Limited symmetry found by comparing calculated magnetic dipole spin and orbital strengths in ^{4}He

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Allowing for $2\hbar\omega$ admixtures in ^{4}He we find that the summed magnetic dipole isovector orbital and spin strengths are equal. This indicates a symmetry which is associated with interchanging the labels of the spin with those of the orbit. Where higher admixtures are included, the orbital sum becomes larger than the spin sum, but the sums over the low energy region are still nearly the same.

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In an LS closed shell nucleus, e.g., ^{4}He, ^{16}O, and ^{40}Ca, the magnetic dipole transition from the J=0^+ ground state to the J=1^+ excited states will vanish unless there are ground state correlations such as two-particle–two-hole (2p-2h) admixtures. Previous theoretical studies of magnetic spin dipole excitations [1] show that the correlations induced by the tensor interaction give a large contribution to the energy-weighted sum rule in a closed shell nucleus. To see the full effect of the tensor interaction, one has to allow excitations up to large values of $n\hbar\omega$. A simplifying feature in the above calculations is the observation, at least for the isoscalar transitions, that the double commutator of the isoscalar magnetic dipole spin operator with the tensor interaction is proportional to the same tensor interaction. Another point made in the above work was that a central interaction had to have a spin dependence in order to generate magnetic dipole strength in a closed LS shell nucleus.

Other works on $M1$'s with sum rule techniques include those of Desplanques, and Noguerra [2], Orlandini *et al.* [3], and Lipparini and Stringari [4].

Some experimental and experiment-theory collaborative works for magnetic dipole transitions in ^{16}O and ^{40}Ca have been performed with the motivation of discerning the nature of ground state correlations. These include the inelastic scattering work of Gross *et al.* [5], Richter and Knüpfer [6], Steffen *et al.* [7], and Brown *et al.* [8] and the proton capture work of Snover *et al.* [9] in ^{16}O.

Whereas in our earlier work [1] we considered only spin transitions, in the present work we wish to consider also *orbital* magnetic dipole transitions and to see if there is any interrelation with the spin transitions in a closed shell nucleus. For example, does the tensor interaction also induce *orbital* excitations comparable to the spin excitations ?

We shall calculate the magnetic dipole strengths from the J=0^+, T=0 of ^{4}He to J=1^+, T=1 excited states. We will calculate separately the total $B(M1)$ rate, $B(M1)_{\rm spin}$ and $B(M1)_{\rm orbit}$ where the operators in question, in units of μ_N, are $9.412\vec{s}\,t_z + \vec{l}\,t_z$, $\vec{s}\,t_z$, and $\vec{l}\,t_z$ where t_z=$+\frac{1}{2}$ for a proton and $-\frac{1}{2}$ for a neutron. Note that we define our isovector spin $B(M1)$ so that it has the same coupling strength as the orbital one; i.e., we drop the factor 9.412. This makes it easier to compare spin and orbital strengths.

As mentioned previously, it is necessary to have ground state correlations in ^{4}He in order to get magnetic dipole transitions. We get these by performing shell model matrix diagonalizations using OXBASH [10]. We have used progressively larger shell model spaces for the J=0^+, T=0 ground state and J=1^+, T=1 states: up to $2\hbar\omega$, up to $4\hbar\omega$, and up to $6\hbar\omega$ admixtures.

We use the interaction of Zheng and Zamick [11] which has central, spin-orbit, and tensor parts:

$$V_{\rm sche} = V_c + xV_{\rm s.o.} + yV_t. \tag{1}$$

For x=1 and y=1 this interaction gives a fairly good fit to the matrix elements of the Bonn A interaction. We can turn the spin-orbit (tensor) interaction off by setting x (y) equal to zero. We can thus isolate and study the effects of the spin-orbit and/or tensor interaction on the magnetic dipole excitations.

In Table I we give the total summed strength $B(M1)_{\rm spin}$ and $B(M1)_{\rm orbit}$ to all (nonspurious) J=1^+, T=1 states corresponding to the operators $\vec{s}\,t_z$ and $\vec{l}\,t_z$, respectively [as mentioned before we drop the isovector factor $5.586 - (-3.826)$=9.412]. We do this for progressively increasing model spaces: up to $2\hbar\omega$, up to $4\hbar\omega$, and up to $6\hbar\omega$.

We perform the calculation with the spin-orbit and tensor interactions off and on.

Examining Table I we find one *a priori* unexpected result. When we restrict the ground state correlations to $2\hbar\omega$, we find that the summed *spin* strengths are virtually equal to the summed *orbital* strengths. This is true for all four cases of (x,y), i.e., whether or not there is a spin-orbit interaction present and whether or not there is a tensor interaction present.

TABLE I. Summed spin and orbital magnetic dipole moment strengths in ^{4}He in units of $10^{-3}\mu_N{}^2$.

Interaction		up to $2\hbar\omega$		up to $4\hbar\omega$		up to $6\hbar\omega$	
x	y	Spin	Orbit	Spin	Orbit	Spin	Orbit
0	0	0.8546	0.8546	1.3357	5.1635	1.5897	7.1474
1	0	0.8569	0.8571	1.3417	5.1851	1.6211	7.2296
0	1	3.8245	3.8239	5.2346	10.937	6.0653	14.607
1	1	3.3944	3.3955	4.8288	10.554	5.6052	14.272

This striking result does not extend to larger configurations. When we allow up to $4\hbar\omega$ excitations, the summed orbital $B(M1)$ strengths become substantially larger than the summed spin strengths. For example, for x=1 and y=1, these are, respectively, 10.6 ($10^{-3}\mu_N{}^2$) and 4.8 ($10^{-3}\mu_N{}^2$). Note that when we go to $6\hbar\omega$ excitations, the summed strengths are even larger, and the convergence in terms of $n\hbar\omega$, if it exists, is very slow.

Let us now consider the systematics of the interaction. We note that, relative to the central (but spin-dependent) force case (x=0, y=0), turning on the spin-orbit interaction scarcely changes the summed strength at all. However, when the tensor interaction is turned on (by changing y from 0 to 1), there is a big jump in the summed spin strength and in the summed orbital strength. In the $2\hbar\omega$ case, the change in both cases (since they are equal) is from 0.86 ($10^{-3}\mu_N{}^2$) to 3.82 ($10^{-3}\mu_N{}^2$).

We gain further insight by examining Table II where the strengths to individual states are given. Consider first the $2\hbar\omega$ calculation and the case of a central interaction (x=0, y=0). There are only seven nonspurious $J=1^+$, T=1 states in this model space with the following excitation energies in MeV: 36.7, 44.0, 45.3, 48.8, 49.2, 53.9, and 56.4. The spin transition strength goes to only one state, the third one at 45.3 MeV with a strength $B(M1)_{\rm spin}$=0.8546 ($10^{-3}\mu_N{}^2$). The orbital transition strength also goes to one state, but to a different one than in the case of the spin. The orbital strength all goes to the highest state (No. 7) at 56.5 MeV with a value $B(M1)_{\rm orbit}$=0.8546 ($10^{-3}\mu_N{}^2$). Thus $B(M1)_{\rm orbit}=B(M1)_{\rm spin}$. For other interactions $x\neq0$ or $y\neq0$, we do not get these sharp results although one can make an approximate association between nearly equal spin and orbital transitions. Nevertheless, the *summed values* of $B(M1)_{\rm spin}$ and $B(M1)_{\rm orbit}$ are virtually the same even in the presence of spin-orbit and tensor interactions.

The above behavior more or less tells us what the symmetry we are dealing with is. For a central interaction, given one eigenstate with certain spin and orbital labels, we can get another eigenstate by interchanging the spin labels with the orbital labels. This symmetry is limited

TABLE II. For the case x=0, y=0 (central interaction, LS limit), we give the energies and $B(M1)$'s of "spin excited" and "orbit excited" states, with up to $2\hbar\omega$ admixtures.

	Energy (MeV)	$B(M1)$ (in units of $10^{-3}\mu_N{}^2$)	
		Spin	Orbit
Nonspurious	36.7	0	0
	44.0	0	0
	45.3	0.855	0
	48.8	0	0
	49.2	0	0
	53.9	0	0
	56.5	0	0.855
Spurious	436.7	0	0
	436.7	0	0
	436.7	0	0
	439.3	0	13.07

to the $2\hbar\omega$ space because in that space the configurations are simple—all are of the form $0s^20p^2$.

We consider the case x=y=0. We are in the LS limit. Since the $0s^4$ closed shell has L=0, S=0, only $2\hbar\omega$ excitations with the same quantum numbers will admix into the ground state. Let us consider two particles excited from the $0s$ shell to the $0p$ shell. We can label the 2p-2h states by $[L_\pi L_\nu]^{L=0}[S_\pi S_\nu]^{S=0}$. There are several cases to be considered:

(1) Two protons are excited. The configurations are $(p_\pi^2)^{L_\pi S_\pi}(s_\nu^2)^{L_\nu S_\nu}$. Since L_ν=0 and S_ν=0 and L and S are zero, we must have L_π=0 and S_π=0. So all in all we get the state $|a\rangle = (p^2)^{L_\pi=0,S_\pi=0}(s^2)^{L_\nu=0,S_\nu=0}$.

(2) Two neutrons are excited. By analogy, the configuration is $|b\rangle = (s^2)^{L_\pi=0,S_\pi=0}(p^2)^{L_\nu=0,S_\nu=0}$.

(3) A neutron and a proton are excited from the s shell to the p shell. The configuration is $[(sp)^{L_\pi S_\pi}(sp)^{L_\nu S_\nu}]^{L=0,S=0}$. There are two possibilities:

$|c\rangle = [L_\pi = 1, L_\nu = 1]^{L=0}[S_\pi = 0, S_\nu = 0]^{S=0}$

and

$|d\rangle = [L_\pi = 1, L_\nu = 1]^{L=0}[S_\pi = 1, S_\nu = 1]^{S=0}$.

We can form an isovector orbital excitation by applying the operator $\vec{L_\pi} - \vec{L_\nu}$ to the $J=0^+$ ground state; likewise we can form an isovector spin excitation by applying the operator $\vec{S_\pi} - \vec{S_\nu}$ to the $J=0^+$ ground state. When acting on the configurations $|a\rangle$ or $|b\rangle$, the orbital operator $\vec{L_\pi} - \vec{L_\nu}$ gives zero; likewise the spin operator $\vec{S_\pi} - \vec{S_\nu}$. That is,

$(\vec{L_\pi} - \vec{L_\nu})|L_\pi = 0, L_\nu = 0\rangle = 0$.

Let us skip to the state $|d\rangle$. Note that the orbital and spin quantum numbers are the same: $L_\pi = S_\pi = 1$ and $L_\nu = S_\nu = 1$. This is enough to prove that, if this were the only state present, we would have the result $B(M1)_{\rm spin}=B(M1)_{\rm orbit}$.

In more detail,

$$(\vec{L_\pi}-\vec{L_\nu})|d\rangle = N[L_\pi = 1, L_\nu = 1]^{L=1}[S_\pi = 1, S_\nu = 1]^{S=0}$$

and

$$(\vec{S_\pi}-\vec{S_\nu})|d\rangle = N[L_\pi = 1, L_\nu = 1]^{L=0}[S_\pi = 1, S_\nu = 1]^{S=1}.$$

There is no reason why these states should be at the same energy and indeed they are not, but the equality of the spin and orbital strengths, *provided* the state $|c\rangle$ were not present, is obvious. However, the presence of the state $|c\rangle$ apparently presents a problem. The isovector spin operator $\vec{S_\pi} - \vec{S_\nu}$ will annihilate this state, whereas the isovector orbital operator $(\vec{L}_\pi - \vec{L}_\nu)$ creates the state $[L_\pi = 1, L_\nu = 1]^{L=1}[S_\pi = 0, S_\nu = 0]^{S=0}$. There should therefore be more orbital strength than spin strength. What saves the day is that this transition is spurious. In the OXBASH program [10] the spurious states are put very high in energy by adding a large constant to the single-particle energies for center of mass motion. We added 100 MeV for each nucleon, thus putting the spurious states in the vicinity of 400 MeV excitation energy. In Table III we show the $2\hbar\omega$ x=0, y=0 calculation in which all the 1^+, T=1 states are shown, both nonspurious and spurious, with the values of $B(M1)_{\rm spin}$ and $B(M1)_{\rm orbit}$.

We see from Table II that our results are consistent

TABLE III. Summed *low energy* spin and orbital magnetic dipole moment strength in ^{4}He with up to $6\hbar\omega$ admixtures, in units of $10^{-3}\mu_N{}^2$.

Interaction		$B(M1)$		Deviation
x	y	Spin	Orbit	(%)
0	0	0.5592	0.6018	7.3
1	0	0.5888	0.6291	6.6
0	1	1.9546	1.8046	-8.0
1	1	1.8199	1.6155	-10.8

with the above discussion. The nonspurious orbital and spin strengths are the same, 0.855 ($10^{-3}\mu_N{}^2$), but the respective states are at different energies 45.3 MeV for the spin state and 56.5 MeV for the orbital state. These correspond to excitations from the configuration $|d\rangle$. There are no more spin excitations but there is an orbital excitation which is quite strong 13.07 ($10^{-3}\mu_N{}^2$) to a spurious state artificially placed at 439.3 MeV. This is consistent with our previous remarks that the configuration $|c\rangle$ allows for an orbital but not a spin excitation.

We can further extend our results to include the tensor interaction. This interaction allows $[L = 2,\ S = 2]^{J=0}$ 2p-2h admixtures into the ground state. For $2\hbar\omega$ excitations the only way to achieve such a state is to excite a proton and neutron from the $0s$ state to the $0p$ state. Thus we have a state

$$|e\rangle \;=\; \{[L_\pi = 1, L_\nu = 1]^{L=2}\;\; [S_\pi = 1, S_\nu = 1]^{S=2}\}^{J=0}.$$

The state $|e\rangle$ is also invariant under the interchange of the spin and orbit labels, and hence preserves the equality of the summed spin and summed orbit strength at the $2\hbar\omega$ level.

Concerning the two-body spin-orbit interaction it should be noted that all matrix elements of the form $\langle 0s\ 0sV_{\rm s.o.}0p\ 0p\rangle$ vanish. The reason is that the spin-orbit interaction does not act in relative $\ell = 0$ states and furthermore does not change the relative orbital angular momentum. However, the $(0s\ 0s)$ state can only have relative orbital angular momentum equal to zero. Thus the spin-orbit interaction does not induce ground state correlations in first-order perturbation theory.

We thus have explained the equality of the summed spin and summed orbital strength in ^{4}He for the entire interaction, central, spin orbit, and tensor. It should be noted that these results are specific to ^{4}He. For larger closed shell nuclei, e.g., ^{16}O, the orbital $B(M1)$ is substantially larger than the spin $B(M1)$ even at the $2\hbar\omega$ level [12].

It is trivial to show that for an *isoscalar* magnetic dipole transition from the $J=0^+$, $T=0$ ground state to a given $J=1^+$, $T=0$ excited state, the matrix element of the spin operator $\vec{S}= \vec{S_\pi} + \vec{S_\nu}$ is equal and opposite to that of the orbital operator $\vec{L}=\vec{L_\pi} + \vec{L_\nu}$. This is because the total angular momentum operator $\vec{J}= \vec{L} + \vec{S}$, when acting on the $J=0^+$ ground state, yields zero. More generally, since any nuclear state is an eigenfunction of $\vec{J}$, this operator cannot induce transitions out of the multiplet.

However, the above argument certainly does not hold for the *isovector* case for which the relevant operators are $\vec{L}=\vec{L_\pi} - \vec{L_\nu}$ and $\vec{S}=\vec{S_\pi} - \vec{S_\nu}$. Furthermore the equality that we obtain between spin and orbit in the isovector case (at the $2\hbar\omega$ level) is for different states, whereas in the isoscalar case it is for the same 1^+ state. It should be further noted that one does not get any isoscalar 1^+ transitions in an LS closed shell like ^{4}He in the case of a central spin-dependent interaction [1]. However, if a tensor interaction is present, we do get finite isoscalar transitions.

In the case x=0, y=0 when we allow up to $4\hbar\omega$ or $6\hbar\omega$ excitations, we no longer have the summed orbital strengths equal. However, some features of the $2\hbar\omega$ case are preserved in the $6\hbar\omega$ calculation. Most transition rates vanish. In the low energy sector (defined more precisely in the next section) only one spin state and only one orbital state get excited, just as in the $2\hbar\omega$ case. The spin state is at 34.4 MeV with $B(M1)$=0.55 ($10^{-3}\mu_N{}^2$) and the orbital state is at 43.5 MeV with $B(M1)$=0.60 ($10^{-3}\mu_N{}^2$). Although the two $B(M1)$'s are not equal they differ by less than 11%, as shown in Table III.

But other states in the $4\hbar\omega$ and $6\hbar\omega$ region also get excited. Indeed, the single largest calculated orbital $B(M1)$ is to a state at 61.4 MeV with a rate $B(M1)_{\rm orbit}$=2.79 ($10^{-3}\mu_N{}^2$). This is more than 4 times larger than the $B(M1)$ in the low energy sector. We show in Table IV for x=0, y=0 all states with $B(M1) \geq 10^{-4}\mu_N{}^2$.

In Fig. 1 we present the cumulative sum of the strength distribution for the spin $B(M1)$ and orbit $B(M1)$ when up to $6\hbar\omega$ excitations are allowed. We consider the case x=1, y=1 (realistic). In Fig. 1 we give the spin distribution. We see some strength starting at about 35 MeV with a plateau from about 41 MeV until about 65 MeV. This is the low-lying strength which one might obtain in a $2\hbar\omega$ restricted space. Then there is a sharp rise corresponding to $4\hbar\omega$ and higher admixtures. The curve ultimately flattens out because we run out of states. The corresponding orbital strength curve also has a plateau from about 46 to 61 MeV. This also can be identified as the low energy part. As mentioned in the previous sec-

TABLE IV. For the case x=0, y=0 (central interaction), we give the energies and $B(M1)$'s of "spin excited" and "orbit excited" states, with strength $\geq 10^{-4}\mu_N{}^2$.

Energy (MeV)	$B(M1)$ (in units of $10^{-3}\mu_N{}^2$)	
	Spin	Orbit
34.369	0.5524	0
43.509	0	0.6015
61.414	0	2.7900
65.253	0.2048	0
66.616	0.2276	0
67.762	0.2265	0
71.837	0.2334	0
71.868	0	0.7696
73.723	0	0.9369
83.071	0	0.7107
96.715	0	0.7761
101.20	0	0.2435
107.35	0	0.1216

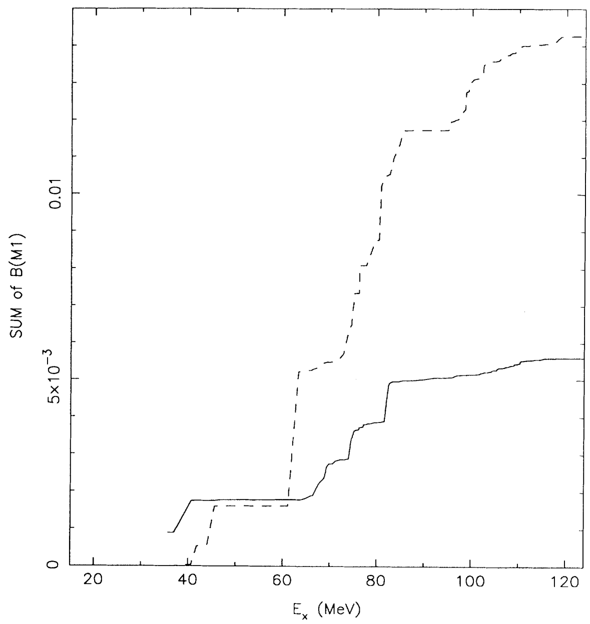

FIG. 1. Sum of $B(M1)_{\rm spin}$ (solid line) and of $B(M1)_{\rm orbit}$ (dashed line) with x=1, y=1 and up to $6\hbar\omega$ admixtures in units of μ_N^2.

tion, the value of $B(M1)_{\rm orbit}$ is very close to the value of $B(M1)_{\rm spin}$ for this plateau. This shows that the symmetry relation for $2\hbar\omega$ is not broken very much in the low energy sector when we extend the calculation to $6\hbar\omega$. The low energy strength is obviously easier to find experimentally than the higher-lying strength.

As we increase the excitation energy in the orbital case, we see a sharp rise at 63 MeV to another plateau. The second orbital plateau is much higher than the second spin plateau. But then, unlike the spin case, there are more sharp rises until we reach a saturation value of 14.3 $(10^{-3}\mu_N{}^2)$. It would certainly be of interest to look for such a strong orbital strength distribution at a very high energy $\sim 3-4\ \hbar\omega$. If we had extended our calculation to $8\hbar\omega$ there might be even further rises.

In closing we point out that we have uncovered a rather unusual symmetry when $2\hbar\omega$ ground state correlations are included in the wave function of ^{4}He. It will be difficult to test this result experimentally because of the large isovector spin coupling for the electromagnetic probe which will drown out the orbital contribution. Possibly, a multiprobe analysis would help. Nevertheless, we feel that the results are of considerable *theoretical* interest. Among the unique features of our findings are the following.

(a) We obtain our symmetry with an "ugly" Hamiltonian—the realistic nucleon-nucleon interaction. This is in contrast to the more prevalent practice of constructing simplified Hamiltonians to display approximate symmetries.

(b) We obtain a simpler result (equality of spin and orbit) for the energy-independent sum than for the energy-weighted sum. In most other cases, the energy-weighted strength gives the simplest results.

(c) We even go beyond $2\hbar\omega$ and show that although the symmetry no longer holds, there is a wide plateau where the cumulative spin and orbit sums are nearly equal.

We obtain this symmetry not in spite of but because of the fact that we remove spurious states. Interest in spurious states is widespread, not only for nuclear structure but also in atomic physics and for the structure of baryons where the degrees of freedom are quarks and gluons. Thus the symmetry we have found here in the nuclear context should be of interest in these other fields. And indeed even in the present context, it may suggest to others that it is worth probing more deeply for unexpected symmetries.

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