$ [8(a)], and the former can be calculated ad tabulated in a rank-independent way, instead of one $m$ and $n$ at a time. In 1991, Chen *et al.* [8(b)] published a book with phase-consistent $SU(mn) \supset SU(m)\times SU(n)$ ISF's for arbitrary $m$ and $n$ and for up to six particles. Because of this, we are now in a position to develop an efficient algorithm for dibaryon calculations based on the fp technique. This paper reports the extension of the fp expansion to the nonrelativistic and relativistic $SU^f(3)$ quark model calculation in line with the work of Harvey [9] and of Chen *et al.* [10].

## II. PHYSICAL AND SYMMETRY BASES

A dibaryon may be a loosely bound two-$q^3$-cluster state like the deuteron, or it may be a tightly bound $q^6$ cluster like Jaffe's version of the $H$ particle. Many cases may be in between. To describe these states, the physical basis is preferable because of its apparent dibaryon content in the asymptotic region without artificial confinement assumptions. The physical basis is nothing else but the cluster model basis developed in the nuclear cluster model [11]. To show the symmetry property explicitly, we follow the notation of Chen *et al.* [8,10], but with a slight modification, because we are working in the $u$, $d$, $s$ three-flavor world instead of $u$, $d$ flavors.

A baryon in the $u$, $d$, $s$ three-flavor world is described by

$$\psi(B) = \left| [\sigma] W \, [\mu] \overset{[\nu]}{[f]} \, YIJ \right\rangle , \tag{1}$$

which is a basis vector belonging to the irreducible representations (irreps)

$$\overset{[1^3]}{SU(36)} \supset \left( \overset{[\nu]}{SU^x(2)} \times \left\{ \overset{[\tilde\nu]}{SU(18)} \supset \overset{[\sigma]}{SU^c(3)} \times \left[ \overset{[\mu]}{SU(6)} \supset \left( \overset{[f]}{SU^f(3)} \supset \overset{I}{SU^\tau(2)} \times \overset{Y}{U^Y(1)} \right) \times \overset{J}{SU^\sigma(2)} \right] \right\} \right) , \tag{2}$$

where the first reduction is to orbital times combined color-flavor-spin symmetry, the second reduces the latter to color times combined flavor spin, and the third reduces the last to flavor (which is itself reduced to isospin times hypercharge) times spin. Here $[\nu]$, etc., are the Young diagrams describing the permutational and $SU(n)$ symmetries. In our calculation, the ground-state baryons are assumed to be in the totally symmetric orbital state $[\nu]=[3]$, while $[\sigma]=[1^3]W$ is the Weyl tableau for the $SU^c(3)$ state due to color confinement; i.e., the baryon is colorless. On the other hand, $[\mu]=[3]$ as a result of the totally antisymmetry requirement $[\sigma]\times[\mu]\to[\tilde\nu]=[1^3]$, $[\tilde\nu]$ being the conjugate Young diagram of $[\nu]$. $[f]$ and $[\sigma_J]$ are restricted by the condition $[f]\times[\sigma_J]\to[\mu]=[3]$, and this leads to $[f]=[\sigma_J]=[3]$ or $[21]$; $[\sigma_J]$ represents the spin symmetry, $[\sigma_J]=[\frac{n}{2}+J,\frac{n}{2}-J]$, and $n$ is the total number of quarks, i.e., the $SU^f(3)$ decuplet and octet baryons $\Delta, \Sigma^*, \Xi^*, \Omega$ and $N, \Lambda, \Sigma, \Xi$. The symbols $Y$, $I$, and $J$ denote the hypercharge, isospin, and spin quantum numbers, respectively. $M_I$ and $M_J$, the magnetic quantum numbers, are omitted in Eq. (1).

A two-baryon physical basis is described by

$$\begin{aligned}\Psi_{\alpha k}(B_1B_2) &= \mathcal{A}[\psi(B_1)\psi(B_2)] \overset{[\sigma]\ I\ \ J}{W M_I M_J} \\ &= \mathcal{A}\left[ \left| [\sigma_1][\mu_1]\overset{[\nu_1]}{[f_1]} Y_1 I_1 J_1 \right\rangle \left| [\sigma_2][\mu_2]\overset{[\nu_2]}{[f_2]} Y_2 I_2 J_2 \right\rangle \right] \overset{[\sigma]\ I\ \ J}{W M_I M_J} ;\end{aligned} \tag{3}$$

here, $[\ \ ] \overset{[\sigma]\ I\ \ J}{W M_I M_J}$ means couplings in terms of the $SU^c(3)$, $SU^\tau(2)$, and $SU^\sigma(2)$ Clebsch-Gordan coefficients (CGC's) so that it has total color symmetry $[\sigma]W$, isospin $IM_I$, and spin $JM_J$. Because of color confinement, only the overall color singlet $[\sigma]=[2^3]$ is allowed. $\mathcal{A}$ is a normalized antisymmetric operator. $\alpha=(YIJ)$ with $Y=Y_1+Y_2, k$ represents the quantum numbers $\nu_i, \sigma_i, \mu_i, f_i, I_i, J_i$ $(i=1,2)$.

To take into account the mutual distortion or the internal orbital excitation of the interacting baryons when they are near one another, the delocalized single-quark state $l(r)$ is used for baryon $B_1$ $(B_2)$ [2,6,7]

$$l = [\phi_L + \epsilon(s)\phi_R]/N(s), \quad r = [\phi_R + \epsilon(s)\phi_L]/N(s) ,$$

$$\phi_L(\vec r) = \left(\frac{1}{\pi b^2}\right)^{\frac{3}{4}} e^{-\frac{(\vec r - \vec s/2)^2}{2b^2}} , \tag{4}$$

$$N^2(s) = 1 + \epsilon^2(s) + 2\epsilon(s)\langle \phi_L | \phi_R \rangle .$$

The $\vec s$ is the separation between two-$q^3$-cluster centers, and $\epsilon(s)$ is a parameter describing the delocalization (or percolation) effect, which is determined variationally by the $q^6$ dynamics. Hidden color channels are not included in Eq. (3), because it has been proven [12] that colorless hadron channels form a complete Hilbert space if excited colorless baryon states are included. Also, the concept of a colorful hadron has not been well defined in QCD models.

Physical bases are not convenient for matrix element calculations. To take advantage of the fp-expansion technique developed in atomic and nuclear physics, one has to use symmetry bases (group chain classification bases). This requires an extension of the $q^3$ state [Eq. (1)] to the $q^6$ case,

$$\Phi_{\alpha K}(q^6) = \left| [\sigma]W[\mu]\beta\overset{[\nu]l^3r^3}{[f]}YIJM_IM_J \right\rangle . \quad (5)$$

Here $K$ represents the quantum numbers $[\nu],[\mu],\beta,[f]$ appearing in Eq. (5). $[\sigma]=[2^3]$ as a result of color confinement. To be consistent with the physical basis choice, the orbital part is truncated to include the $l^3r^3$ configuration only. $[\nu]$ is restricted to be $[\nu]=[3]\times[3]=[6]+[51]+[42]+[33]$. $\beta$ is the inner multiplicity index in the reduction $[\mu]\to[f]\times[\sigma_J]$.

Physical and symmetry bases both form a complete set in a truncated Hilbert space and are related by a unitary transformation. Harvey [9] first calculated the transformation coefficients for the $u$, $d$ two-flavor case. Chen *et al.* [10] proved that the transformation coefficients are just a product of $(6\to 3+3)\mathrm{SU}(mn)\supset\mathrm{SU}(m)\times\mathrm{SU}(n)$ isoscalar factors. Here we extend them to the $\mathrm{SU}^f(3)$ case,

$$\mathcal{A}\left[\left|[\sigma_1][\mu_1]\overset{[\nu_1]l^3}{[f_1]}Y_1I_1J_1\right\rangle\left|[\sigma_2][\mu_2]\overset{[\nu_2]r^3}{[f_2]}Y_2I_2J_2\right\rangle\right]\overset{[\sigma]\ I\ \ J}{WM_IM_J}$$

$$=\sum_{\bar\nu\mu\beta f\gamma} C^{[\bar\nu][\sigma][\mu]}_{[\bar\nu_1][\sigma_1][\mu_1][\bar\nu_2][\sigma_2][\mu_2]}C^{[\mu]\beta[f]\gamma J}_{[\mu_1][f_1]J_1,[\mu_2][f_2]J_2}C^{[f]\gamma YI}_{[f_1]Y_1I_1[f_2]Y_2I_2}\left|[\sigma]W[\mu]\beta\overset{[\nu]l^3r^3}{[f]}YIJM_IM_J\right\rangle . \quad (6)$$

This expression is written simply as

$$\Psi_{\alpha k}(B_1B_2)=\sum_K C_{kK}\Phi_{\alpha K}(q^6) \ ; \quad (7)$$

here, $\gamma$ is an outer multiplicity index in the reduction $[f_1]\times[f_2]\to[f]$. The first two $C$ factors in Eq. (6) are the $\mathrm{SU}(18)\supset\mathrm{SU}^c(3)\times\mathrm{SU}(6)$ and $\mathrm{SU}(6)\supset\mathrm{SU}^f(3)\times\mathrm{SU}^\sigma(2)$ isoscalar factors, respectively, and the third one is the $\mathrm{SU}^f(3)\supset\mathrm{SU}^\tau(2)\times\mathrm{U}^Y(1)$ isoscalar factor. All these isoscalar factors can be found in Ref. [8(b)]. The calculated transformation coefficients are listed in Table I. The $Y=2$ part is a revised version (phase consistent and simplified for $I=J=1$ case) of Harvey's Table 11 [9]. (The relationship between our tables and those of Harvey is discussed in the Appendix.) The $Y\neq 2$ part is an extension of Harvey's two-flavor case to a three-flavor case. Because the hidden color channels are not included, this table can be used to expand the physical bases in terms of the symmetry bases only. If one wants to expand the symmetry bases in terms of the physical bases, then the hidden color physical bases (or other equivalent set of bases) should be added. One example is

$$\begin{aligned}|H\rangle = &\sqrt{\tfrac{1}{5}}\left(\sqrt{\tfrac{3}{8}}|\Sigma\Sigma\rangle-\sqrt{\tfrac{4}{8}}|\bar N\Xi\rangle-\sqrt{\tfrac{1}{8}}|\Lambda\Lambda\rangle\right)\\ &-\sqrt{\tfrac{6}{40}}|\Sigma\Sigma\rangle_c+\sqrt{\tfrac{8}{40}}|\bar N\Xi\rangle_c+\sqrt{\tfrac{2}{40}}|\Lambda\Lambda\rangle_c\\ &+\sqrt{\tfrac{3}{40}}|\Sigma'\Sigma'\rangle_c-\sqrt{\tfrac{4}{40}}|\bar N'\Xi'\rangle_c-\sqrt{\tfrac{1}{40}}|\Lambda'\Lambda'\rangle_c\\ &-\sqrt{\tfrac{8}{40}}|\Lambda_s\Lambda_s\rangle_c \ . \end{aligned} \quad (8)$$

Here $|\overline{XY}\}$ means the symmetric channel of baryons $X$ and $Y$, $|XY\rangle_c$ means hidden color channel of colorful baryons $X$ and $Y$, $\Lambda_s$ is the flavor singlet $\Lambda$, and $X'$ represents an excited colorful baryon with spin $\frac{3}{2}$. In the prevailing literature, only the first three colorless channels are given [13]. See the Appendix for a description of the difference between our meaning for symmetry and that of Harvey [9].

## III. FRACTIONAL PARENTAGE EXPANSION

A physical six-quark state with quantum number $\alpha=(YIJ)$ is expressed as a channel coupling wave function (WF)

$$\Psi_\alpha=\sum_k C_k\Psi_{\alpha k}(B_1B_2) \ . \quad (9)$$

The channel coupling coefficients $C_k$ are determined by the diagonalization of the six-quark Hamiltonian as usual.

To calculate the six-quark Hamiltonian matrix elements in the physical basis,

$$H_{kk'}=\langle\Psi_{\alpha k}||H||\Psi_{\alpha k'}\rangle \ , \quad (10)$$

is tedious. We first express the physical basis in terms of the symmetry basis by the transformation [Eq. (6)], and the matrix element [Eq. (10)] is transformed into a sum of matrix elements in the symmetry basis

$$H_{kk'}=\sum_{K,K'}C_{kK}C_{k'K'}\langle\Phi_{\alpha K}|H|\Phi_{\alpha K'}\rangle \ . \quad (11)$$

The matrix elements $\{\Phi_{\alpha K}|H|\Phi_{\alpha K'}\}$ can be calculated by the well-known fp expansion method,

TABLE I. Transformation coefficients between physical bases and symmetry bases. The column labels are $[\nu], [\mu], [f]$: where 1 stands for the symmetry label [6]; 2, [51]; 3, [42]; 4, [33]; 5, [411]; 6, [321]; 7, [222]. The row labels are $B_1B_2$: 1 stand for the nucleon $N$; 2, $\Sigma$; 3, $\Xi$; 4, $\Lambda$; 5, $\Delta$; 6, $\Sigma^*$; 7, $\Xi^*$; 8, $\Omega$. $\overline{xy}$ ($\widetilde{xy}$) means symmetric (antisymmetric) channel of baryons $x$ and $y$. The transformation coefficients should be the square root of the entries, and a negative sign means to take the negative square root.

(a) $Y = 2$

| $IJ = 33$ | 411 |
|---|---|
| 55 | 1 |

| $IJ = 32$ | 321 |
|---|---|
| 55 | 1 |

| $IJ = 31$ | 231 | 431 |
|---|---|---|
| 55 | $-\frac{5}{9}$ | $-\frac{4}{9}$ |

| $IJ = 30$ | 141 | 341 |
|---|---|---|
| 55 | $-\frac{1}{5}$ | $-\frac{4}{5}$ |

| $IJ = 23$ | 322 |
|---|---|
| 55 | 1 |

| $IJ = 22$ | 232 | 322 | 412 | 432 |
|---|---|---|---|---|
| $\overline{15}$ | $\frac{1}{9}$ | 0 | $\frac{4}{5}$ | $\frac{4}{45}$ |
| $\widetilde{51}$ | 0 | $-1$ | 0 | 0 |
| 55 | $-\frac{4}{9}$ | 0 | $\frac{1}{5}$ | $-\frac{16}{45}$ |

| $IJ = 21$ | 142 | 232 | 322 | 342 | 432 |
|---|---|---|---|---|---|
| $\overline{15}$ | $\frac{4}{45}$ | 0 | $\frac{5}{9}$ | $\frac{16}{45}$ | 0 |
| $\widetilde{15}$ | 0 | $\frac{5}{9}$ | 0 | 0 | $\frac{4}{9}$ |
| 55 | $-\frac{1}{9}$ | 0 | $\frac{4}{9}$ | $-\frac{4}{9}$ | 0 |

| $IJ = 20$ | 232 | 432 |
|---|---|---|
| 55 | $-\frac{5}{9}$ | $-\frac{4}{9}$ |

| $IJ = 13$ | 233 | 433 |
|---|---|---|
| 55 | $-\frac{5}{9}$ | $-\frac{4}{9}$ |

| $IJ = 12$ | 143 | 233 | 323 | 343 | 4133 |
|---|---|---|---|---|---|
| $\overline{15}$ | $\frac{4}{45}$ | 0 | $\frac{5}{9}$ | $\frac{16}{45}$ | 0 |
| $\widetilde{15}$ | 0 | $\frac{5}{9}$ | 0 | 0 | $\frac{4}{9}$ |
| 55 | $-\frac{1}{9}$ | 0 | $\frac{4}{9}$ | $-\frac{4}{9}$ | 0 |

| $IJ = 11$ | $2_133$ | $2_233$ | 323 | 413 | $4_133$ | $4_233$ |
|---|---|---|---|---|---|---|
| 11 | $\frac{5}{81}$ | $\frac{20}{81}$ | 0 | $\frac{4}{9}$ | $\frac{4}{81}$ | $\frac{16}{81}$ |
| $\overline{15}$ | $-\frac{20}{81}$ | $-\frac{5}{81}$ | 0 | $\frac{4}{9}$ | $-\frac{16}{81}$ | $-\frac{4}{81}$ |
| $\widetilde{15}$ | 0 | 0 | $-1$ | 0 | 0 | 0 |
| 55 | $\frac{20}{81}$ | $-\frac{20}{81}$ | 0 | $\frac{1}{9}$ | $\frac{16}{81}$ | $-\frac{16}{81}$ |

| $IJ = 10$ | 143 | 323 | 343 |
|---|---|---|---|
| 11 | $\frac{1}{9}$ | $\frac{4}{9}$ | $\frac{4}{9}$ |
| 55 | $-\frac{4}{45}$ | $\frac{5}{9}$ | $-\frac{16}{45}$ |

| $IJ = 03$ | 144 | 344 |
|---|---|---|
| 55 | $-\frac{1}{5}$ | $-\frac{4}{5}$ |

| $IJ = 02$ | 234 | 434 |
|---|---|---|
| 55 | $-\frac{5}{9}$ | $-\frac{4}{9}$ |

TABLE I. *(Continued).*

| $IJ=01$ | 144 | 324 | 344 |
|---|---|---|---|
| 11 | $\frac{1}{9}$ | $\frac{4}{9}$ | $\frac{4}{9}$ |
| 55 | $-\frac{4}{45}$ | $\frac{5}{9}$ | $-\frac{16}{45}$ |

| $IJ=00$ | 234 | 414 | 434 |
|---|---|---|---|
| 11 | $\frac{1}{9}$ | $\frac{4}{5}$ | $\frac{4}{45}$ |
| 55 | $-\frac{4}{9}$ | $\frac{1}{5}$ | $-\frac{16}{45}$ |

(b) $Y=1$

| $IJ=\frac{5}{2}3$ | 322 | 411 |
|---|---|---|
| $\overline{56}$ | 0 | 1 |
| $\widetilde{56}$ | 1 | 0 |

| $IJ=\frac{5}{2}2$ | 232 | 321 | 322 | 412 | 432 |
|---|---|---|---|---|---|
| $\overline{25}$ | $\frac{1}{9}$ | 0 | 0 | $\frac{4}{5}$ | $\frac{4}{45}$ |
| $\widetilde{25}$ | 0 | 0 | $-1$ | 0 | 0 |
| $\overline{56}$ | 0 | 1 | 0 | 0 | 0 |
| $\widetilde{56}$ | $-\frac{4}{9}$ | 0 | 0 | $\frac{1}{5}$ | $-\frac{16}{45}$ |

| $IJ=\frac{5}{2}1$ | 142 | 231 | 232 | 322 | 342 | 431 | 432 |
|---|---|---|---|---|---|---|---|
| $\overline{25}$ | $\frac{4}{45}$ | 0 | 0 | $\frac{5}{9}$ | $\frac{16}{45}$ | 0 | 0 |
| $\widetilde{25}$ | 0 | 0 | $\frac{5}{9}$ | 0 | 0 | 0 | $\frac{4}{9}$ |
| $\overline{56}$ | 0 | $-\frac{5}{9}$ | 0 | 0 | 0 | $-\frac{4}{9}$ | 0 |
| $\widetilde{56}$ | $-\frac{1}{9}$ | 0 | 0 | $\frac{4}{9}$ | $-\frac{4}{9}$ | 0 | 0 |

| $IJ=\frac{3}{2}0$ | 141 | 232 | 341 | 432 |
|---|---|---|---|---|
| $\overline{56}$ | $-\frac{1}{5}$ | 0 | $-\frac{4}{5}$ | 0 |
| $\widetilde{56}$ | 0 | $-\frac{5}{9}$ | 0 | $-\frac{4}{9}$ |

| $IJ=\frac{3}{2}3$ | 233 | 322 | 433 |
|---|---|---|---|
| $\overline{56}$ | 0 | 1 | 0 |
| $\widetilde{56}$ | $-\frac{5}{9}$ | 0 | $-\frac{4}{9}$ |

| $IJ=\frac{3}{2}2$ | 143 | 232 | 233 | 235 | 322 | 323 | 325 | 343 | 412 | 432 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\overline{16}$ | 0 | $\frac{5}{72}$ | $\frac{5}{72}$ | $-\frac{5}{36}$ | 0 | 0 | 0 | 0 | $\frac{1}{2}$ | $\frac{1}{18}$ |
| $\overline{25}$ | $\frac{1}{36}$ | 0 | 0 | 0 | $-\frac{1}{16}$ | $\frac{25}{144}$ | $-\frac{5}{8}$ | $\frac{1}{9}$ | 0 | 0 |
| $\overline{45}$ | 0 | $\frac{5}{144}$ | $-\frac{5}{16}$ | $\frac{5}{72}$ | 0 | 0 | 0 | 0 | $\frac{1}{4}$ | $\frac{1}{36}$ |
| $\widetilde{25}$ | 0 | $\frac{1}{144}$ | $\frac{25}{144}$ | $\frac{25}{72}$ | 0 | 0 | 0 | 0 | $\frac{1}{20}$ | $\frac{1}{180}$ |
| $\widetilde{15}$ | $-\frac{1}{20}$ | 0 | 0 | 0 | $-\frac{5}{16}$ | $-\frac{5}{16}$ | $-\frac{1}{8}$ | $-\frac{1}{5}$ | 0 | 0 |
| $\overline{56}$ | 0 | $-\frac{4}{9}$ | 0 | 0 | 0 | 0 | 0 | 0 | $\frac{1}{5}$ | $-\frac{16}{45}$ |
| $\widetilde{16}$ | $\frac{1}{90}$ | 0 | 0 | 0 | $-\frac{5}{8}$ | $\frac{5}{72}$ | $\frac{1}{4}$ | $\frac{2}{45}$ | 0 | 0 |
| $\widetilde{56}$ | $-\frac{1}{9}$ | 0 | 0 | 0 | 0 | $\frac{4}{9}$ | 0 | $-\frac{4}{9}$ | 0 | 0 |

| | 433 | 435 |
|---|---|---|
| $\overline{16}$ | $\frac{1}{18}$ | $-\frac{1}{9}$ |
| $\overline{25}$ | 0 | 0 |
| $\overline{45}$ | $-\frac{1}{4}$ | $\frac{1}{18}$ |
| $\widetilde{25}$ | $\frac{5}{36}$ | $\frac{5}{18}$ |
| $\widetilde{45}$ | 0 | 0 |
| $\overline{56}$ | 0 | 0 |
| $\widetilde{16}$ | 0 | 0 |
| $\widetilde{56}$ | 0 | 0 |

| $IJ=\frac{3}{2}1$ | 142 | 145 | 232 | $23_13$ | $23_23$ | 235 | 322 | 323 | 325 | 342 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\overline{16}$ | $\frac{1}{18}$ | $-\frac{2}{45}$ | 0 | 0 | 0 | 0 | $\frac{25}{72}$ | $-\frac{1}{8}$ | $\frac{1}{36}$ | $\frac{2}{9}$ |

TABLE I. *(Continued).*

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\overline{25}$ | 0 | 0 | $\frac{5}{144}$ | $-\frac{25}{324}$ | $-\frac{25}{1296}$ | $\frac{25}{72}$ | 0 | 0 | 0 | 0 |
| $\overline{45}$ | $\frac{1}{36}$ | $\frac{1}{45}$ | 0 | 0 | 0 | 0 | $\frac{25}{144}$ | $\frac{9}{16}$ | $-\frac{1}{72}$ | $\frac{1}{9}$ |
| $\widetilde{25}$ | $\frac{1}{180}$ | $\frac{1}{9}$ | 0 | 0 | 0 | 0 | $\frac{5}{144}$ | $-\frac{5}{16}$ | $-\frac{5}{72}$ | $\frac{1}{45}$ |
| $\widetilde{45}$ | 0 | 0 | $\frac{25}{144}$ | $\frac{5}{36}$ | $\frac{5}{144}$ | $\frac{5}{72}$ | 0 | 0 | 0 | 0 |
| $\overline{56}$ | $-\frac{1}{9}$ | 0 | 0 | 0 | 0 | 0 | $\frac{4}{9}$ | 0 | 0 | $-\frac{4}{9}$ |
| $\widetilde{16}$ | 0 | 0 | $\frac{25}{72}$ | $-\frac{5}{162}$ | $-\frac{5}{648}$ | $-\frac{5}{36}$ | 0 | 0 | 0 | 0 |
| $\widetilde{56}$ | 0 | 0 | 0 | $\frac{20}{81}$ | $-\frac{20}{81}$ | 0 | 0 | 0 | 0 | 0 |
| $\overline{12}$ | 0 | 0 | 0 | $\frac{5}{81}$ | $\frac{20}{81}$ | 0 | 0 | 0 | 0 | 0 |
| $\widetilde{12}$ | 0 | $\frac{1}{45}$ | 0 | 0 | 0 | 0 | 0 | 0 | $\frac{8}{9}$ | 0 |

| | 345 | 413 | 432 | $43_13$ | $43_23$ | 435 |
|---|---|---|---|---|---|---|
| $\overline{16}$ | $-\frac{3}{45}$ | 0 | 0 | 0 | 0 | 0 |
| $\overline{25}$ | 0 | $\frac{5}{36}$ | $\frac{1}{36}$ | $-\frac{5}{81}$ | $-\frac{5}{324}$ | $\frac{5}{18}$ |
| $\overline{45}$ | $\frac{4}{45}$ | 0 | 0 | 0 | 0 | 0 |
| $\widetilde{25}$ | $\frac{4}{9}$ | 0 | 0 | 0 | 0 | 0 |
| $\widetilde{45}$ | 0 | $-\frac{1}{4}$ | $\frac{5}{36}$ | $\frac{1}{9}$ | $\frac{1}{36}$ | $\frac{1}{18}$ |
| $\overline{56}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\widetilde{16}$ | 0 | $\frac{1}{18}$ | $\frac{5}{18}$ | $-\frac{2}{81}$ | $-\frac{1}{162}$ | $-\frac{1}{9}$ |
| $\widetilde{56}$ | 0 | $\frac{1}{9}$ | 0 | $\frac{16}{81}$ | $-\frac{16}{81}$ | 0 |
| $\overline{12}$ | 0 | $\frac{4}{9}$ | 0 | $\frac{4}{81}$ | $\frac{16}{81}$ | 0 |
| $\widetilde{12}$ | $\frac{4}{45}$ | 0 | 0 | 0 | 0 | 0 |

| $IJ=\frac{3}{2}0$ | 143 | 232 | 235 | 323 | 343 | 432 | 435 |
|---|---|---|---|---|---|---|---|
| $\overline{12}$ | $\frac{1}{9}$ | 0 | 0 | $\frac{4}{9}$ | $\frac{4}{9}$ | 0 | 0 |
| $\widetilde{12}$ | 0 | 0 | $-\frac{5}{9}$ | 0 | 0 | 0 | $-\frac{4}{9}$ |
| $\overline{56}$ | 0 | $-\frac{5}{9}$ | 0 | 0 | 0 | $-\frac{4}{9}$ | 0 |
| $\widetilde{56}$ | $-\frac{4}{45}$ | 0 | 0 | $\frac{5}{9}$ | $-\frac{16}{45}$ | 0 | 0 |

| $IJ=\frac{1}{2}3$ | 144 | 233 | 344 | 433 |
|---|---|---|---|---|
| $\overline{56}$ | 0 | $-\frac{5}{9}$ | 0 | $-\frac{4}{9}$ |
| $\widetilde{56}$ | $-\frac{1}{5}$ | 0 | $-\frac{4}{5}$ | 0 |

| $IJ=\frac{1}{2}2$ | 143 | 146 | 233 | 234 | 236 | 323 | 343 | 346 | 433 | 434 | 436 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\overline{16}$ | $\frac{16}{255}$ | $-\frac{1}{25}$ | 0 | 0 | 0 | $\frac{4}{9}$ | $\frac{64}{225}$ | $-\frac{4}{25}$ | 0 | 0 | 0 |
| $\overline{25}$ | 0 | 0 | $\frac{1}{9}$ | 0 | $\frac{4}{9}$ | 0 | 0 | 0 | $\frac{4}{45}$ | 0 | $\frac{16}{45}$ |
| $\widetilde{25}$ | $\frac{4}{225}$ | $\frac{4}{25}$ | 0 | 0 | 0 | $\frac{1}{9}$ | $\frac{16}{225}$ | $\frac{16}{25}$ | 0 | 0 | 0 |
| $\overline{56}$ | $-\frac{1}{9}$ | 0 | 0 | 0 | 0 | $\frac{4}{9}$ | $-\frac{4}{9}$ | 0 | 0 | 0 | 0 |
| $\widetilde{16}$ | 0 | 0 | $\frac{4}{9}$ | 0 | $-\frac{1}{9}$ | 0 | 0 | 0 | $\frac{16}{45}$ | 0 | $-\frac{4}{45}$ |
| $\widetilde{56}$ | 0 | 0 | 0 | $-\frac{5}{9}$ | 0 | 0 | 0 | 0 | 0 | $-\frac{4}{9}$ | 0 |

| $IJ=\frac{1}{2}1$ | 144 | 146 | $23_13$ | $23_23$ | $23_16$ | $23_26$ | 323 | 324 | 326 | 344 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\overline{12}$ | $\frac{1}{18}$ | $\frac{2}{45}$ | 0 | 0 | 0 | 0 | 0 | $\frac{2}{9}$ | $-\frac{5}{18}$ | $\frac{2}{9}$ |
| $\overline{14}$ | 0 | 0 | $\frac{1}{18}$ | $\frac{2}{9}$ | $-\frac{1}{36}$ | $-\frac{1}{36}$ | 0 | 0 | 0 | 0 |
| $\widetilde{12}$ | 0 | 0 | $-\frac{1}{162}$ | $-\frac{2}{81}$ | $-\frac{1}{4}$ | $-\frac{1}{4}$ | 0 | 0 | 0 | 0 |
| $\widetilde{14}$ | $\frac{1}{18}$ | $-\frac{2}{25}$ | 0 | 0 | 0 | 0 | 0 | $\frac{2}{9}$ | $\frac{5}{18}$ | $\frac{2}{9}$ |
| $\overline{56}$ | 0 | 0 | $\frac{20}{81}$ | $-\frac{20}{81}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\widetilde{56}$ | $-\frac{4}{45}$ | 0 | 0 | 0 | 0 | 0 | 0 | $\frac{5}{9}$ | 0 | $-\frac{16}{45}$ |
| $\overline{16}$ | 0 | 0 | $-\frac{16}{81}$ | $-\frac{4}{81}$ | $-\frac{1}{18}$ | $\frac{1}{18}$ | 0 | 0 | 0 | 0 |
| $\overline{25}$ | 0 | $-\frac{4}{45}$ | 0 | 0 | 0 | 0 | $-\frac{1}{5}$ | 0 | $-\frac{16}{45}$ | 0 |
| $\widetilde{25}$ | 0 | 0 | $-\frac{4}{81}$ | $-\frac{1}{81}$ | $\frac{2}{9}$ | $-\frac{2}{9}$ | 0 | 0 | 0 | 0 |
| $\widetilde{16}$ | 0 | $\frac{1}{45}$ | 0 | 0 | 0 | 0 | $-\frac{4}{5}$ | 0 | $\frac{4}{45}$ | 0 |

| | 346 | 413 | $43_13$ | $43_23$ | $43_16$ | $43_26$ |
|---|---|---|---|---|---|---|
| $\overline{12}$ | $\frac{8}{45}$ | 0 | 0 | 0 | 0 | 0 |
| $\overline{14}$ | 0 | $\frac{2}{5}$ | $\frac{2}{45}$ | $\frac{8}{45}$ | $-\frac{1}{45}$ | $-\frac{1}{45}$ |
| $\widetilde{12}$ | 0 | $-\frac{2}{45}$ | $-\frac{2}{405}$ | $-\frac{8}{405}$ | $-\frac{1}{5}$ | $-\frac{1}{5}$ |

TABLE I. *(Continued)*.

| | | | | | | |
|---|---|---|---|---|---|---|
| $\widetilde{14}$ | $-\frac{8}{45}$ | 0 | 0 | 0 | 0 | 0 |
| $\overline{56}$ | 0 | $\frac{1}{9}$ | $\frac{16}{81}$ | $-\frac{16}{81}$ | 0 | 0 |
| $\widetilde{56}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\overline{16}$ | 0 | $\frac{16}{45}$ | $-\frac{64}{405}$ | $-\frac{16}{405}$ | $-\frac{2}{45}$ | $\frac{2}{45}$ |
| $\overline{25}$ | $-\frac{16}{45}$ | 0 | 0 | 0 | 0 | 0 |
| $\widetilde{25}$ | 0 | $\frac{4}{45}$ | $-\frac{16}{405}$ | $-\frac{4}{405}$ | $\frac{8}{45}$ | $-\frac{8}{45}$ |
| $\widetilde{16}$ | $\frac{4}{45}$ | 0 | 0 | 0 | 0 | 0 |

| $IJ=\frac{1}{2}0$ | 143 | 234 | 236 | 323 | 326 | 343 | 414 | 434 | 436 |
|---|---|---|---|---|---|---|---|---|---|
| $\overline{12}$ | 0 | $\frac{1}{18}$ | $\frac{5}{18}$ | 0 | 0 | 0 | $\frac{2}{5}$ | $\frac{2}{45}$ | $\frac{2}{9}$ |
| $\overline{14}$ | $\frac{1}{10}$ | 0 | 0 | $\frac{2}{5}$ | $\frac{1}{10}$ | $\frac{2}{5}$ | 0 | 0 | 0 |
| $\widetilde{12}$ | $-\frac{1}{90}$ | 0 | 0 | $-\frac{2}{45}$ | $\frac{9}{10}$ | $-\frac{2}{45}$ | 0 | 0 | 0 |
| $\widetilde{14}$ | 0 | $\frac{1}{18}$ | $-\frac{5}{18}$ | 0 | 0 | 0 | $\frac{2}{5}$ | $\frac{2}{45}$ | $-\frac{2}{9}$ |
| $\overline{56}$ | $-\frac{4}{45}$ | 0 | 0 | $\frac{5}{9}$ | 0 | $-\frac{16}{45}$ | 0 | 0 | 0 |
| $\widetilde{56}$ | 0 | $-\frac{4}{9}$ | 0 | 0 | 0 | 0 | $\frac{1}{5}$ | $-\frac{16}{45}$ | 0 |

(c) $Y=0$

| $IJ=23$ | 233 | 322 | 411 | 433 |
|---|---|---|---|---|
| $\overline{57}$ | $-\frac{1}{3}$ | 0 | $\frac{2}{5}$ | $-\frac{4}{15}$ |
| 66 | $\frac{2}{9}$ | 0 | $\frac{3}{5}$ | $\frac{8}{45}$ |
| $\widetilde{57}$ | 0 | 1 | 0 | 0 |

| $IJ=22$ | 143 | 232 | 233 | 321 | 322' | 323 | 343 | 412 | 432 | 433 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\overline{26}$ | 0 | $\frac{1}{12}$ | $\frac{5}{36}$ | 0 | 0 | 0 | 0 | $\frac{3}{5}$ | $\frac{1}{15}$ | $\frac{1}{9}$ |
| $\overline{35}$ | 0 | $\frac{1}{36}$ | $-\frac{5}{12}$ | 0 | 0 | 0 | 0 | $\frac{1}{5}$ | $\frac{1}{45}$ | $-\frac{1}{3}$ |
| $\widetilde{35}$ | $-\frac{1}{15}$ | 0 | 0 | 0 | $-\frac{1}{4}$ | $-\frac{5}{12}$ | $-\frac{4}{15}$ | 0 | 0 | 0 |
| $\overline{57}$ | $-\frac{1}{15}$ | 0 | 0 | $\frac{2}{5}$ | 0 | $\frac{4}{15}$ | $-\frac{4}{15}$ | 0 | 0 | 0 |
| $\widetilde{26}$ | $\frac{1}{45}$ | 0 | 0 | 0 | $-\frac{3}{4}$ | $\frac{5}{36}$ | $\frac{4}{45}$ | 0 | 0 | 0 |
| 66 | $\frac{2}{45}$ | 0 | 0 | $\frac{3}{5}$ | 0 | $-\frac{8}{45}$ | $\frac{8}{45}$ | 0 | 0 | 0 |
| $\widetilde{57}$ | 0 | $-\frac{4}{9}$ | 0 | 0 | 0 | 0 | 0 | $\frac{1}{5}$ | $-\frac{16}{45}$ | 0 |

| $IJ=21$ | 142 | 231 | 232 | $23_13$ | $23_23$ | 322 | 323 | 342 | 413 | 431 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\overline{26}$ | $\frac{1}{15}$ | 0 | 0 | 0 | 0 | $\frac{5}{12}$ | $-\frac{1}{4}$ | $\frac{4}{15}$ | 0 | 0 |
| $\overline{35}$ | $\frac{1}{45}$ | 0 | 0 | 0 | 0 | $\frac{5}{36}$ | $\frac{3}{4}$ | $\frac{4}{45}$ | 0 | 0 |
| $\widetilde{35}$ | 0 | 0 | $\frac{5}{36}$ | $\frac{5}{27}$ | $\frac{5}{108}$ | 0 | 0 | 0 | $-\frac{1}{3}$ | 0 |
| $\overline{57}$ | 0 | $-\frac{2}{9}$ | 0 | $\frac{4}{27}$ | $-\frac{4}{27}$ | 0 | 0 | 0 | $\frac{1}{15}$ | $-\frac{8}{45}$ |
| $\widetilde{26}$ | 0 | 0 | $\frac{5}{12}$ | $-\frac{5}{81}$ | $-\frac{5}{324}$ | 0 | 0 | 0 | $\frac{1}{9}$ | 0 |
| $\widetilde{57}$ | $-\frac{1}{9}$ | 0 | 0 | 0 | 0 | $\frac{4}{9}$ | 0 | $-\frac{4}{9}$ | 0 | 0 |
| 66 | 0 | $-\frac{1}{3}$ | 0 | $-\frac{8}{81}$ | $\frac{8}{81}$ | 0 | 0 | 0 | $-\frac{2}{45}$ | $-\frac{4}{15}$ |
| 22 | 0 | 0 | 0 | $\frac{5}{81}$ | $\frac{20}{81}$ | 0 | 0 | 0 | $\frac{4}{9}$ | 0 |

| | 432 | $43_13$ | $43_23$ |
|---|---|---|---|
| $\overline{26}$ | 0 | 0 | 0 |
| $\overline{35}$ | 0 | 0 | 0 |
| $\widetilde{35}$ | $\frac{1}{9}$ | $\frac{4}{27}$ | $\frac{1}{27}$ |
| $\overline{57}$ | 0 | $\frac{16}{135}$ | $-\frac{16}{135}$ |
| $\widetilde{26}$ | $\frac{1}{3}$ | $-\frac{4}{81}$ | $-\frac{1}{81}$ |
| $\widetilde{57}$ | 0 | 0 | 0 |
| 66 | 0 | $-\frac{32}{405}$ | $\frac{32}{405}$ |
| 22 | 0 | $\frac{4}{31}$ | $\frac{16}{31}$ |

| $IJ=20$ | 141 | 143 | 232 | 323 | 341 | 343 | 432 |
|---|---|---|---|---|---|---|---|
| $\overline{57}$ | $-\frac{2}{25}$ | $-\frac{4}{75}$ | 0 | $\frac{1}{3}$ | $-\frac{8}{25}$ | $-\frac{16}{75}$ | 0 |
| 66 | $-\frac{3}{25}$ | $\frac{8}{225}$ | 0 | $-\frac{2}{9}$ | $-\frac{12}{25}$ | $\frac{32}{225}$ | 0 |
| $\widetilde{57}$ | 0 | 0 | $-\frac{5}{9}$ | 0 | 0 | 0 | $-\frac{4}{9}$ |
| 22 | 0 | $\frac{1}{9}$ | 0 | $\frac{4}{9}$ | 0 | $\frac{4}{9}$ | 0 |

TABLE I. *(Continued).*

| $IJ=13$ | 144 | 233 | 322 | 344 | 433 |
|---|---|---|---|---|---|
| $\overline{57}$ | $-\frac{2}{15}$ | 0 | $\frac{1}{3}$ | $-\frac{3}{15}$ | 0 |
| 66 | $\frac{1}{15}$ | 0 | $\frac{2}{3}$ | $\frac{4}{15}$ | 0 |
| $\widetilde{57}$ | 0 | $\frac{5}{9}$ | 0 | 0 | $-\frac{4}{9}$ |

| $IJ=12$ | 143 | 146 | 232 | 233 | 234 | 235 | 236 | 322 | 323 | 325 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\overline{17}$ | 0 | 0 | $\frac{1}{27}$ | $\frac{1}{9}$ | 0 | $-\frac{5}{27}$ | $-\frac{2}{27}$ | 0 | 0 | 0 |
| $\overline{26}$ | $\frac{1}{25}$ | $\frac{2}{75}$ | 0 | 0 | 0 | 0 | 0 | $-\frac{1}{12}$ | $\frac{1}{4}$ | $-\frac{1}{3}$ |
| $\overline{35}$ | $-\frac{1}{225}$ | $-\frac{8}{75}$ | 0 | 0 | 0 | 0 | 0 | $-\frac{1}{12}$ | $-\frac{1}{36}$ | $-\frac{1}{3}$ |
| $\overline{46}$ | 0 | 0 | $\frac{1}{18}$ | $-\frac{1}{6}$ | 0 | 0 | $\frac{1}{9}$ | 0 | 0 | 0 |
| $\widetilde{35}$ | 0 | 0 | $\frac{1}{108}$ | $-\frac{1}{36}$ | 0 | $\frac{5}{27}$ | $-\frac{8}{27}$ | 0 | 0 | 0 |
| $\overline{57}$ | 0 | 0 | $-\frac{4}{27}$ | 0 | $-\frac{10}{27}$ | 0 | 0 | 0 | 0 | 0 |
| $\widetilde{26}$ | 0 | 0 | $\frac{1}{108}$ | $\frac{1}{4}$ | 0 | $\frac{5}{27}$ | $\frac{2}{27}$ | 0 | 0 | 0 |
| $\widetilde{46}$ | $-\frac{2}{75}$ | $\frac{1}{25}$ | 0 | 0 | 0 | 0 | 0 | $-\frac{1}{2}$ | $-\frac{1}{6}$ | 0 |
| $\widetilde{17}$ | $\frac{4}{225}$ | $-\frac{2}{75}$ | 0 | 0 | 0 | 0 | 0 | $-\frac{1}{3}$ | $\frac{1}{9}$ | $\frac{1}{3}$ |
| $\widetilde{57}$ | $-\frac{1}{9}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $\frac{4}{9}$ | 0 |
| 66 | 0 | 0 | $-\frac{8}{27}$ | 0 | $\frac{5}{27}$ | 0 | 0 | 0 | 0 | 0 |

| | 343 | 346 | 412 | 432 | 433 | 434 | 435 | 436 |
|---|---|---|---|---|---|---|---|---|
| $\overline{17}$ | 0 | 0 | $\frac{4}{15}$ | $\frac{4}{135}$ | $\frac{4}{45}$ | 0 | $-\frac{4}{27}$ | $-\frac{8}{135}$ |
| $\overline{26}$ | $\frac{4}{25}$ | $\frac{8}{75}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\overline{35}$ | $-\frac{4}{225}$ | $-\frac{32}{75}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\overline{46}$ | 0 | 0 | $\frac{2}{5}$ | $\frac{2}{45}$ | $-\frac{2}{15}$ | 0 | 0 | $\frac{4}{45}$ |
| $\widetilde{35}$ | 0 | 0 | $\frac{1}{15}$ | $\frac{1}{135}$ | $-\frac{1}{45}$ | 0 | $\frac{4}{27}$ | $-\frac{32}{135}$ |
| $\overline{57}$ | 0 | 0 | $\frac{1}{15}$ | $-\frac{16}{135}$ | 0 | $-\frac{8}{27}$ | 0 | 0 |
| $\widetilde{26}$ | 0 | 0 | $\frac{1}{15}$ | $\frac{1}{135}$ | $\frac{1}{5}$ | 0 | $\frac{4}{27}$ | $\frac{8}{135}$ |
| $\widetilde{46}$ | $-\frac{8}{75}$ | $\frac{4}{25}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\widetilde{17}$ | $\frac{16}{225}$ | $-\frac{8}{75}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\widetilde{57}$ | $-\frac{4}{9}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 66 | 0 | 0 | $\frac{2}{15}$ | $-\frac{32}{135}$ | 0 | $\frac{4}{27}$ | 0 | 0 |

| $IJ=11$ | 142 | 144 | 145 | 146 | 232 | $23_13$ | $23_23$ | 235 | $23_16$ | $23_26$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $\overline{17}$ | $\frac{4}{135}$ | 0 | $-\frac{8}{135}$ | $\frac{2}{135}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\overline{26}$ | 0 | 0 | 0 | 0 | $\frac{5}{108}$ | $-\frac{1}{9}$ | $-\frac{1}{36}$ | $\frac{5}{27}$ | $\frac{1}{27}$ | $-\frac{1}{27}$ |
| $\overline{35}$ | 0 | 0 | 0 | 0 | $\frac{5}{108}$ | $\frac{1}{81}$ | $\frac{1}{324}$ | $\frac{5}{27}$ | $-\frac{4}{27}$ | $\frac{4}{27}$ |
| $\overline{46}$ | $\frac{2}{45}$ | 0 | 0 | $-\frac{1}{45}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\widetilde{35}$ | $\frac{1}{135}$ | 0 | $\frac{8}{135}$ | $\frac{8}{135}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\overline{57}$ | $-\frac{1}{27}$ | $-\frac{8}{135}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\widetilde{26}$ | $\frac{1}{135}$ | 0 | $\frac{8}{135}$ | $-\frac{2}{135}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\widetilde{46}$ | 0 | 0 | 0 | 0 | $\frac{5}{18}$ | $\frac{2}{27}$ | $\frac{1}{54}$ | 0 | $\frac{1}{18}$ | $-\frac{1}{18}$ |
| 66 | $-\frac{2}{27}$ | $\frac{4}{135}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\widetilde{17}$ | 0 | 0 | 0 | 0 | $\frac{5}{27}$ | $-\frac{4}{81}$ | $-\frac{1}{81}$ | $-\frac{5}{27}$ | $-\frac{1}{27}$ | $\frac{1}{27}$ |
| $\widetilde{57}$ | 0 | 0 | 0 | 0 | 0 | $\frac{20}{81}$ | $-\frac{20}{81}$ | 0 | 0 | 0 |
| $\overline{13}$ | 0 | 0 | 0 | 0 | 0 | $\frac{2}{81}$ | $\frac{8}{81}$ | 0 | $-\frac{1}{6}$ | $-\frac{1}{6}$ |
| 22 | 0 | $\frac{1}{54}$ | $\frac{1}{270}$ | $\frac{8}{135}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\overline{24}$ | 0 | 0 | 0 | 0 | 0 | $\frac{1}{27}$ | $\frac{4}{27}$ | 0 | $\frac{1}{9}$ | $\frac{1}{9}$ |
| $\widetilde{13}$ | 0 | $\frac{1}{27}$ | $\frac{1}{135}$ | $-\frac{4}{135}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\widetilde{42}$ | 0 | $\frac{1}{18}$ | $-\frac{1}{90}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | 322 | 323 | 324 | 325 | 326 | 342 | 344 | 345 | 346 | 413 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\overline{17}$ | $\frac{5}{27}$ | $-\frac{1}{5}$ | 0 | $\frac{1}{27}$ | $\frac{8}{135}$ | $\frac{16}{135}$ | 0 | $-\frac{32}{135}$ | $\frac{8}{135}$ | 0 |
| $\overline{26}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $\frac{1}{5}$ |
| $\overline{35}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{1}{45}$ |
| $\overline{46}$ | $\frac{5}{18}$ | $\frac{3}{10}$ | 0 | 0 | $-\frac{4}{45}$ | $\frac{8}{45}$ | 0 | 0 | $-\frac{4}{45}$ | 0 |
| $\widetilde{35}$ | $\frac{5}{108}$ | $\frac{1}{20}$ | 0 | $-\frac{1}{27}$ | $\frac{32}{135}$ | $\frac{4}{135}$ | 0 | $\frac{32}{135}$ | $\frac{32}{135}$ | 0 |
| $\overline{57}$ | $\frac{4}{27}$ | 0 | $\frac{10}{27}$ | 0 | 0 | $-\frac{4}{27}$ | $-\frac{32}{135}$ | 0 | 0 | 0 |
| $\widetilde{26}$ | $\frac{5}{108}$ | $-\frac{9}{20}$ | 0 | $-\frac{1}{27}$ | $-\frac{8}{135}$ | $\frac{24}{135}$ | 0 | $\frac{32}{135}$ | $-\frac{8}{135}$ | 0 |
| $\widetilde{46}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{2}{15}$ |

TABLE I. *(Continued)*.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 66 | $\frac{8}{27}$ | 0 | $-\frac{5}{27}$ | 0 | 0 | $-\frac{8}{27}$ | $\frac{16}{135}$ | 0 | 0 | 0 |
| $\widetilde{17}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $\frac{4}{45}$ |
| $\widetilde{57}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $\frac{1}{9}$ |
| $\overline{13}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $\frac{8}{45}$ |
| 22 | 0 | 0 | $\frac{2}{27}$ | $\frac{4}{27}$ | $-\frac{10}{27}$ | 0 | $\frac{2}{27}$ | $\frac{2}{135}$ | $\frac{32}{135}$ | 0 |
| $\overline{24}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $\frac{4}{15}$ |
| $\widetilde{13}$ | 0 | 0 | $\frac{4}{27}$ | $\frac{8}{27}$ | $\frac{5}{27}$ | 0 | $\frac{4}{27}$ | $\frac{4}{135}$ | $-\frac{16}{135}$ | 0 |
| $\widetilde{42}$ | 0 | 0 | $\frac{2}{9}$ | $-\frac{4}{9}$ | 0 | 0 | $\frac{2}{9}$ | $-\frac{2}{45}$ | 0 | 0 |
| | 432 | $43_13$ | $43_23$ | 435 | $43_16$ | $43_26$ | | | | |
| $\overline{17}$ | 0 | 0 | 0 | 0 | 0 | 0 | | | | |
| $\overline{26}$ | $\frac{1}{27}$ | $-\frac{4}{45}$ | $-\frac{1}{45}$ | $\frac{4}{27}$ | $\frac{4}{135}$ | $-\frac{4}{135}$ | | | | |
| $\overline{35}$ | $\frac{1}{27}$ | $\frac{4}{405}$ | $\frac{1}{405}$ | $\frac{4}{27}$ | $-\frac{16}{135}$ | $\frac{16}{135}$ | | | | |
| $\overline{46}$ | 0 | 0 | 0 | 0 | 0 | 0 | | | | |
| $\widetilde{35}$ | 0 | 0 | 0 | 0 | 0 | 0 | | | | |
| $\overline{57}$ | 0 | 0 | 0 | 0 | 0 | 0 | | | | |
| $\widetilde{26}$ | 0 | 0 | 0 | 0 | 0 | 0 | | | | |
| $\widetilde{46}$ | $\frac{2}{9}$ | $\frac{8}{135}$ | $\frac{2}{135}$ | 0 | $\frac{2}{45}$ | $-\frac{2}{45}$ | | | | |
| 66 | 0 | 0 | 0 | 0 | 0 | 0 | | | | |
| $\widetilde{17}$ | $\frac{4}{27}$ | $-\frac{16}{405}$ | $-\frac{4}{405}$ | $-\frac{4}{27}$ | $-\frac{4}{135}$ | $\frac{4}{135}$ | | | | |
| $\widetilde{57}$ | 0 | $\frac{16}{81}$ | $-\frac{16}{81}$ | 0 | 0 | 0 | | | | |
| $\overline{13}$ | 0 | $\frac{8}{405}$ | $\frac{32}{405}$ | 0 | $-\frac{2}{15}$ | $-\frac{2}{15}$ | | | | |
| 22 | 0 | 0 | 0 | 0 | 0 | 0 | | | | |
| $\overline{24}$ | 0 | $\frac{4}{135}$ | $\frac{16}{135}$ | 0 | $\frac{4}{45}$ | $\frac{4}{45}$ | | | | |
| $\widetilde{13}$ | 0 | 0 | 0 | 0 | 0 | 0 | | | | |
| $\widetilde{42}$ | 0 | 0 | 0 | 0 | 0 | 0 | | | | |
| $IJ=10$ | 143 | 232 | 234 | 235 | 236 | 323 | 326 | 343 | 414 | 432 |
| $\overline{13}$ | $\frac{2}{45}$ | 0 | 0 | 0 | 0 | $\frac{8}{45}$ | $\frac{3}{5}$ | $\frac{8}{45}$ | 0 | 0 |
| $\overline{24}$ | $\frac{1}{15}$ | 0 | 0 | 0 | 0 | $\frac{4}{15}$ | $-\frac{2}{5}$ | $\frac{4}{15}$ | 0 | 0 |
| $\widetilde{13}$ | 0 | 0 | $\frac{1}{27}$ | $-\frac{5}{27}$ | $-\frac{5}{27}$ | 0 | 0 | 0 | $\frac{4}{15}$ | 0 |
| $\widetilde{42}$ | 0 | 0 | $\frac{1}{18}$ | $\frac{5}{18}$ | 0 | 0 | 0 | 0 | $\frac{2}{5}$ | 0 |
| $\overline{57}$ | 0 | $-\frac{5}{27}$ | $-\frac{8}{27}$ | 0 | 0 | 0 | 0 | 0 | $\frac{2}{15}$ | $-\frac{4}{27}$ |
| $\widetilde{57}$ | $-\frac{4}{45}$ | 0 | 0 | 0 | 0 | $\frac{5}{9}$ | 0 | $-\frac{16}{45}$ | 0 | 0 |
| 66 | 0 | $-\frac{10}{27}$ | $\frac{4}{27}$ | 0 | 0 | 0 | 0 | 0 | $-\frac{1}{15}$ | $-\frac{8}{27}$ |
| 22 | 0 | 0 | $\frac{1}{54}$ | $-\frac{5}{54}$ | $\frac{10}{27}$ | 0 | 0 | 0 | $\frac{2}{15}$ | 0 |
| | 434 | 435 | 436 | | | | | | | |
| $\overline{13}$ | 0 | 0 | 0 | | | | | | | |
| $\overline{24}$ | 0 | 0 | 0 | | | | | | | |
| $\widetilde{13}$ | $\frac{4}{135}$ | $-\frac{4}{27}$ | $-\frac{4}{27}$ | | | | | | | |
| $\widetilde{42}$ | $\frac{2}{45}$ | $\frac{2}{9}$ | 0 | | | | | | | |
| $\overline{57}$ | $-\frac{32}{135}$ | 0 | 0 | | | | | | | |
| $\widetilde{57}$ | 0 | 0 | 0 | | | | | | | |
| 66 | $\frac{16}{135}$ | 0 | 0 | | | | | | | |
| 22 | $\frac{2}{135}$ | $-\frac{2}{27}$ | $\frac{8}{27}$ | | | | | | | |
| $IJ=03$ | 233 | 433 | | | | | | | | |
| 66 | $-\frac{5}{9}$ | $-\frac{4}{9}$ | | | | | | | | |
| $IJ=02$ | 143 | 146 | 233 | 236 | 323 | 343 | 346 | 433 | 436 | |
| $\overline{17}$ | $\frac{4}{75}$ | $-\frac{2}{25}$ | 0 | 0 | $\frac{1}{3}$ | $\frac{16}{75}$ | $-\frac{8}{25}$ | 0 | 0 | |
| $\overline{26}$ | 0 | 0 | $\frac{2}{9}$ | $\frac{1}{3}$ | 0 | 0 | 0 | $\frac{8}{45}$ | $\frac{4}{15}$ | |
| $\widetilde{26}$ | $\frac{8}{225}$ | $\frac{3}{25}$ | 0 | 0 | $\frac{2}{9}$ | $\frac{32}{225}$ | $\frac{12}{25}$ | 0 | 0 | |
| 66 | $-\frac{1}{9}$ | 0 | 0 | 0 | $\frac{4}{9}$ | $-\frac{4}{9}$ | 0 | 0 | 0 | |
| $\widetilde{17}$ | 0 | 0 | $\frac{1}{3}$ | $-\frac{2}{9}$ | 0 | 0 | 0 | $\frac{4}{15}$ | $-\frac{8}{45}$ | |

TABLE I. *(Continued).*

| $IJ=01$ | 146 | $23_13$ | $23_23$ | $23_16$ | $23_26$ | 237 | 323 | 326 | 346 | 413 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\overline{13}$ | $\frac{4}{45}$ | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{5}{9}$ | $\frac{16}{45}$ | 0 |
| $\overline{17}$ | 0 | $-\frac{4}{27}$ | $-\frac{1}{27}$ | $-\frac{1}{9}$ | $\frac{1}{9}$ | 0 | 0 | 0 | 0 | $\frac{4}{15}$ |
| $\overline{26}$ | $-\frac{1}{15}$ | 0 | 0 | 0 | 0 | 0 | $-\frac{2}{5}$ | $-\frac{4}{15}$ | $-\frac{4}{15}$ | 0 |
| $\widetilde{13}$ | 0 | $-\frac{1}{54}$ | $-\frac{2}{27}$ | $-\frac{1}{18}$ | $-\frac{1}{18}$ | $\frac{5}{18}$ | 0 | 0 | 0 | $-\frac{2}{15}$ |
| $\widetilde{26}$ | 0 | $-\frac{8}{81}$ | $-\frac{2}{81}$ | $\frac{1}{6}$ | $-\frac{1}{6}$ | 0 | 0 | 0 | 0 | $\frac{8}{45}$ |
| $\widetilde{17}$ | $\frac{2}{45}$ | 0 | 0 | 0 | 0 | 0 | $-\frac{3}{5}$ | $\frac{8}{45}$ | $\frac{8}{45}$ | 0 |
| 22 | 0 | $-\frac{1}{648}$ | $-\frac{1}{162}$ | $-\frac{1}{6}$ | $-\frac{1}{6}$ | $-\frac{5}{24}$ | 0 | 0 | 0 | $-\frac{1}{90}$ |
| 44 | 0 | $\frac{1}{24}$ | $\frac{1}{6}$ | $-\frac{1}{18}$ | $-\frac{1}{18}$ | $\frac{5}{72}$ | 0 | 0 | 0 | $\frac{3}{10}$ |
| 66 | 0 | $\frac{20}{81}$ | $-\frac{20}{81}$ | 0 | 0 | 0 | 0 | 0 | 0 | $\frac{1}{9}$ |

| | $43_13$ | $43_23$ | $43_16$ | $43_26$ | 437 |
|---|---|---|---|---|---|
| $\overline{13}$ | 0 | 0 | 0 | 0 | 0 |
| $\overline{17}$ | $-\frac{16}{135}$ | $-\frac{4}{135}$ | $-\frac{4}{45}$ | $\frac{4}{45}$ | 0 |
| $\overline{26}$ | 0 | 0 | 0 | 0 | 0 |
| $\widetilde{13}$ | $-\frac{2}{135}$ | $-\frac{8}{135}$ | $-\frac{2}{45}$ | $-\frac{2}{45}$ | $\frac{2}{9}$ |
| $\widetilde{26}$ | $-\frac{32}{405}$ | $-\frac{8}{405}$ | $\frac{2}{15}$ | $-\frac{2}{15}$ | 0 |
| $\widetilde{17}$ | 0 | 0 | 0 | 0 | 0 |
| 22 | $-\frac{1}{810}$ | $-\frac{2}{405}$ | $-\frac{2}{15}$ | $-\frac{2}{15}$ | $-\frac{1}{6}$ |
| 44 | $\frac{1}{30}$ | $\frac{2}{15}$ | $-\frac{2}{45}$ | $-\frac{2}{45}$ | $\frac{1}{18}$ |
| 66 | $\frac{16}{81}$ | $-\frac{16}{81}$ | 0 | 0 | 0 |

| $IJ=00$ | 143 | 147 | 236 | 323 | 326 | 343 | 347 | 436 |
|---|---|---|---|---|---|---|---|---|
| $\overline{13}$ | 0 | 0 | $\frac{5}{9}$ | 0 | 0 | 0 | 0 | $\frac{4}{9}$ |
| 22 | $-\frac{1}{360}$ | $\frac{3}{40}$ | 0 | $-\frac{1}{90}$ | $\frac{3}{5}$ | $-\frac{1}{90}$ | $\frac{3}{10}$ | 0 |
| $\widetilde{13}$ | $-\frac{1}{30}$ | $-\frac{1}{10}$ | 0 | $-\frac{2}{15}$ | $\frac{1}{5}$ | $-\frac{2}{15}$ | $-\frac{2}{5}$ | 0 |
| 44 | $\frac{3}{40}$ | $-\frac{1}{40}$ | 0 | $\frac{3}{10}$ | $\frac{1}{5}$ | $\frac{3}{10}$ | $-\frac{1}{10}$ | 0 |
| 66 | $-\frac{4}{45}$ | 0 | 0 | $\frac{5}{9}$ | 0 | $-\frac{16}{45}$ | 0 | 0 |

(d) $Y=-1$

| $IJ=\frac{3}{2}3$ | 144 | 233 | 322 | 344 | 411 | 433 |
|---|---|---|---|---|---|---|
| $\overline{58}$ | 0 | $-\frac{1}{2}$ | 0 | 0 | $\frac{1}{10}$ | $-\frac{2}{5}$ |
| $\overline{67}$ | 0 | $\frac{1}{18}$ | 0 | 0 | $\frac{9}{10}$ | $\frac{2}{45}$ |
| $\widetilde{67}$ | $\frac{1}{10}$ | 0 | $\frac{1}{2}$ | $\frac{2}{5}$ | 0 | 0 |
| $\widetilde{58}$ | $-\frac{1}{10}$ | 0 | $\frac{1}{2}$ | $-\frac{2}{5}$ | 0 | 0 |

| $IJ=\frac{3}{2}2$ | 143 | 232 | 233 | 234 | 321 | 322 | 323 | 343 | 412 | 432 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\overline{27}$ | 0 | $\frac{1}{18}$ | $\frac{5}{18}$ | 0 | 0 | 0 | 0 | 0 | $\frac{2}{5}$ | $\frac{2}{45}$ |
| $\overline{36}$ | 0 | $\frac{1}{18}$ | $-\frac{5}{18}$ | 0 | 0 | 0 | 0 | 0 | $\frac{2}{5}$ | $\frac{2}{45}$ |
| $\overline{58}$ | $-\frac{1}{10}$ | 0 | 0 | 0 | $\frac{1}{10}$ | 0 | $\frac{2}{5}$ | $-\frac{2}{5}$ | 0 | 0 |
| $\widetilde{36}$ | $-\frac{2}{45}$ | 0 | 0 | 0 | 0 | $-\frac{1}{2}$ | $-\frac{5}{18}$ | $-\frac{8}{45}$ | 0 | 0 |
| $\overline{67}$ | $\frac{1}{90}$ | 0 | 0 | 0 | $\frac{9}{10}$ | 0 | $-\frac{2}{45}$ | $\frac{2}{45}$ | 0 | 0 |
| $\widetilde{27}$ | $\frac{2}{45}$ | 0 | 0 | 0 | 0 | $-\frac{1}{2}$ | $\frac{5}{18}$ | $\frac{8}{45}$ | 0 | 0 |
| $\widetilde{67}$ | 0 | $-\frac{2}{9}$ | 0 | $\frac{5}{18}$ | 0 | 0 | 0 | 0 | $\frac{1}{10}$ | $-\frac{8}{45}$ |
| $\widetilde{58}$ | 0 | $-\frac{2}{9}$ | 0 | $-\frac{5}{18}$ | 0 | 0 | 0 | 0 | $\frac{1}{10}$ | $-\frac{8}{45}$ |

| | 433 | 434 |
|---|---|---|
| $\overline{27}$ | $\frac{2}{9}$ | 0 |
| $\overline{36}$ | $-\frac{2}{9}$ | 0 |
| $\overline{58}$ | 0 | 0 |
| $\widetilde{36}$ | 0 | 0 |
| $\overline{67}$ | 0 | 0 |
| $\widetilde{27}$ | 0 | 0 |
| $\widetilde{67}$ | 0 | $\frac{2}{9}$ |
| $\widetilde{58}$ | 0 | $-\frac{2}{9}$ |

TABLE I. *(Continued).*

| $IJ=\frac{3}{2}1$ | 142 | 144 | 231 | 232 | $23_13$ | $23_23$ | 322 | 323 | 324 | 342 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\overline{27}$ | $\frac{2}{45}$ | 0 | 0 | 0 | 0 | 0 | $\frac{5}{18}$ | $-\frac{1}{2}$ | 0 | $\frac{8}{45}$ |
| $\overline{36}$ | $\frac{2}{45}$ | 0 | 0 | 0 | 0 | 0 | $\frac{5}{18}$ | $\frac{1}{2}$ | 0 | $\frac{8}{45}$ |
| $\overline{58}$ | 0 | 0 | $-\frac{1}{18}$ | 0 | $\frac{2}{9}$ | $-\frac{2}{9}$ | 0 | 0 | 0 | 0 |
| $\widetilde{36}$ | 0 | 0 | 0 | $\frac{5}{18}$ | $\frac{10}{81}$ | $\frac{5}{162}$ | 0 | 0 | 0 | 0 |
| $\overline{67}$ | 0 | 0 | $-\frac{1}{2}$ | 0 | $-\frac{2}{81}$ | $\frac{2}{81}$ | 0 | 0 | 0 | 0 |
| $\widetilde{27}$ | 0 | 0 | 0 | $\frac{5}{18}$ | $-\frac{10}{81}$ | $-\frac{5}{162}$ | 0 | 0 | 0 | 0 |
| $\widetilde{67}$ | $-\frac{1}{18}$ | $\frac{2}{45}$ | 0 | 0 | 0 | 0 | $\frac{2}{9}$ | 0 | $-\frac{5}{18}$ | $-\frac{2}{9}$ |
| $\widetilde{58}$ | $-\frac{1}{18}$ | $-\frac{2}{45}$ | 0 | 0 | 0 | 0 | $\frac{2}{9}$ | 0 | $\frac{5}{18}$ | $-\frac{2}{9}$ |
| $\overline{23}$ | 0 | 0 | 0 | 0 | $\frac{5}{81}$ | $\frac{20}{81}$ | 0 | 0 | 0 | 0 |
| $\widetilde{23}$ | 0 | $\frac{1}{9}$ | 0 | 0 | 0 | 0 | 0 | 0 | $\frac{4}{9}$ | 0 |

| | 344 | 413 | 431 | 432 | $43_13$ | $43_23$ |
|---|---|---|---|---|---|---|
| $\overline{27}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\overline{36}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\overline{58}$ | 0 | $\frac{1}{10}$ | $-\frac{2}{45}$ | 0 | $\frac{8}{45}$ | $-\frac{8}{45}$ |
| $\widetilde{36}$ | 0 | $-\frac{2}{9}$ | 0 | $\frac{2}{9}$ | $\frac{8}{81}$ | $\frac{2}{81}$ |
| $\overline{67}$ | 0 | $-\frac{1}{90}$ | $-\frac{2}{5}$ | 0 | $-\frac{8}{405}$ | $\frac{8}{405}$ |
| $\widetilde{27}$ | 0 | $\frac{2}{9}$ | 0 | $\frac{2}{9}$ | $-\frac{8}{81}$ | $-\frac{2}{81}$ |
| $\widetilde{67}$ | $\frac{8}{45}$ | 0 | 0 | 0 | 0 | 0 |
| $\widetilde{58}$ | $-\frac{8}{45}$ | 0 | 0 | 0 | 0 | 0 |
| $\overline{23}$ | 0 | $\frac{4}{9}$ | 0 | 0 | $\frac{4}{81}$ | $\frac{16}{81}$ |
| $\widetilde{23}$ | $\frac{4}{9}$ | 0 | 0 | 0 | 0 | 0 |

| $IJ=\frac{3}{2}0$ | 141 | 143 | 232 | 234 | 323 | 341 | 343 | 414 | 432 | 434 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\overline{58}$ | $-\frac{1}{50}$ | $-\frac{2}{25}$ | 0 | 0 | $\frac{1}{2}$ | $-\frac{2}{25}$ | $-\frac{8}{25}$ | 0 | 0 | 0 |
| $\overline{67}$ | $-\frac{9}{50}$ | $\frac{2}{225}$ | 0 | 0 | $-\frac{1}{18}$ | $-\frac{18}{25}$ | $\frac{8}{225}$ | 0 | 0 | 0 |
| $\widetilde{67}$ | 0 | 0 | $-\frac{5}{18}$ | $\frac{2}{9}$ | 0 | 0 | 0 | $-\frac{1}{10}$ | $-\frac{2}{9}$ | $\frac{8}{45}$ |
| $\widetilde{58}$ | 0 | 0 | $-\frac{5}{18}$ | $-\frac{2}{9}$ | 0 | 0 | 0 | $\frac{1}{10}$ | $-\frac{2}{9}$ | $-\frac{8}{45}$ |
| $\overline{23}$ | 0 | $\frac{1}{9}$ | 0 | 0 | $\frac{4}{9}$ | 0 | $\frac{4}{9}$ | 0 | 0 | 0 |
| $\widetilde{23}$ | 0 | 0 | 0 | $\frac{1}{9}$ | 0 | 0 | 0 | $\frac{4}{5}$ | 0 | $\frac{4}{45}$ |

| $IJ=\frac{1}{2}3$ | 233 | 322 | 433 |
|---|---|---|---|
| $\overline{67}$ | 0 | 1 | 0 |
| $\widetilde{67}$ | $-\frac{5}{9}$ | 0 | $-\frac{4}{9}$ |

| $IJ=\frac{1}{2}2$ | 143 | 146 | 232 | 233 | 235 | 236 | 322 | 323 | 325 | 343 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\overline{18}$ | 0 | 0 | $\frac{1}{72}$ | $\frac{1}{8}$ | $-\frac{5}{36}$ | $-\frac{2}{9}$ | 0 | 0 | 0 | 0 |
| $\overline{27}$ | $\frac{49}{900}$ | $\frac{1}{25}$ | 0 | 0 | 0 | 0 | $-\frac{1}{16}$ | $\frac{49}{144}$ | $-\frac{1}{8}$ | $\frac{49}{225}$ |
| $\overline{36}$ | $-\frac{1}{225}$ | $-\frac{1}{25}$ | 0 | 0 | 0 | 0 | $-\frac{1}{4}$ | $-\frac{1}{36}$ | $-\frac{1}{2}$ | $-\frac{4}{225}$ |
| $\overline{47}$ | 0 | 0 | $\frac{1}{16}$ | $-\frac{1}{16}$ | $-\frac{5}{72}$ | $\frac{1}{9}$ | 0 | 0 | 0 | 0 |
| $\widetilde{36}$ | 0 | 0 | $\frac{1}{36}$ | $-\frac{1}{36}$ | $\frac{5}{18}$ | $-\frac{1}{9}$ | 0 | 0 | 0 | 0 |
| $\overline{67}$ | 0 | 0 | $-\frac{4}{9}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\widetilde{27}$ | 0 | 0 | $\frac{1}{144}$ | $\frac{49}{144}$ | $\frac{5}{72}$ | $\frac{1}{9}$ | 0 | 0 | 0 | 0 |
| $\widetilde{47}$ | $-\frac{1}{100}$ | $\frac{1}{25}$ | 0 | 0 | 0 | 0 | $-\frac{9}{16}$ | $-\frac{1}{16}$ | $\frac{1}{8}$ | $-\frac{1}{25}$ |
| $\widetilde{67}$ | $-\frac{1}{9}$ | 0 | 0 | 0 | 0 | 0 | 0 | $\frac{4}{9}$ | 0 | $-\frac{4}{9}$ |
| $\widetilde{18}$ | $\frac{1}{50}$ | $-\frac{2}{25}$ | 0 | 0 | 0 | 0 | $-\frac{1}{8}$ | $\frac{1}{8}$ | $\frac{1}{4}$ | $\frac{2}{25}$ |

| | 346 | 412 | 432 | 433 | 435 | 436 |
|---|---|---|---|---|---|---|
| $\overline{18}$ | 0 | $\frac{1}{10}$ | $\frac{1}{90}$ | $\frac{1}{10}$ | $-\frac{1}{9}$ | $-\frac{8}{45}$ |
| $\overline{27}$ | $\frac{4}{25}$ | 0 | 0 | 0 | 0 | 0 |
| $\overline{36}$ | $-\frac{4}{25}$ | 0 | 0 | 0 | 0 | 0 |
| $\overline{47}$ | 0 | $\frac{9}{20}$ | $\frac{1}{20}$ | $-\frac{1}{20}$ | $-\frac{1}{18}$ | $\frac{4}{45}$ |
| $\widetilde{36}$ | 0 | $\frac{1}{5}$ | $\frac{1}{45}$ | $-\frac{1}{45}$ | $\frac{2}{9}$ | $-\frac{4}{45}$ |
| $\overline{67}$ | 0 | $\frac{1}{5}$ | $-\frac{16}{45}$ | 0 | 0 | 0 |
| $\widetilde{27}$ | 0 | $\frac{1}{20}$ | $\frac{1}{180}$ | $\frac{49}{180}$ | $\frac{1}{18}$ | $\frac{4}{45}$ |

TABLE I. *(Continued).*

| | | | | | | |
|---|---|---|---|---|---|---|
| $\widetilde{47}$ | $\frac{4}{25}$ | 0 | 0 | 0 | 0 | 0 |
| $\widetilde{67}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\widetilde{18}$ | $-\frac{8}{25}$ | 0 | 0 | 0 | 0 | 0 |

| $IJ=\frac{1}{2}0$ | 143 | 232 | 235 | 236 | 323 | 326 | 343 | 432 | 435 | 436 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\overline{23}$ | 0 | 0 | $-\frac{5}{18}$ | $\frac{5}{18}$ | 0 | 0 | 0 | 0 | $-\frac{2}{9}$ | $\frac{2}{9}$ |
| $\widetilde{23}$ | $-\frac{1}{90}$ | 0 | 0 | 0 | $-\frac{2}{45}$ | $\frac{9}{10}$ | $-\frac{2}{45}$ | 0 | 0 | 0 |
| $\overline{34}$ | $\frac{1}{10}$ | 0 | 0 | 0 | $\frac{2}{5}$ | $\frac{1}{10}$ | $\frac{2}{5}$ | 0 | 0 | 0 |
| $\widetilde{34}$ | 0 | 0 | $\frac{5}{18}$ | $\frac{5}{18}$ | 0 | 0 | 0 | 0 | $\frac{2}{9}$ | $\frac{2}{9}$ |
| $\overline{67}$ | 0 | $-\frac{5}{9}$ | 0 | 0 | 0 | 0 | 0 | $-\frac{4}{9}$ | 0 | 0 |
| $\widetilde{67}$ | $-\frac{4}{45}$ | 0 | 0 | 0 | $\frac{5}{9}$ | 0 | $-\frac{16}{45}$ | 0 | 0 | 0 |

| $IJ=\frac{1}{2}1$ | 142 | 145 | 146 | 232 | $23_13$ | $23_23$ | 235 | $23_16$ | $23_26$ | 322 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\overline{18}$ | $\frac{1}{90}$ | $-\frac{2}{45}$ | $\frac{2}{45}$ | 0 | 0 | 0 | 0 | 0 | 0 | $\frac{5}{72}$ |
| $\overline{27}$ | 0 | 0 | 0 | $\frac{5}{144}$ | $-\frac{49}{324}$ | $-\frac{49}{1296}$ | $\frac{5}{72}$ | $\frac{1}{18}$ | $-\frac{1}{18}$ | 0 |
| $\overline{36}$ | 0 | 0 | 0 | $\frac{5}{36}$ | $\frac{1}{81}$ | $\frac{1}{324}$ | $\frac{5}{18}$ | $-\frac{1}{18}$ | $\frac{1}{18}$ | 0 |
| $\overline{47}$ | $\frac{1}{20}$ | $-\frac{1}{45}$ | $-\frac{1}{45}$ | 0 | 0 | 0 | 0 | 0 | 0 | $\frac{5}{16}$ |
| $\widetilde{36}$ | $\frac{1}{45}$ | $\frac{4}{45}$ | $\frac{1}{45}$ | 0 | 0 | 0 | 0 | 0 | 0 | $\frac{5}{36}$ |
| $\overline{67}$ | $-\frac{1}{9}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $\frac{4}{9}$ |
| $\widetilde{27}$ | $\frac{1}{180}$ | $\frac{1}{45}$ | $-\frac{1}{45}$ | 0 | 0 | 0 | 0 | 0 | 0 | $\frac{5}{144}$ |
| $\widetilde{47}$ | 0 | 0 | 0 | $\frac{5}{16}$ | $\frac{1}{36}$ | $\frac{1}{144}$ | $-\frac{5}{72}$ | $\frac{1}{18}$ | $-\frac{1}{18}$ | 0 |
| $\widetilde{67}$ | 0 | 0 | 0 | 0 | $\frac{20}{81}$ | $-\frac{20}{81}$ | 0 | 0 | 0 | 0 |
| $\widetilde{18}$ | 0 | 0 | 0 | $\frac{5}{72}$ | $-\frac{1}{18}$ | $-\frac{1}{72}$ | $-\frac{5}{36}$ | $-\frac{1}{9}$ | $\frac{1}{9}$ | 0 |
| $\overline{23}$ | 0 | $\frac{1}{90}$ | $\frac{2}{45}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\widetilde{23}$ | 0 | 0 | 0 | 0 | $-\frac{1}{162}$ | $-\frac{2}{81}$ | 0 | $-\frac{1}{4}$ | $-\frac{1}{4}$ | 0 |
| $\overline{34}$ | 0 | 0 | 0 | 0 | $\frac{1}{18}$ | $\frac{2}{9}$ | 0 | $-\frac{1}{36}$ | $-\frac{1}{36}$ | 0 |
| $\widetilde{34}$ | 0 | $-\frac{1}{90}$ | $\frac{2}{45}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | 323 | 325 | 326 | 342 | 345 | 346 | 413 | 432 | $43_13$ | $43_23$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $\overline{18}$ | $-\frac{9}{40}$ | $\frac{1}{36}$ | $\frac{2}{45}$ | $-\frac{8}{45}$ | $\frac{8}{45}$ | 0 | 0 | 0 | 0 | 0 |
| $\overline{27}$ | 0 | 0 | 0 | 0 | 0 | $\frac{49}{180}$ | $\frac{1}{36}$ | $-\frac{49}{405}$ | $-\frac{49}{1620}$ | $\frac{1}{18}$ |
| $\overline{36}$ | 0 | 0 | 0 | 0 | 0 | $-\frac{1}{45}$ | $\frac{1}{9}$ | $\frac{4}{405}$ | $\frac{1}{405}$ | $\frac{2}{9}$ |
| $\overline{47}$ | $\frac{9}{80}$ | $\frac{1}{72}$ | $\frac{1}{5}$ | $-\frac{4}{45}$ | $-\frac{4}{45}$ | 0 | 0 | 0 | 0 | 0 |
| $\widetilde{36}$ | $\frac{1}{20}$ | $-\frac{1}{18}$ | $\frac{4}{45}$ | $\frac{16}{45}$ | $\frac{4}{45}$ | 0 | 0 | 0 | 0 | 0 |
| $\overline{67}$ | 0 | 0 | $-\frac{4}{9}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\widetilde{27}$ | $-\frac{49}{80}$ | $-\frac{1}{72}$ | $\frac{1}{45}$ | $\frac{4}{45}$ | $-\frac{4}{45}$ | 0 | 0 | 0 | 0 | 0 |
| $\widetilde{47}$ | 0 | 0 | 0 | 0 | 0 | $-\frac{1}{20}$ | $\frac{1}{4}$ | $\frac{1}{45}$ | $\frac{1}{180}$ | $-\frac{1}{18}$ |
| $\widetilde{67}$ | 0 | 0 | 0 | 0 | 0 | $\frac{1}{9}$ | 0 | $\frac{16}{81}$ | $-\frac{16}{81}$ | 0 |
| $\widetilde{18}$ | 0 | 0 | 0 | 0 | 0 | $\frac{1}{10}$ | $\frac{1}{18}$ | $-\frac{2}{45}$ | $-\frac{1}{90}$ | $-\frac{1}{9}$ |
| $\overline{23}$ | 0 | $\frac{4}{9}$ | 0 | $\frac{2}{45}$ | $\frac{8}{45}$ | 0 | 0 | 0 | 0 | 0 |
| $\widetilde{23}$ | 0 | 0 | 0 | 0 | 0 | $-\frac{2}{45}$ | 0 | $-\frac{2}{405}$ | $-\frac{8}{405}$ | 0 |
| $\overline{34}$ | 0 | 0 | 0 | 0 | 0 | $\frac{2}{5}$ | 0 | $\frac{2}{45}$ | $\frac{8}{45}$ | 0 |
| $\widetilde{34}$ | 0 | $-\frac{4}{9}$ | 0 | $-\frac{2}{45}$ | $\frac{8}{45}$ | 0 | 0 | 0 | 0 | 0 |

| | 435 | $43_16$ | $43_26$ |
|---|---|---|---|
| $\overline{18}$ | 0 | 0 | $\frac{8}{45}$ |
| $\overline{27}$ | $\frac{2}{45}$ | $-\frac{2}{45}$ | 0 |
| $\overline{36}$ | $-\frac{2}{45}$ | $\frac{2}{45}$ | 0 |
| $\overline{47}$ | 0 | 0 | $-\frac{4}{45}$ |
| $\widetilde{36}$ | 0 | 0 | $\frac{4}{45}$ |
| $\overline{67}$ | 0 | 0 | 0 |
| $\widetilde{27}$ | 0 | 0 | $-\frac{4}{45}$ |
| $\widetilde{47}$ | $\frac{2}{45}$ | $-\frac{2}{45}$ | 0 |
| $\widetilde{67}$ | 0 | 0 | 0 |
| $\widetilde{18}$ | $-\frac{4}{45}$ | $\frac{4}{45}$ | 0 |
| $\overline{23}$ | 0 | 0 | $-\frac{5}{18}$ |

TABLE I. *(Continued).*

| | | | |
|---|---|---|---|
| $\widetilde{23}$ | $-\frac{1}{5}$ | $-\frac{1}{5}$ | 0 |
| $\overline{34}$ | $-\frac{1}{45}$ | $-\frac{1}{45}$ | 0 |
| $\widetilde{34}$ | 0 | 0 | $-\frac{5}{18}$ |

(e) $Y = -2$

| $IJ = 13$ | 233 | 322 | 411 | 433 |
|---|---|---|---|---|
| $\overline{68}$ | $-\frac{1}{3}$ | 0 | $\frac{2}{5}$ | $-\frac{4}{15}$ |
| 77 | $\frac{2}{9}$ | 0 | $\frac{3}{5}$ | $\frac{8}{45}$ |
| $\widetilde{68}$ | 0 | 1 | 0 | 0 |

| $IJ = 12$ | 143 | 232 | 233 | 321 | 322 | 323 | 343 | 412 | 432 | 433 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\overline{28}$ | 0 | $\frac{1}{36}$ | $\frac{5}{12}$ | 0 | 0 | 0 | 0 | $\frac{1}{5}$ | $\frac{1}{45}$ | $\frac{1}{3}$ |
| $\overline{37}$ | 0 | $\frac{1}{12}$ | $-\frac{5}{36}$ | 0 | 0 | 0 | 0 | $\frac{3}{5}$ | $\frac{1}{15}$ | $-\frac{1}{9}$ |
| $\overline{68}$ | $-\frac{1}{15}$ | 0 | 0 | $\frac{2}{5}$ | 0 | $\frac{4}{15}$ | $-\frac{4}{15}$ | 0 | 0 | 0 |
| $\widetilde{37}$ | $-\frac{1}{45}$ | 0 | 0 | 0 | $-\frac{3}{4}$ | $-\frac{5}{36}$ | $-\frac{4}{45}$ | 0 | 0 | 0 |
| 77 | $\frac{2}{45}$ | 0 | 0 | $\frac{3}{5}$ | 0 | $-\frac{8}{45}$ | $\frac{8}{45}$ | 0 | 0 | 0 |
| $\widetilde{28}$ | $\frac{1}{15}$ | 0 | 0 | 0 | $-\frac{1}{4}$ | $\frac{5}{12}$ | $\frac{4}{15}$ | 0 | 0 | 0 |
| $\widetilde{68}$ | 0 | $-\frac{4}{9}$ | 0 | 0 | 0 | 0 | 0 | $\frac{1}{5}$ | $-\frac{16}{45}$ | 0 |

| $IJ = 11$ | 142 | 231 | 232 | $23_13$ | $23_23$ | 322 | 323 | 342 | 413 | 431 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\overline{28}$ | $\frac{1}{45}$ | 0 | 0 | 0 | 0 | $\frac{5}{36}$ | $-\frac{3}{4}$ | $\frac{4}{45}$ | 0 | 0 |
| $\overline{37}$ | $\frac{1}{15}$ | 0 | 0 | 0 | 0 | $\frac{5}{12}$ | $\frac{1}{4}$ | $\frac{4}{15}$ | 0 | 0 |
| $\overline{68}$ | 0 | $-\frac{2}{9}$ | 0 | $\frac{4}{27}$ | $-\frac{4}{27}$ | 0 | 0 | 0 | $\frac{1}{15}$ | $-\frac{8}{45}$ |
| $\widetilde{37}$ | 0 | 0 | $\frac{5}{12}$ | $\frac{5}{81}$ | $\frac{5}{324}$ | 0 | 0 | 0 | $-\frac{1}{9}$ | 0 |
| $\widetilde{28}$ | 0 | 0 | $\frac{5}{36}$ | $-\frac{5}{27}$ | $-\frac{5}{108}$ | 0 | 0 | 0 | $\frac{1}{3}$ | 0 |
| $\widetilde{68}$ | $-\frac{1}{9}$ | 0 | 0 | 0 | 0 | $\frac{4}{9}$ | 0 | $-\frac{4}{9}$ | 0 | 0 |
| 77 | 0 | $-\frac{1}{3}$ | 0 | $-\frac{8}{81}$ | $\frac{8}{81}$ | 0 | 0 | 0 | $-\frac{2}{45}$ | $-\frac{4}{15}$ |
| 33 | 0 | 0 | 0 | $\frac{5}{81}$ | $\frac{20}{81}$ | 0 | 0 | 0 | $\frac{4}{9}$ | 0 |

| | 432 | $43_13$ | $43_23$ |
|---|---|---|---|
| $\overline{28}$ | 0 | 0 | 0 |
| $\overline{37}$ | 0 | 0 | 0 |
| $\overline{68}$ | 0 | $\frac{16}{135}$ | $-\frac{16}{135}$ |
| $\widetilde{37}$ | $\frac{1}{3}$ | $\frac{4}{81}$ | $\frac{1}{81}$ |
| $\widetilde{28}$ | $\frac{1}{9}$ | $-\frac{4}{27}$ | $-\frac{1}{27}$ |
| $\widetilde{68}$ | 0 | 0 | 0 |
| 77 | 0 | $-\frac{32}{405}$ | $\frac{32}{405}$ |
| 33 | 0 | $\frac{4}{81}$ | $\frac{16}{81}$ |

| $IJ = 10$ | 141 | 143 | 232 | 323 | 341 | 343 | 432 |
|---|---|---|---|---|---|---|---|
| $\overline{68}$ | $-\frac{2}{25}$ | $-\frac{4}{75}$ | 0 | $\frac{1}{3}$ | $-\frac{8}{25}$ | $-\frac{16}{75}$ | 0 |
| 77 | $-\frac{3}{25}$ | $\frac{8}{225}$ | 0 | $-\frac{2}{9}$ | $-\frac{12}{25}$ | $\frac{32}{225}$ | 0 |
| $\widetilde{68}$ | 0 | 0 | $-\frac{5}{9}$ | 0 | 0 | 0 | $-\frac{4}{9}$ |
| 33 | 0 | $\frac{1}{9}$ | 0 | $\frac{4}{9}$ | 0 | $\frac{4}{9}$ | 0 |

| $IJ = 03$ | 322 |
|---|---|
| 77 | 1 |

| $IJ = 02$ | 232 | 235 | 322 | 325 | 412 | 432 | 435 |
|---|---|---|---|---|---|---|---|
| $\overline{37}$ | 0 | 0 | $-\frac{1}{2}$ | $-\frac{1}{2}$ | 0 | 0 | 0 |
| $\overline{48}$ | $\frac{1}{18}$ | $-\frac{5}{18}$ | 0 | 0 | $\frac{2}{5}$ | $\frac{2}{45}$ | $-\frac{2}{9}$ |
| $\widetilde{37}$ | $\frac{1}{18}$ | $\frac{5}{18}$ | 0 | 0 | $\frac{2}{5}$ | $\frac{2}{45}$ | $\frac{2}{9}$ |
| 77 | $-\frac{4}{9}$ | 0 | 0 | 0 | $\frac{1}{5}$ | $-\frac{16}{45}$ | 0 |
| $\widetilde{48}$ | 0 | 0 | $-\frac{1}{2}$ | $\frac{1}{2}$ | 0 | 0 | 0 |

TABLE I. *(Continued).*

| $IJ=01$ | 142 | 145 | 232 | 235 | 322 | 325 | 342 | 345 | 432 | 435 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\overline{37}$ | 0 | 0 | $\frac{5}{18}$ | $\frac{5}{18}$ | 0 | 0 | 0 | 0 | $\frac{2}{9}$ | $\frac{2}{9}$ |
| $\overline{48}$ | $\frac{2}{45}$ | $-\frac{4}{45}$ | 0 | 0 | $\frac{5}{18}$ | $\frac{1}{18}$ | $\frac{8}{45}$ | $-\frac{16}{45}$ | 0 | 0 |
| $\widetilde{37}$ | $\frac{2}{45}$ | $\frac{4}{45}$ | 0 | 0 | $\frac{5}{18}$ | $-\frac{1}{18}$ | $\frac{8}{45}$ | $\frac{16}{45}$ | 0 | 0 |
| 77 | $-\frac{1}{9}$ | 0 | 0 | 0 | $\frac{4}{9}$ | 0 | $-\frac{4}{9}$ | 0 | 0 | 0 |
| $\widetilde{48}$ | 0 | 0 | $\frac{5}{18}$ | $-\frac{5}{18}$ | 0 | 0 | 0 | 0 | $\frac{2}{9}$ | $-\frac{2}{9}$ |
| 33 | 0 | $\frac{1}{45}$ | 0 | 0 | 0 | $\frac{8}{9}$ | 0 | $\frac{4}{45}$ | 0 | 0 |

| $IJ=00$ | 232 | 235 | 432 | 435 |
|---|---|---|---|---|
| 77 | $-\frac{5}{9}$ | 0 | $-\frac{4}{9}$ | 0 |
| 33 | 0 | $-\frac{5}{9}$ | 0 | $-\frac{4}{9}$ |

(f) $Y=-3$

| $IJ=\frac{1}{2}3$ | 322 | 411 |
|---|---|---|
| $\overline{78}$ | 0 | 1 |
| $\widetilde{78}$ | 1 | 0 |

| $IJ=\frac{1}{2}2$ | 232 | 321 | 322 | 412 | 432 |
|---|---|---|---|---|---|
| $\overline{38}$ | $\frac{1}{9}$ | 0 | 0 | $\frac{4}{5}$ | $\frac{4}{45}$ |
| $\overline{78}$ | 0 | 1 | 0 | 0 | 0 |
| $\widetilde{38}$ | 0 | 0 | $-1$ | 0 | 0 |
| $\widetilde{78}$ | $-\frac{4}{9}$ | 0 | 0 | $\frac{1}{5}$ | $-\frac{16}{45}$ |

| $IJ=\frac{1}{2}1$ | 142 | 231 | 232 | 322 | 342 | 431 | 432 |
|---|---|---|---|---|---|---|---|
| $\overline{38}$ | $\frac{4}{45}$ | 0 | 0 | $\frac{5}{9}$ | $\frac{16}{45}$ | 0 | 0 |
| $\overline{78}$ | 0 | $-\frac{5}{9}$ | 0 | 0 | 0 | $-\frac{4}{9}$ | 0 |
| $\widetilde{38}$ | 0 | 0 | $\frac{5}{9}$ | 0 | 0 | 0 | $\frac{4}{9}$ |
| $\widetilde{78}$ | $-\frac{1}{9}$ | 0 | 0 | $\frac{4}{9}$ | $-\frac{4}{9}$ | 0 | 0 |

| $IJ=\frac{1}{2}0$ | 141 | 232 | 341 | 342 |
|---|---|---|---|---|
| $\overline{78}$ | $-\frac{1}{5}$ | 0 | $-\frac{4}{5}$ | 0 |
| $\widetilde{78}$ | 0 | $-\frac{5}{9}$ | 0 | $-\frac{4}{9}$ |

(g) $Y=-4$

| $IJ=03$ | 411 |
|---|---|
| 88 | 1 |

| $IJ=02$ | 321 |
|---|---|
| 88 | 1 |

| $IJ=01$ | 231 | 431 |
|---|---|---|
| 88 | $-\frac{5}{9}$ | $-\frac{4}{9}$ |

| $IJ=00$ | 141 | 341 |
|---|---|---|
| 88 | $-\frac{1}{5}$ | $-\frac{4}{5}$ |

$$\langle\Phi_{\alpha K}|H|\Phi_{\alpha K'}\rangle=\sum\binom{6}{2}\langle\Phi_{\alpha K}|\alpha_1K_1,\alpha_2K_2\rangle\langle\alpha_1'K_1',\alpha_2'K_2'|\Phi_{\alpha K'}\rangle\langle\alpha_1K_1|\alpha_1'K_1'\rangle\langle\alpha_2K_2|H_{56}|\alpha_2'K_2'\rangle\ . \qquad (12)$$

Here $\langle\alpha_1K_1|\alpha_1'K_1'\rangle$ is the four-quark overlap and is a little more complicated than the atomic and nuclear shell model case because of the nonorthogonal property of the single-quark orbital state (see below). $\langle\alpha_2K_2|H_{56}|\alpha_2'K_2'\rangle$ is the two-body matrix element, and $H_{56}$ represents the two-body operator for the last pair. $\binom{6}{2}=15$ is the interacting pair number. To simplify the computer program, the one-body operator matrix elements are calculated by the same expansion [Eq. (12)] with the obvious substitution $H_{56} \to H_5 + H_6$ and $\binom{6}{2} \to \frac{6}{2} = 3$ (only 6 one-body operators altogether instead of 15 pair interactions).

$\langle \Psi_{\alpha K} | \alpha_1 K_1, \alpha_2 K_2 \rangle$ and $\langle \alpha_1' K_1', \alpha_2' K_2' | \Phi_{\alpha K'} \rangle$ are the total Clebsch-Gordan coefficients (CGC's). They are calculated as follows [8(b)]:

$$\left\langle [\sigma] W \overset{[\nu] l^3 r^3}{[\mu] \beta [f]} Y I J M_I M_J \,\middle|\, [\sigma_1 W_1^c, \overset{[\nu_1] W_1^x}{[\mu_1][f_1]} Y_1 I_1 J_1 M_{I_1} M_{J_1}, [\sigma_2] W_2^c \overset{[\nu_2] W_2^x}{[\mu_2][f_2]} Y_2 I_2 J_2 M_{I_2} M_{J_2} \right\rangle$$

$$= \sum_\gamma C^{[\sigma]W}_{[\sigma_1]W_1^c, [\sigma_2]W_2^c} C^{IM_I}_{I_1 M_{I_1}, I_2 M_{I_2}} C^{JM_J}_{J_1 M_{J_1}, J_2 M_{J_2}} C^{[\nu] l^3 r^3}_{[\nu_1] W_1^x, [\nu_2] W_2^x}$$
$$\times C^{[1^6][\nu][\tilde{\nu}]}_{[1^4][\nu_1][\tilde{\nu}_1],[1^2][\nu_2][\tilde{\nu}_2]} C^{[\tilde{\nu}][\sigma][\mu]}_{[\tilde{\nu}_1][\sigma_1][\mu_1],[\tilde{\nu}_2][\sigma_2][\mu_2]} C^{[\mu]\beta[f]\gamma J}_{[\mu_1][f_1][J_1],[\mu_2][f_2][J_2]} C^{[f]\gamma Y I}_{[f_1] Y_1 I_1, [f_2] Y_2 I_2} . \quad (13)$$

The first four $C$'s are the $SU^c(3)$, $SU^\tau(2)$, $SU^\sigma(2)$, and $SU^x(2)$ CGC's, the next three $C$'s are the $SU(36) \supset SU^x(2) \times SU(18)$, $SU(18) \supset SU^c(3) \times SU(6)$, and $SU(6) \supset SU^f(3) \times SU^\sigma(2)$ isoscalar factors, and the last one is the $SU^f(3) \supset SU^\tau(2) \times U^Y(1)$ isoscalar factor. All these isoscalar factors (for particle number $\leq 6$) can be found in Ref. [8(b)]. The $SU^x(2)$ orbital CGC $C^{[\nu] l^3 r^3}_{[\nu_1] W_1^x, [\nu_2] W_x^x}$ is called the orbital two-body cfp by Harvey and listed in his Table 4 [9]. It is obvious that it is better to use the standard phase convention of the SU(2) CGC. Then the entries under [4]: $[11]a^2b^2 : \tilde{a}b$, $[31] : [2]ab^3 : a^2$, and $[31] : [2]a^2b^2 : \bar{a}b$ should be assigned opposite signs.

The four-quark state $|\alpha_1 K_1\rangle$ can be expressed as [10]

$$|\alpha_1 K_1\rangle = \left| [\sigma_1] W_1^c \overset{[\nu_1] W_1^x}{[\mu_1][f_1]} Y_1 I_1 J_1 M_{I_1} M_{J_1} \right\rangle$$
$$= \sum_m (h_{\nu_1})^{-\frac{1}{2}} \Lambda_m^{\nu_1} \left| \begin{matrix} [\nu_1] W_1^x \\ m \end{matrix} \right\rangle \left| \begin{matrix} [\tilde{\nu}_1] \\ \tilde{m} \end{matrix} [\sigma_1] W_1^c [\mu_1][f_1] Y_1 I_1 M_{I_1} M_{J_1} \right\rangle ; \quad (14)$$

here, $m(\tilde{m})$ is the Yamanouchi number of the Young tableau, $(h_{\nu_1})^{-\frac{1}{2}} \Lambda_m^{\nu_1}$ is the CGC for $[\nu_1] \times [\tilde{\nu}_1] \to [1^4]$ of the permutation group. The color-flavor-spin part

$$\left| \begin{matrix} [\tilde{\nu}_1] \\ \tilde{m} \end{matrix} [\sigma_1] W_1^c [\mu_1][f_1] Y_1 I_1 J_1 M_{I_1} M_{J_1} \right\rangle$$

is orthogonal as usual,

$$\left\langle \begin{matrix} [\tilde{\nu}_1] \\ \tilde{m} \end{matrix} [\sigma_1] W_1^c [\mu_1][f_1] Y_1 I_1 J_1 M_{I_1} M_{J_1} \middle| \begin{matrix} [\tilde{\nu}_1'] \\ \tilde{m}' \end{matrix} [\sigma_1'] W_1'^c [\mu_1'][f_1'] Y_1' I_1' J_1' M_{I_1}' M_{J_1}' \right\rangle = \delta_{11'} ; \quad (15)$$

here, $\delta_{11'}$ is a product of $\delta_{\nu_1 \nu_1'}, \delta_{mm'}, \ldots$, which includes every pair of quantum numbers. The only complication is caused by the nonorthogonality of the single-quark orbital state,

$$\left\langle \begin{matrix} [\nu_1] W_1^x \\ m \end{matrix} \middle| \begin{matrix} [\nu_1'] W_1'^x \\ m' \end{matrix} \right\rangle = \delta_{\nu_1 \nu_1'} \delta_{mm'} \left\langle \begin{matrix} [\nu_1] W_1^x \\ m \end{matrix} \middle| \begin{matrix} [\nu_1'] W_1'^x \\ m' \end{matrix} \right\rangle . \quad (16)$$

Finally, we have the four-quark overlap

$$\langle \alpha_1 K_1 | \alpha_1' K_1' \rangle = \delta_{11'} h_{\nu_1}^{-1} \sum_m \left\langle \begin{matrix} [\nu_1] W_1^x \\ m \end{matrix} \middle| \begin{matrix} [\nu_1] W_1'^x \\ m \end{matrix} \right\rangle$$
$$= \delta_{11'} \left\langle \begin{matrix} [\nu_1] W_1^x \\ m \end{matrix} \middle| \begin{matrix} [\nu_1] W_1'^x \\ m \end{matrix} \right\rangle \text{ (any } m) \quad (17)$$

This four-body overlap is listed by Harvey in his Table 6 [9]. To be consistent with the standard $SU^x(2)$ CGC phase convention, all the entries in his Table 6 should have positive signs. Another modification is caused by the delocalized orbit [Eq. (4)]: The $m$ in Harvey's Table 6 should be replaced by

$$m \to [2\epsilon + (1+\epsilon^2)F]/(1+\epsilon^2+2\epsilon F), \quad F = \langle \phi_L | \phi_R \rangle .$$

Harvey's result is our $\epsilon = 0$ limit.

The two-quark state

$$|\alpha_2 K_2\rangle = \left| [\sigma_2] W_2^c \overset{[\nu_2] W_2^x}{[\mu_2][f_2]} Y_2 I_2 J_2 M_{I_2} M_{J_2} \right\rangle \quad (18)$$

can be expressed in a similar form as Eq. (14). But the $[\nu_2]$, $[\sigma_2]$, $[\mu_2]$, and $[f_2]$ are either symmetric [2] or antisymmetric $[1^2]$, and Eq. (18) is in fact just a product of the orbital, color, flavor, and spin parts. The two-body interaction matrix elements can be factorized too,

$$\langle\alpha_2K_2|H_{56}|\alpha_2'K_2'\rangle = \left\langle \begin{matrix}[\nu_2]\\ W_2^x\end{matrix} \middle| H_{56}^x \middle| \begin{matrix}[\nu_2']\\ W_2'^x\end{matrix} \right\rangle \left\langle \begin{matrix}[\sigma_2]\\ W_2^c\end{matrix} \middle| H_{56}^c \middle| \begin{matrix}[\sigma_2']\\ W'^x_{W_2'^c}\end{matrix} \right\rangle \left\langle \begin{matrix}[f_2]\\ Y_2I_2M_{I_2}\end{matrix} \middle| H_{56}^f \middle| \begin{matrix}[f_2']\\ Y_2'I_2'M_{I_2}'\end{matrix} \right\rangle \langle J_2M_{J_2}|H_{56}^\sigma|J_2'M_{J_2}'\rangle \ . \tag{19}$$

Here we have used the fact that the two-body interaction is a sum of terms of the form which we take as a single term for simplicity:

$$H_{56} = H_{56}^c \times H_{56}^f \times H_{56}^\sigma \times H_{56}^x \ . \tag{20}$$

For the nonrelativistic case, $H$ is a scalar of $SU^c(3)$, $SU^\tau(2)$, and $SU^\sigma(2)$, the two-body matrix elements are $W_2^c$, $M_{I_2}$, and $M_{J_2}$ independent, and the first three CGC's in Eq. (13) will disappear in the matrix element $\langle\Phi_{\alpha K}|H|\Phi_{\alpha K'}\rangle$ of Eq. (12) as a result of the orthonormal property of CGC's.

For the one-body operator (kinetic energy in a nonrelativistic model, kinetic energy and mean field in a relativistic model),

$$H_{56} = H_5 + H_6 \ ,$$

by expanding the coupled state into the product of two-particle states with CGC's and using the orthonormal property of CGC's, the 2 one-body operator matrix elements can be calculated very easily. The $6 \to 5 + 1fp$ expansion can be avoided, and only the $6 \to 4 + 2fp$ coefficients need to be included in a computer program package.

## IV. RELATIVISTIC EXTENSION

It is commonly believed that the classification scheme [Eq. (2)] can be applied to the nonrelativistic quark only, because the spin and orbital parts are intrinsically coupled into a Dirac spinor for a relativistic quark. However, in a Dirac cluster model, only the lowest Dirac state is used and the lowest state of a Dirac particle moving in a central potential can be expressed as a product of a pseudo orbit and a Pauli spinor [14],

$$\phi_\sigma(\mathbf{r}') = \begin{pmatrix} \phi_u(\mathbf{r}') \\ -i\boldsymbol{\sigma}\cdot\mathbf{r}'\phi_{\alpha'}(\mathbf{r}') \end{pmatrix} \chi_\sigma Y_{00}(\theta',\phi') \ . \tag{21}$$

Here $\mathbf{r}' = \mathbf{r} - \mathbf{s}/2$ or $\mathbf{r} + \mathbf{s}/2$ depends on the confinement center, $\chi_\sigma$ is the usual Pauli spinor, $\sigma = j_z = \pm\frac{1}{2}$, $Y_{00} = \sqrt{\frac{1}{4\pi}}$, and $\phi_u$ and $\phi_d$ are the upper and lower (down) components of the Dirac WF. Taking the $\sqrt{\frac{1}{4\pi}}\begin{pmatrix} \phi_u(\mathbf{r}') \\ -i\boldsymbol{\sigma}\cdot\mathbf{r}'\phi_{\alpha'}(\mathbf{r}') \end{pmatrix}$ as a pseudo-orbit WF equivalent to that for the nonrelativistic orbital WF, we obtain two linear independent states as the bases of a pseudo-orbit $SU^x(2)$ for the Dirac quark. In this way we can use the same classification scheme [Eq. (2)] to describe the six-Dirac-quark system [15]. The whole calculation method discussed in Secs. II and III can be extended to a Dirac quark cluster model directly. The only difference is that when we calculate the one- and two-body matrix elements, we have to recombine the pseudo orbit and the Pauli spinor together to be a Dirac spinor. For the four-quark overlap calculation, recombination of the pseudo orbit and Pauli spinor seems to be needed too. However, because we only use the lowest Dirac state WF [Eq. (21)], the single-particle overlap still can be separated into a pseudo-orbit part and a Pauli spinor part,

$$\begin{aligned}\langle\psi_{\sigma_1}(\mathbf{r}_1)|\psi_{\sigma_2}(\mathbf{r}_2)\rangle &= \chi_{\sigma_1}^\dagger \frac{1}{4\pi}\int d\mathbf{r}(\psi_u(\mathbf{r}_1), i\boldsymbol{\sigma}\cdot\mathbf{r}_1\psi_d(\mathbf{r}_1))(\psi_u(\mathbf{r}_2), -i\boldsymbol{\sigma}\cdot\mathbf{r}_2\psi_d(\mathbf{r}_2))\chi_{\sigma_2} \\ &= \chi_{\sigma_1}^\dagger \frac{1}{4\pi}\int d\mathbf{r}[\psi_u(\mathbf{r}_1)\psi_u(\mathbf{r}_2) + \boldsymbol{\sigma}\cdot\mathbf{r}_1\boldsymbol{\sigma}\cdot\mathbf{r}_2\psi_d(\mathbf{r}_1)\psi_d(\mathbf{r}_2)]\chi_{\sigma_2} \\ &= \chi_{\sigma_1}^\dagger\chi_{\sigma_2} \frac{1}{4\pi}\int d\mathbf{r}[\psi_u(\mathbf{r}_1)\psi_u(\mathbf{r}_2) + \mathbf{r}_1\cdot\mathbf{r}_2\psi_d(\mathbf{r}_1)\psi_d(\mathbf{r}_2)] \ . \end{aligned} \tag{22}$$

The spin-dependent part is identically zero [16],

$$\frac{1}{4\pi}\int i\boldsymbol{\sigma}\cdot(\hat{\mathbf{r}}_1\times\hat{\mathbf{r}}_2)\psi_d(\mathbf{r}_1)\psi_d(\mathbf{r}_2)d\mathbf{r} \equiv 0 \ . \tag{23}$$

Therefore the four-quark overlap calculation can be done in exactly the same way as that for the nonrelativistic case, i.e., separated into a pseudo-orbital part and a spin part.

## V. COMPUTERIZED FRACTIONAL PARENTAGE EXPANSION

All the needed $SU(mn) \supset SU(m)\times SU(n)$ isoscalar factors can be found directly from Ref. [8(b)]; the needed $SU(3)\supset SU(2)\times U(1)$ isoscalar factors can be obtained from the SU(3) CGC of Chen *et al.* [8(b)] and the standard SU(2) CGC. [Some SU(3) CGC's not explicitly listed there can be obtained by the symmetry properties from the listed ones. Table II gives the additional needed phase factors $\epsilon_2$ which are missing in Table 5 of Sec. II of Ref. [8(b)].]

It is time consuming and requires a good grasp of group theory to combine the individual isoscalar factors into the transformation coefficients between physical bases and symmetry bases and the $6 \to 4 + 2$ cfp for the matrix element calculations.

In order to make the calculation automatic and to facilitate others using this fp-expansion technique, a com-

TABLE II. Additional phase factor $\epsilon_2(\nu_1\nu_2\nu)$.

| $\nu_1\nu_2$ | $\nu$ | $\epsilon_2$ | $\nu_1\nu_2$ | $\nu$ | $\epsilon_2$ | $\nu_1\nu_2$ | $\nu$ | $\epsilon_2$ |
|---|---|---|---|---|---|---|---|---|
| [4] [11] | [51] | 1 | [22] [11] | [33] | 1 | [211] [11] | [2211] | 1 |
| | [411] | 1 | | [321] | −1 | | $[21^4]$ | 1 |
| [31] [11] | [42] | 1 | | [2211] | −1 | $[1^4]$ [11] | [2211] | 1 |
| | [321] | 1 | [211] [11] | [321] | 1 | | $[21^4]$ | −1 |
| | $[31^3]$ | −1 | | [222] | −1 | | $[1^6]$ | 1 |
| | [411] | −1 | | $[31^3]$ | 1 | | | |

puter program has been written. All the needed isoscalar factors are stored in the program. After inputting the quantum numbers $\alpha = (YIJ)$, the program will automatically yield the physical bases, symmetry bases, and the transformation coefficients between these two bases, and the $6 \to 4+2$ cfp for the symmetry bases. This part may be useful for other dibaryon model practitioners if they want to use fp-expansion methods. For our own problem, the program continues on to calculate the one- and two-body matrix elements and the four-body overlap, combine them together into the six-quark Hamiltonian matrix elements in the physical bases, diagonalize the Hamiltonian in the nonorthogonal physical basis space, minimize the eigenenergy, fix the eigen-WF with respect to the delocalization parameter $\epsilon(s)$, repeat this calculation for different separations $s$ between two $q^3$ clusters from $s = 0.1$ to 3 fm, and finally output the adiabatic potential $V_\alpha(s)$. This program greatly reduced the labor involved in the systematic search of dibaryon candidates in the $u$, $d$, and $s$ three-flavor world. Only minor modification of the subroutine for the one- and two-body matrix elements calculations suffices to adapt the program to a relativistic quark model dibaryon search. We expect it is also easy to apply this program to other nonrelativistic and relativistic dibaryon calculations with minor modifications, particularly as the fp-expansion part is universal for this kind of dibaryon model calculation.

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## APPENDIX

Our tables for the symmetry decompositions might appear to contradict Harvey's results [9]. This is due to a difference in terminology: Harvey used "symmetric" and "antisymmetric" to refer only to the *orbital* components when discussing nonidentical particles. We prefer to use the more inclusive definition below, since we believe it allows for a more natural relation to the identical particle case.

The symmetric (antisymmetric) combination $\bar{x}y$ $(\tilde{x}y)$ of two baryon state is defined as

$$\overline{xy} = \frac{xy + yx}{\sqrt{2}}, \quad \widetilde{xy} = \frac{xy - yx}{\sqrt{2}} . \tag{A1}$$

Let us use the $N\Delta$ two-baryon state as an example to show the symmetry property. Below, $\chi_c$ is the color-singlet three-quark state $N_{m_N\tau_N}(\Delta_{m_\Delta\tau_\Delta})$ is a three-quark $N(\Delta)$ spin-isospin symmetric state with spin-isospin projection quantum numbers $m_N\tau_N(m_\Delta\tau_\Delta)$, $l(123)$ is a product orbital state $l(1)l(2)l(3)$, where $l$ is defined in Eq. (4), $r(456)$ has the parallel meaning, and $C^{Kk}_{K_ak_a,K_bk_b}$ is the spin (isospin) CGC. Then

$$\begin{aligned}(N\Delta)_{IJ} &= A\Sigma C^{Jm}_{J_Nm_N,J_\Delta m_\Delta} C^{I\tau}_{I_N\tau_N,I_\Delta\tau_\Delta} \chi_c(123) N_{m_N\tau_N}(123) l(123) \chi_c(456) \Delta_{m_\Delta\tau_\Delta}(456) r(456) \\ &= \frac{1}{\sqrt{20}} \Sigma C^{Jm}_{J_Nm_N,J_\Delta m_\Delta} C^{I\tau}_{I_N\tau_N,I_\Delta\tau_\Delta} \{\chi_c(123)\chi_c(456)[N_{m_N\tau_N}(123)\Delta_{m_\Delta\tau_\Delta}(456) l(123) r(456) \\ &\quad - N_{m_N\tau_N}(456)\Delta_{m_\Delta\tau_\Delta}(123) l(456) r(123)] + \cdots\} ,\end{aligned} \tag{A2}$$

$$\begin{aligned}(\Delta N)_{IJ} &= A\Sigma C^{Jm}_{J_\Delta m_\Delta,J_Nm_N} C^{I\tau}_{I_\Delta\tau_\Delta,I_N\tau_N} \chi_c(123) \Delta_{m_\Delta\tau_\Delta}(123) l(123) \chi_c(456) N_{m_N\tau_N}(456) r(456) \\ &= \frac{1}{\sqrt{20}} \Sigma C^{Jm}_{J_\Delta m_\Delta,J_Nm_N} C^{I\tau}_{I_\Delta\tau_\Delta,I_N\tau_N} \{\chi_c(123)\chi_c(456)[\Delta_{m_\Delta\tau_\Delta}(123) N_{m_N\tau_N}(456) l(123) r(456) \\ &\quad - \Delta_{m_\Delta\tau_\Delta}(456) N_{m_N\tau_N}(123) l(456) r(123)] + \cdots\} ,\end{aligned} \tag{A3}$$

where $+\cdots$ represents all of the other permutations:

$$(\overline{N\Delta})_{IJ} = \frac{(N\Delta)_{IJ} + (\Delta N)_{IJ}}{\sqrt{2}}$$
$$= \frac{1}{\sqrt{40}} \Sigma C^{Jm}_{J_N m_N, J_\Delta m_\Delta} C^{I\tau}_{I_N \tau_N, I_\Delta \tau_\Delta} (\chi_c(123)\chi_c(456)$$
$$\times \{N_{m_N \tau_N}(123)\Delta_{m_\Delta \tau_\Delta}(456)[l(123)r(456) - (-)^{J_N+J_\Delta-J+I_N+I_\Delta-I} l(456)r(123)]$$
$$-N_{m_N \tau_N}(456)\Delta_{m_\Delta \tau_\Delta}(123)[l(456)r(123) - (-)^{J_N+J_\Delta-J+I_N+I_\Delta-I} l(123)r(456)]\} + \cdots) . \quad \text{(A4)}$$

The orbital symmetry property (the parity) of a two-baryon state under the permutation $\begin{pmatrix} 123\,456 \\ 456\,123 \end{pmatrix}$ is dependent on the spin-isospin quantum numbers, instead of directly related to the symmetry (antisymmetry) $\overline{xy}$ ($\widetilde{xy}$) combination as explained in [9].

In deriving (A4), we have used the well-known SU(2) relation

$$C^{Kk}_{K_a k_a K_b k_b} = (-)^{K_a+K_b-K} C^{Kk}_{K_b k_b K_a k_a} . \quad \text{(A5)}$$

Note, for example, that if the $\Delta$ were replaced by a second $N$ and $I + J$ is even, the first and fourth terms become identical as do the second and third terms, etc. The result has *only* antisymmetric orbital parts. The $(\overline{NN})_{IJ} \equiv 0$ for odd $I + J$. Conversely, for $(\overline{NN})_{IJ}$, only the odd $I + J$ symmetric orbital parts exist (as, for example, in the deuteron).

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