Behavior of negative parity spin modes as a function of the strength of the tensor interaction: Shell model vs one particle one hole (or random-phase approximation)

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We consider negative parity excitations in ^{4}He, J^π=0^-, 1^-, and 2^- with isospin T=0 and 1. In a one-particle one-hole calculation the unnatural parity states, J=0^- and 2^- come down in energy linearly with the increasing strength of the tensor interaction, ultimately coming below the original J=0^+ ground state. However in a shell model diagonalization in a sufficiently large space, although the states may start to come down, there is a turnaround so that the excitation energies of these states thereafter increases with increasing tensor strength, and there is no energy inversion with the J=0^+ state.

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I. INTRODUCTION

In this paper we wish to study the behavior of negative parity excitations in a closed shell nucleus as a function of the strength of the tensor interaction. We are especially interested in spin modes, such as the J=0^- states which in LS coupling have the quantum numbers L=1^-, S=1, J=0. We will also consider J=1^- states which are partly spin mode and partly not and J=2^- which again are pure spin modes. The J=0^- and 2^- states are called unnatural parity states.

One motivation for studying these states is that their properties have been proposed as signatures of precursors to pion condensation. If the T=1 spin mode states were to come down very low in energy, this would be a good signature.

In this paper we will not deal directly with the problem of precursors to pion condensation or pion condensation itself. Rather we will consider a related problem of studying how the energies of these states respond to increasing the strength of the tensor interaction. We feel that in the consideration of problems like precursors, the nuclear structure has often been oversimplified to such an extent that wrong conclusions could be drawn. We will therefore focus on a problem over which we have very good control and make a comparison of simplified pictures of the spin mode excitations, e.g., $1p$-$1h$ excitations [or the random-phase approximation (RPA) generalization] and the more exact large shell model diagonalizations.

II. METHOD

A. Lowest order

In lowest order, the configuration for the ground state of ^{4}He is $(0s)^4$, and the excited states mentioned above are one-particle one-hole configurations. The 0^- state in lowest order has the unique configuration $(0p\frac{1}{2}\ 0s\frac{1}{2}^{-1})$. The energy of this state is given by

$$E(0^-,T) = \epsilon_{0p_{\frac{1}{2}}} - \epsilon_{0s_{\frac{1}{2}}} + \langle (0p_{\frac{1}{2}} 0s_{\frac{1}{2}}{}^{-1})|V|(0p_{\frac{1}{2}} 0s_{\frac{1}{2}}{}^{-1})\rangle^{0^-,T} , \quad (1)$$

where ϵ_i are the single-particle energies and the last term is the particle-hole interaction. It should be mentioned that we calculate the single-particle energies with the same interaction that is used to calculate the two-particle matrix elements. We used the schematic (or democratic) interaction described in a work by Zheng and Zamick [1]. It is of the form

$$V = V_c + xV_{so} + yV_t , \quad (2)$$

where $c \equiv$ central, $so \equiv$ spin orbit, and $t \equiv$ tensor. For $x = 1$, $y = 1$ one gets a fairly good fit to the two-body matrix elements of more realistic interactions like Bonn A. We focus on the effects of the *tensor interaction* on the energies of the unnatural parity states in ^{4}He. We do this by varying y, the strength of the tensor interaction. In the simple one-particle one-hole picture, the single-particle energies do not depend on y (i.e., the first-order tensor contribution to these energies is zero) and only the particle-hole matrix element is affected. In the full shell model calculation the situation is more complicated—there are many configurations.

In a $1p$-$1h$ calculation, the J=0^- states have unique configurations $(0p_{\frac{1}{2}} 0s_{\frac{1}{2}}{}^{-1})^{J=0,T}$ with T=0 or 1. Using Eq. (1) we note that as we increase y, the single-particle energies ϵ do not change. Obviously, the particle-hole interaction

$$V_{ph} = \langle (0p_{\frac{1}{2}} 0s_{\frac{1}{2}}{}^{-1})|V|(0p_{\frac{1}{2}} 0s_{\frac{1}{2}}{}^{-1})\rangle^{0^-,T}$$

will be linear in y. We find

$$V_{ph}(T=0) = 2.575 - 3.820y \quad \text{MeV} ,$$

$$V_{ph}(T=1) = 3.445 - 1.270y \quad \text{MeV} .$$

Note that the coefficient of y for T=1, i.e., the slope,

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is $\frac{1}{3}$ of that for T=0. The excitation energy will decrease linearly in y and we clearly can get the 0^- states coming below the ground state by making y sufficiently large. The T=1 state in this model is always higher in energy than the T=0, J=0^- states. We have not performed RPA calculations but we know from experience that when states come down in energy in Tamm-Dancoff approximation (TDA) calculations they come down even faster in RPA calculations. Beyond the point where the energy of the unnatural parity state has zero excitation energy the RPA energies become imaginary.

It should be remarked that a tensor force instability, such as the one described above, was noted by Bleuler in his 1966 Varenna Lectures, "Parity Mixing in Spherical Nuclei" [2]. The results we have so far are much as he describes. The linear behavior that we get for the energy of J=0^- is also similar to that shown in the book by Eisenberg and Koltun [3] in the section where they present a schematic model of pion condensation. However, the parameter they vary is the pion-nucleon coupling constant, not the overall strength of the tensor interaction.

We further cite the work of Meyer-ter-Vehn [4] who shows that an approach to pion condensation manifests itself by having states coming down to zero energy in RPA calculations. He uses $J = 2^-$ states in ^{16}O as an example. This work is discussed in the book by Ericson and Weise [5]. We should also mention the works of Oset *et al.* [6] and Goeke and Speth [7].

As discussed by Ericson and Weise, an interaction of the following type is used (in momentum space):

$$V_{\sigma\tau} = \frac{f^2}{m_\pi^2}\left(g'\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 - \frac{(\boldsymbol{\sigma}_1\cdot\mathbf{q})(\boldsymbol{\sigma}_2\cdot\mathbf{q})}{\mathbf{q}^2+m_\pi^2}\right)\tau_1\cdot\tau_2\ . \quad (3)$$

From information on Gamow-Teller resonances the repulsive delta interaction parameter g' is large, about 0.7. If, however, g' is decreased to 0.3, a 2^- state comes crashing down to zero energy in an RPA calculation. Barshay *et al.* [8] provide a justification for a large value of g'. Part of the contribution comes from short-range correlations in the nucleus due to a repulsive hard core in the nucleon-nucleon interaction. We reemphasize that the model we use in this work is different from the above "pion condensation" models.

B. Matrix diagonalization

Thus far in one-particle one-hole calculations with no ground-state correlations we find that the excitation energies of J=0^-, T=0, and T=1 states decrease linearly in y, the strength of the tensor interaction. We next perform shell model diagonalizations for ^{4}He using the OXBASH code [9] for the J=0^+ ground state and J=0^- states with isospin T=0 and T=1. We present the T=0 results in Fig. 1 where the excitation energy of the $J = 0^-$ state is plotted versus y, the strength of the tensor interaction. We consider four spaces.

(a) The smallest space: for J=0^+ the configuration is $(0s)^4$. For J=0^- we allow one nucleon to be excited from $0s$ to $0p$. This leads to the unique configuration $(0p_{\frac{1}{2}}0s_{\frac{1}{2}}{}^{-1})^{J=0^-}$. The results are shown as a solid line.

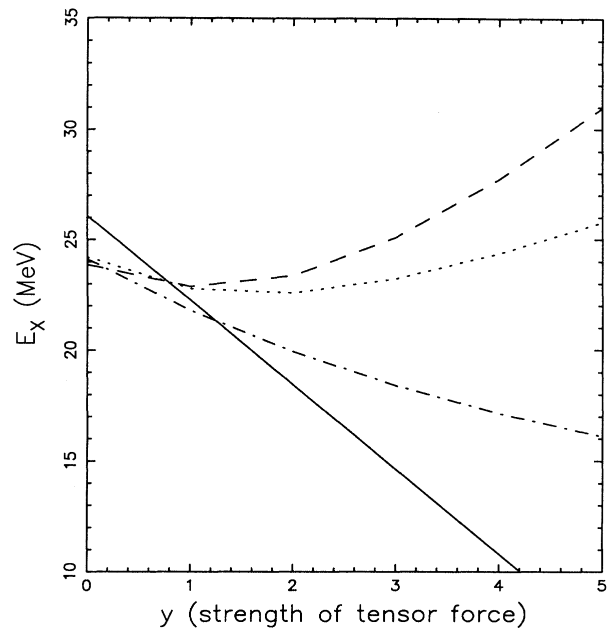

FIG. 1. Solid line corresponds to the smallest space ($0p_{\frac{1}{2}}$-$0s_{\frac{1}{2}}{}^{-1}$). Dashed line corresponds to the largest space (S, P, SD, PF). Dash-dot line corresponds to 1 and 3 $\hbar\omega$ excitations for 0^- and 0 and 2 $\hbar\omega$ for the ground state. Dotted line corresponds to 1, 3, and 5 $\hbar\omega$ excitations for 0^- and 0, 2, and 4 $\hbar\omega$ for the ground state.

(b) For the J=0^+ state we allow all "$0\hbar\omega$" and "$2\hbar\omega$" excitations. For the J=0^- state we allow all "$1\hbar\omega$" and "$3\hbar\omega$" excitations. The results are given by a dash-dot line.

(c) For J=0^+ we allow up to "$4\hbar\omega$" and for J=0^- we allow up to "$5\hbar\omega$" excitations. The results are given by a dotted line.

(d) Largest space: all nucleons can be anywhere in the first four major shells. The results are given by a dashed line.

The results in Fig. 1 show that as we increase the size of the shell model space, the behavior of the excitation energy versus y changes in a dramatic but systematic way. In the smallest space we get a linear decrease in excitation energy vs y as given by Eq. (1). The energy of the J=0^-, T=0 state goes to zero at y=4.2. This is the instability point.

When $2\hbar\omega$ excitations are included for J=0^+ and $3\hbar\omega$ for J=0^- (dash-dot line) we find that the descent is not so rapid and there is a deviation from linearity.

When $4\hbar\omega$ excitations for J=0^+ and $5\hbar\omega$ for J=0^- are included (dotted line) we find that there is a turnaround and the excitation energy never goes to zero. Although the excitation energy decreases slightly from y=0 to y=1, for higher values of y the excitation energy goes up. The J=0^- states never comes below or even near the J=0^+ ground state.

The effect is even more pronounced in our largest space calculation (dashed line) in which all four nucleons can

TABLE I. The excitation energies of the lowest-lying negative parity states in ^{4}He as a function of the strength of the tensor interaction y from matrix diagonalization in different configuration spaces.

Space	y	$J=0^-$ $T=0$	$J=0^-$ $T=1$	$J=1^-$ $T=0$	$J=1^-$ $T=1$	$J=2^-$ $T=0$	$J=2^-$ $T=1$
$1p$-$1h$	0	26.07	26.94	24.95	25.26	22.70	23.57
	1	22.26	25.67	26.86	25.73	22.32	23.44
	2	18.44	24.40	28.77	26.11	21.94	23.32
	3	14.62	23.13	30.68	26.41	21.55	23.19
	4	10.80	21.85	32.59	26.62	21.17	23.06
	5	6.98	20.58	34.50	27.76	20.79	22.93
S, P	0	26.42	27.24	25.29	25.54	23.04	23.87
	1	23.47	26.91	28.15	26.83	23.59	24.65
	2	22.31	28.28	32.88	29.75	25.99	27.24
	3	22.72	31.11	39.21	34.00	29.98	31.37
	4	24.27	35.01	46.72	39.21	35.14	36.61
	5	26.58	39.60	55.01	45.00	41.07	42.57
S, P, SD	0	28.03	29.53	27.04	28.03	25.00	26.35
	1	25.34	28.61	28.75	28.61	25.28	26.64
	2	23.18	28.12	31.13	29.58	26.73	27.78
	3	21.77	28.60	34.34	31.02	29.23	29.80
	4	21.19	30.08	38.30	33.00	32.56	32.60
	5	21.34	32.40	42.82	35.52	36.52	36.06
$S, P, SD,$ PF	0	23.87	24.96	23.48	24.39	22.56	23.63
	1	22.88	26.25	25.46	25.00	23.39	24.51
	2	23.37	28.82	29.65	27.35	26.27	27.09
	3	25.09	32.72	35.72	31.05	30.81	31.18
	4	27.72	37.68	42.96	35.75	36.46	36.39
	5	30.97	43.43	50.79	41.18	42.79	42.39

be anywhere in the first four major shells.

We now wish to show that the results in Fig. 1 for the $J=0^-$, $T=0$ modes also hold for other J, T values. In Table I we present results for $J=0^-$, 1^-, and 2^-, $T=0$, and $T=1$. The results are given first in the smallest space ($1p$-$1h$). Then we allow all four nucleons to be anywhere in the first two major shells *S, P.*

Next, they can be anywhere in the first three major shells *S, P, SD,* and finally anywhere in the first four major shells *S, P, SD, PF.*

Whereas in the smallest space the $J=0^-$, $T=0$, and $T=1$, and $J=2^-$, $T=0$, and $T=1$ mode excitation energies decrease linearly in y, in all the larger spaces the energies ultimately increase with increasing y.

We note that the ground-state binding energy changes as we increase y. In lowest order, i.e, $(0s)^4$, there is no contribution to the binding energy due to the tensor interaction (this holds for any *major* shell). However, the nucleon-nucleon interaction induces configuration mixing into the ground-state wave function. For this more complicated ground state, the tensor interaction does contribute to the binding energy.

The change in binding energy in MeV of the ground state relative to the case $y=0$, for the largest space calculation, is as follows:

$$y = 0, \quad 0.00; \quad y = 1, \quad 2.22\ ;$$
$$y = 2, \quad 8.72; \quad y = 3, \quad 18.84\ ;$$
$$y = 4, \quad 31.66; \quad y = 5, \quad 46.42\ .$$

We see that this change of energy starts out quadratic in y as we would expect from a second-order tensor effect.

Although the full matrix diagonalization does not lead to the negative parity excitations sinking below the $J=0^+$ state, the ground-state wave function does change as the tensor strength y is increased. The occupancy of shells higher than $0s$ increases with increasing y. From $y = 0$ to $y = 5$, $0s$ occupancies are 3.78, 3.71, 3.53, 3.31, 3.10, and 2.91. The corresponding $0p_{\frac{1}{2}}$ occupancies are 0.03, 0.07, 0.17, 0.29, 0.40, and 0.50. Hence the nature of the ground state does change but it does so in a continuous manner.

It should be noted that in this matrix diagonalization calculation in the effects of spurious states have been removed. More precisely, in OXBASH the spurious states are pushed up to a very high energy.

C. Second-order perturbation theory

Can we understand the shell model results of Table I by going to higher order perturbation theory? To answer this we have carried out a complete second-order calculation of the shift in energy of the $J=0^-$, $T=0$, and $T=1$ states. The results in which we allow excitations up to $4\hbar\omega$ are shown in Table II, and the corresponding diagrams are shown in Fig. 2. We have not done anything

TABLE II. Second-order perturbation theory results (MeV) : $J=0^-$, $T=0$.

	Strength of tensor interaction y					
Diagram	0	1	2	3	4	5
A: Bare	23.83	20.01	16.19	12.37	8.55	4.73
B: $1p$-$1h$ scattering	-0.07	-0.04	-0.02	-0.01	-0.003	0.000
C: RPA	-0.10	-1.49	-4.48	-9.07	-15.28	-23.09
D: Bubble	-0.08	2.32	6.23	11.64	18.55	26.97
E: $2p$-$2h$ scattering	0.08	-0.70	-2.35	-4.85	-8.22	-12.45
F: Single-particle renorm.	1.68	2.10	3.16	4.87	7.23	10.23
TOTAL	25.33	22.19	18.73	14.94	10.83	6.40

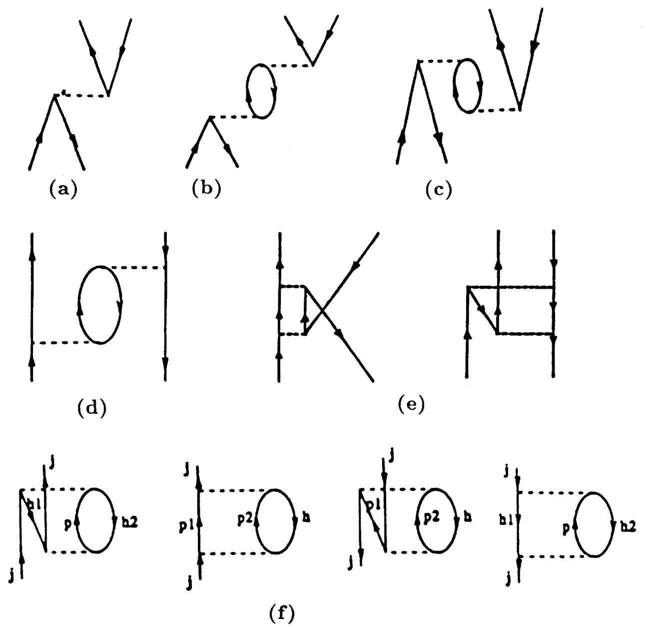

FIG. 2. (a) Bare; (b) $1p$-$1h$ scattering; (c) RPA; (d) bubble; (e) $2p$-$2h$ scattering; (f) single-particle and single-hole renormalization.

to remove the spurious center-of-mass motion in this perturbation theory calculation. For $J=0^-$, $T=0$ we see that we are still getting a monotonic decrease in energy of the states as y increases. The total results are completely different quantitatively and qualitatively from the shell model results in Table I. A similar behavior is seen for $J=0^-$, $T=1$ (see Table III). Note that the individual diagrams in second order can be very large but there is a large cancellation. For example, for $J=0^-$, $T=0$, $y=5$, diagram C, the phonon exchange between the particle and hole is 26.97 MeV but the RPA diagram is -23.09 MeV. The single particle–single hole energy correction is 16.23 MeV, but diagram F is -12.45 MeV. Indeed the *total* of all results up to second-order look embarrassingly similar to the bare results. That there would be large cancellations between RPA and other diagrams was especially noted some time ago by Kuo and Osnes [10]. There is also related work by Zamick on monopole vibrations [11]. The emphasis in these works was that the added diagrams make it more difficult for RPA instabilities to

TABLE III. Second-order perturbation theory results (MeV) : $J=0^-$, $T=1$.

	Strength of tensor interaction y					
Diagram	0	1	2	3	4	5
A: Bare	24.70	23.42	22.15	20.88	19.60	18.33
B: $1p$-$1h$ scattering	-0.14	-0.000	-0.13	-0.52	-1.19	-2.12
C: RPA	-0.34	-0.10	-1.47	-4.45	-9.03	-15.23
D: Bubble	-0.10	-0.74	-1.25	-1.64	-1.89	-2.01
E: $2p$-$2h$ scattering	0.88	1.93	3.63	5.98	8.98	12.63
F: Single-particle renorm.	1.68	2.10	3.16	4.87	7.23	10.23
TOTAL	26.67	26.61	26.09	25.13	23.71	21.83

occur.

So when all is said and done, the complete second-order calculations yield results such that the energies of $J=0^-$ vibrations decrease with increasing y, just like the bare ones. Matrix diagonalization, which implicitly contains even higher order diagrams, causes a qualitative change so that the energies of these states ultimately increase with increasing y. This is our main result.

Lipkin model calculations of phase transition have been studied in the past, for example, the work of Shuck and Ethofer [12] as discussed in the book of Ring and Shuck [13]. Without discussing the details of their Hamiltonian we note that their coupling strength χ is chosen to give an RPA stability at $\chi = 1$. A "self-consistent RPA" model gets rid of the instability, the energy reaches a plateau versus χ, whereas the exact solution is such that there is still an instability but at a higher value of the coupling constant $\chi \approx 1.5$.

In our model the behavior is somewhat different. Our large space calculations not only remove the instability but cause the excitation energies to increase with increasing coupling strength.

III. ADDITIONAL REMARKS

Whereas simple $1p$-$1h$ calculations with sufficiently strong tensor interactions can cause unnatural parity excitations to come below the $J=0^+$ state, we find that in superior full shell model calculations this is not the case—ultimately the energies get higher with increasing tensor interaction strength. Strangely, the second-order perturbation theory calculations do not help to resolve the discrepancy between the bare results and matrix diagonalization results. The second-order results are much like the bare ones because of large cancellations between different diagrams. We therefore must conclude that the higher order diagrams implicitly contained in the matrix calculation are responsible for preventing the excitation energies of the unnatural parity states from monotonically decreasing when the strength of the tensor interaction is increased. This is a new result deserving of further study.

There are other subtleties. When all $\Delta N=2$ configurations are included, there is still a crossover of the $J=0^-$ state with the $J=0^+$ state but it occurs at a much larger value of y than in the $1p$-$1h$ case. Only when up to $\Delta N=4$ excitations are included is there a turnaround so that the excitation energy ultimately increases with y.

We have obviously chosen the nucleus ^{4}He for practical reasons—it is easier to perform large space shell model calculations in lighter nuclei. However, with the advent of faster shell model programs it may be possible to perform such calculations in heavier nuclei. The impetus for doing this is certainly present in this work.

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