Comparison of different Skyrme forces: Fusion barriers and fusion cross sections

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Fusion barriers and fusion cross sections are calculated using the Skyrme energy-density formalism. To study the role of different parametrizations of the Skyrme interaction, we use two typical forces, i.e., the original Skyrme force S and the widely used Skyrme force SIII. Our calculations show that, in the reactions considered here, the Skyrme force S gives higher fusion cross sections compared to that of the Skyrme force SIII. The main part of this difference can be associated with the presence of the spin-density contribution in the Skyrme force SIII.

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Heavy-ion collisions from a few MeV/nucleon to a few GeV/nucleon provide a variety of phenomena which occur at different bombarding energies. Due to the lack of free phase space at low energies, almost all nucleon-nucleon collisions are absent and thus the collision of two heavy nuclei at low energies leads to phenomena like complete fusion or deep inelastic collisions, depending on the impact parameter. As one goes to higher energies, frequent nucleon-nucleon collisions govern the fate of heavy-ion dynamics and, therefore, processes like incomplete fusion, multifragmentation, particle-production, etc. can be observed. In the low and intermediate energy regions, the role of mean field (which individual nucleons feel) is very important. A large amount of work has been done using the phenomenological forces which account for the mean field potential of nucleons. In a phenomenological force, one fits the parameters to known ground state properties of nuclei and thus can generate several sets of parameters which can reproduce the ground state properties of nuclei with similar accuracy. Among various phenomenological forces, the Skyrme interaction is widely used to study the heavy-ion collisions from low to intermediate energies.

The Skyrme force has been fitted to generate the standard hard and soft parametrizations depending upon the incompressibility [1]. These different parametrizations of the Skyrme interaction (the hard and soft) are used to extract the so-called equation of state (EOS) [1]. In past many years, extensive attempts have been made using these soft and hard parametrizations of the Skyrme force to understand the heavy-ion dynamics at intermediate energies [1–13]. In contrast, very few attempts have been made using different Skyrme forces to study the low energy phenomena like fusion of two nuclei, their excitation functions, etc. [14–20]. Therefore, it is very interesting and also important to compare the predictions of different Skyrme forces for heavy-ion phenomena at low energies. In this paper, we attempt to do so for the fusion barriers and fusion cross sections. In the following, we first summarize the important points of the model used and then present the results of our calculations. The details of the model can be found in [14].

In the Skyrme energy density formalism (SEDF) [21], the interaction potential $V_N(R)$ is defined as the difference between the energy expectation value E of the colliding system at a finite separation distance R and at infinity,

$$V_N(R) = E(R) - E(\infty) \ . \tag{1}$$

The two nuclei overlap at a distance R and are completely separated at infinity. The energy expectation value E for the energy-density functional $H(\mathbf{r})$ of Vautherin and Brink [21] is given by

$$E = \int H(\mathbf{r})d\mathbf{r} \ , \tag{2}$$

where the Hamiltonian $H(\mathbf{r})$ for an even-even spherical nucleus ($N = Z$) is given by

$$H(\rho,\tau,\mathbf{J}) = \frac{\hbar^2}{2m}\tau + \frac{3}{8}t_0\rho^2 + \frac{1}{16}t_3\rho^3 + \frac{1}{16}(3t_1+5t_2)\rho\tau + \frac{1}{64}(9t_1-5t_2)(\nabla\rho)^2 - \frac{3}{4}W_0\rho\nabla\cdot\mathbf{J} \ . \tag{3}$$

Using Eqs. (1)–(3), the interaction potential $V_N(R)$ reads as

$$V_N(R) = \int\{H(\rho,\tau,\mathbf{J}) - H_1(\rho_1,\tau_1,\mathbf{J}_1) - H_2(\rho_2,\tau_2,\mathbf{J}_2)\}d\mathbf{r} \ . \tag{4}$$

In Eq. (3), ρ_i, τ_i, and $\mathbf{J}_i$ are the nucleon density, kinetic energy density, and spin density of individual nuclei, respectively. Under sudden approximation (which is used in the present study), the nucleon density ρ, kinetic energy density τ, and spin density $\mathbf{J}$ for the composite system read, respectively, as $\rho = \rho_1 + \rho_2$, $\tau = \tau_1 + \tau_2$, and $\mathbf{J} = \mathbf{J}_1 + \mathbf{J}_2$. The five parameters [$t_0, t_1, t_2, t_3$, and W_0] appearing in Eq. (3) have been fitted by various authors in a self-consistent manner to reproduce the correct single particle properties of nuclei. First, the set of

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parametrization was given by Skyrme himself [22] (labeled as Skyrme force S). Later, two forces SI and SII were fitted by Vautherin and Brink [21]. Beiner *et al.* [23] succeeded in reproducing the radii and binding energies of the whole Periodic Table using four new Skyrme forces, namely, SIII, SIV, SV, and SVI. Note that of all these Skyrme forces, force SIII is found to give remarkable agreement with the experimental nuclear radii, binding energy, etc. [23], and has been used quite extensively to calculate the heavy-ion potentials and cross sections [14–20]. In the following, we compare the fusion barriers and fusion cross sections obtained by using the original Skyrme force S and the widely used force SIII, where parameters are listed in Table I. The choice of forces S and SIII is based on the idea that the Skyrme force S has no spin density dependence ($W_0 = 0$ MeV fm^5), whereas SIII does have it ($W_0 = 120.0$ MeV fm^5). Thus, the comparison of fusion barriers and fusion cross sections using forces S and SIII, should give us the possibility of examining the role of the spin-density term in Skyrme forces. It may be noted here that these forces represent the low density region and fusion is a low density phenomenon.

In Eq. (4), the nucleon density ρ_i is calculated by using the two-parameter Fermi density distribution; the kinetic energy density τ_i and spin density $\mathbf{J}_i$ are given, respectively, by [14]

$$\tau_i = \frac{3}{5}\left(\frac{3}{2}\pi^2\right)^{2/3}\rho_i^{5/3} + \lambda(\nabla\rho_i)^2/\rho_i \tag{5}$$

and

$$\mathbf{J}_i(\mathbf{r}) = \frac{\mathbf{r}}{4\pi r^4}\sum_\alpha (2j_\alpha+1)[j_\alpha(j_\alpha+1) - l_\alpha(l_\alpha+1) - \tfrac{3}{4}]R_\alpha^2(r) \ . \tag{6}$$

The value of constant λ in Eq. (5) varies between 1/36 and 9/36 and is a point of controversy [24]. Therefore, in present calculations, we take $\lambda = 0$. Note that the summation α in Eq. (6) runs over all occupied levels and $R(r)$ are the harmonic oscillator radial wave functions. From Eq. (6), one recognizes that this equation is valid only for closed shell nuclei and must be generalized for fusion of nuclei with valence particles (or holes) outside the closed core. Following Gupta and collaborators [14], the contribution of spin density $\mathbf{J}_i$ for even-even nuclei with valence particles (or holes) outside a closed core is calculated by dividing the contribution of $\mathbf{J}_i(\mathbf{r})$ in two parts: One due to the core consisting of closed shells and another due to the valence n_v particles (or holes):

$$\mathbf{J}_i(\mathbf{r}) = \mathbf{J}_i^c(\mathbf{r}) \pm \mathbf{J}_i^{n_v}(\mathbf{r}) \ . \tag{7}$$

The + sign is for the particles whereas − stands for holes. The first term in Eq. (7) is apparently the same as Eq. (6), whereas the second term, representing the contribution from valence particles (or holes), is given as

$$\mathbf{J}_i^{n_v}(\mathbf{r}) = \frac{n_v\mathbf{r}}{4\pi r^4}[j(j+1) - l(l+1) - \tfrac{3}{4}]R_l^2(r) \ . \tag{8}$$

For more details, we refer the reader to [14].

Furthermore, Eq. (4) is solved by separating the terms containing the spin-density dependent part and the spin-density independent part, as

$$V_N(R) = V_P(R) + V_J(R) \ , \tag{9}$$

with

$$V_P(R) = \int [H(\rho,\tau) - \{H_1(\rho_1,\tau_1) + H_2(\rho_2,\tau_2)\}]d\mathbf{r} \tag{10}$$

and

$$V_J(R) = \int [H(\rho,\mathbf{J}) - \{H_1(\rho_1,\mathbf{J}_1) + H_2(\rho_2,\mathbf{J}_2)\}]d\mathbf{r} \ . \tag{11}$$

Here, $V_P(R)$ is evaluated in sudden approximation, using the proximity theorem treatment of Chattopadhyay and Gupta [15]. By adding the Coulomb interaction to the nuclear interaction V_N, we get the total interaction potential

$$V_T(R) = V_N(R) + Z_1Z_2e^2/R \ . \tag{12}$$

The fusion barrier is defined as the height of the interaction potential $V_T(R)$ where the slope

$$\left[\frac{dV_T(R)}{dR}\right]_{R=R_B} = 0, \quad \left[\frac{d^2V_T(R)}{dR^2}\right]_{R=R_B} < 0 \ . \tag{13}$$

In other words, the fusion barrier is defined by its position at the distance $R = R_B$, and the height $V(R = R_B) = V_B$. From the knowledge of the height and the position of the barrier, the fusion cross section can be calculated by using the so-called sharp-cutoff model,

$$\sigma_{\text{fus}} = \pi R_B^2\{1 - V_B/E_{\text{c.m.}}\} \ . \tag{14}$$

Table II shows our calculated barrier heights and positions for a number of colliding nuclei using the forces S and SIII and also for a special version of SIII with the spin-density contribution neglected (i.e., $V_J(R) = 0$ MeV). We note that the collisions between two spin saturated nuclei, like ^{16}O and ^{40}Ca, result in fusion barrier heights and positions that are nearly the same for both S and SIII forces. However, both the barrier heights and

TABLE I. The Skyrme force parameters used in present calculations.

Skyrme interaction	t_0 (MeV fm^3)	t_1 (MeV fm^5)	t_2 (MeV fm^5)	t_3 (MeV fm^6)	W_0 (MeV fm^5)
S	−1072.00	461.00	−40.00	8027.00	
SIII	−1128.75	395.00	−95.00	14 000.00	120.00

TABLE II. Calculated fusion barrier heights and positions for spin saturated and spin unsaturated colliding nuclei using Skyrme forces S, SIII, and SIII without spin density contribution ($V_J = 0$ Mev). The systems are listed with respect to their increasing Z_1Z_2 and $\lambda = 0$.

System	Skyrme force S		Skyrme force SIII		Skyrme force SIII ($V_J = 0$ MeV)	
	V_B (MeV)	R_B (fm)	V_B (MeV)	R_B (fm)	V_B (MeV)	R_B (fm)
^{16}O-^{16}O	10.95	7.90	10.97	7.90	10.97	7.90
^{16}O-^{24}Mg	15.02	8.53	15.07	8.43	15.05	8.53
^{16}O-^{40}Ca	23.94	8.94	23.98	8.94	23.98	8.94
^{26}Mg-^{32}S	27.12	9.30	27.29	9.20	27.13	9.30
^{28}Si-^{28}Si	27.88	9.20	28.12	9.10	27.89	9.20
^{20}Ne-^{40}Ca	28.58	9.31	28.64	9.21	28.59	9.21
^{40}Ca-^{40}Ca	54.21	9.78	54.26	9.68	54.26	9.68
^{32}S-^{58}Ni	59.23	10.04	59.70	9.84	59.29	9.94
^{58}Ni-^{58}Ni	98.00	10.56	99.03	10.26	98.15	10.46

positions begin to differ as we go from spin saturated nuclei to spin unsaturated nuclei. This happens because the force SIII has a strong spin-density dependence (see Table I), which is found to increase the barrier heights and decrease the barrier positions [16]. If one neglects this contribution of the spin-density dependent part in SIII, then once again the Skyrme forces S and SIII give nearly the same barrier heights and positions. This clearly means that the spin-density contribution would play an important role for fusion cross section calculations.

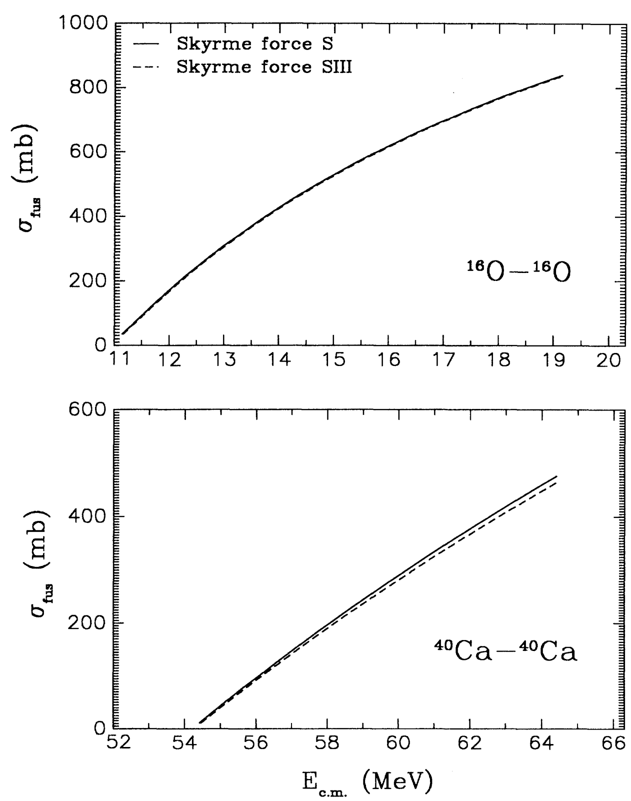

FIG. 1. Calculated excitation functions for reactions of spin-saturated nuclei ^{16}O-^{16}O, ^{40}Ca-^{40}Ca, using Skyrme forces S and SIII.

To study the role of different Skyrme forces for fusion cross sections, we display in Fig. 1 the excitation functions $\sigma_{\text{fus}}(E_{\text{c.m.}})$ for collisions of only the spin-saturated nuclei, ^{16}O-^{16}O and ^{40}Ca-^{40}Ca. It is impressive to note that for the spin saturated nuclei, both Skyrme forces S and SIII give nearly the same excitation functions. This comparison is striking particularly because Skyrme force S gives very small radii for heavy nuclei [22], but is able to reproduce the excitation functions as good as in the

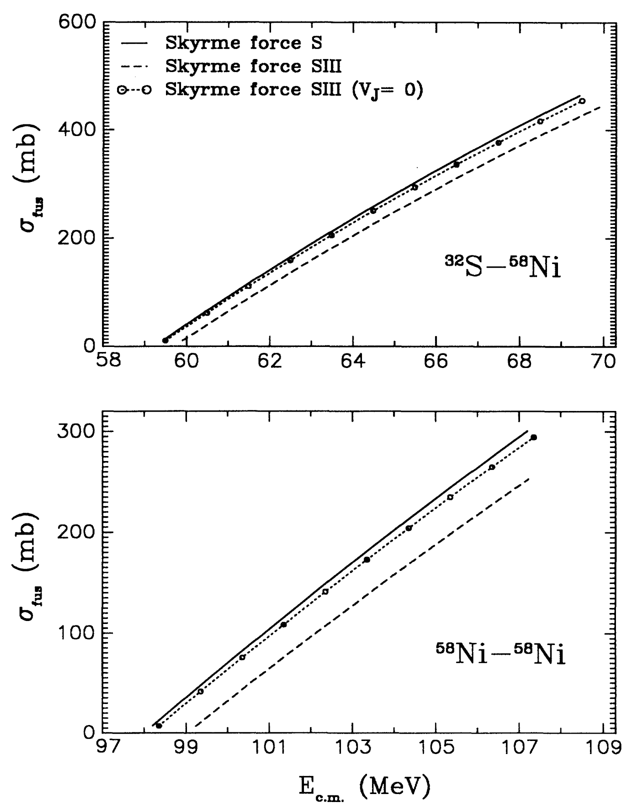

FIG. 2. Calculated excitation functions for reactions of spin-unsaturated nuclei ^{32}S-^{58}Ni, ^{58}Ni-^{58}Ni, using Skyrme forces S, SIII and a special version of SIII where the spin-density contribution V_J is neglected.

case of Skyrme force SIII.

In Fig. 2, we study the variation of the excitation functions for reactions which are the cases of either the weakly or the highly spin-unsaturated nuclei. Comparing this figure with Fig. 1, one sees clearly that as we go from spin-saturated nuclei to spin-unsaturated nuclei, the differences in excitation functions for force S and SIII start appearing and become stronger and stronger. The original Skyrme force S (without spin density term) gives higher fusion cross sections. As we neglect the contribution of spin density potential in SIII also, we see that the forces S and SIII give nearly the same excitation functions. This clearly demonstrates that for the fusion barriers and hence the fusion cross sections, the spin density plays an important role. This result can be understood when one looks at the behavior of energy per nucleon as a function of nucleon density (equation of state). One finds that for the spin-saturated nuclear matter, different parametrizations (the hard and soft) of the equation of state result in nearly the same value for energy per nucleon in the low density region [1]. In the present case, the low density behavior of the potential is dominated by the second term (i.e., $3/8t_0\rho^2$) of the Hamiltonian [3]. The value of parameter t_0 is nearly the same for the two Skyrme forces S and SIII and hence it results in similar fusion barriers for spin-saturated nuclei.

Concluding, we have calculated the fusion barriers and the excitation functions, using the original Skyrme force S and the widely used force SIII. For spin saturated nuclei, both Skyrme forces S and SIII give nearly the same fusion barriers and fusion cross sections. However, the deviations start appearing as one goes to spin-unsaturated nuclei. Neglecting the spin-density contribution to the interaction potential based on force SIII, we obtain nearly the same fusion barrier heights, positions, and the excitation functions as are given by the original Skyrme force S. In other words, the above results show that the low density phenomena for spin-saturated systems are not affected much by different parametrizations of the Skyrme force.

[1] J. Aichelin, Phys. Rep. **202**, 233 (1991) and references therein.
[2] S. A. Bass, C. Hartnack, H. Stöcker, and W. Greiner, Phys. Rev. Lett. **71**, 1144 (1993).
[3] G. Batko, J. Randrup, and T. Vetter, Nucl. Phys. **A536**, 786 (1992).
[4] G. F. Bertsch and S. Das Gupta, Phys. Rep. **160**, 189 (1988).
[5] W. Cassing, V. Metag, U. Mosel, and K. Niita, Phys. Rep. **188**, 363 (1990).
[6] C. Hartnack, GSI Report No. GSI-93-05, GSI Darmstadt, Germany, 1993.
[7] J. Jänicke and J. Aichelin, Nucl. Phys. **A547**, 542 (1992).
[8] D. T. Khoa, N. Ohtsuka, M. A. Matin, A. Faessler, S. W. Huang, E. Lehmann, and R. K. Puri, Nucl. Phys. **A548**, 102 (1992).
[9] E. Lehmann, R. K. Puri, A. Faessler, T. Maruyama, G. Q. Li, N. Ohtsuka, S. W. Huang, D. T. Khoa, and M. A. Matin, Prog. Part. Nucl. Phys. **30**, 219 (1993).
[10] E. Lehmann, R. K. Puri, A. Faessler, G. Batko, and S. W. Huang, Nucl. Phys. A (to be published).
[11] J. J. Molitoris, H. Stöcker, and B. L. Winer, Phys. Rev. C **36**, 220 (1987).
[12] R. K. Puri, N. Ohtsuka, E. Lehmann, A. Faessler, M. A. Matin, D. T. Khoa, G. Batko, and S. W. Huang, Nucl. Phys. **A575**, 733 (1994).
[13] R. K. Puri, E. Lehmann, A. Faessler, and S. W. Huang, Z. Phys. A (to be published).
[14] R. K. Puri, P. Chattopadhyay, and R. K. Gupta, Phys. Rev. C **43**, 315 (1991).
[15] P. Chattopadhyay and R. K. Gupta, Phys. Rev. C **30**, 1191 (1984).
[16] R. K. Puri and R. K. Gupta, Phys. Rev. C **45**, 1837 (1992).
[17] R. K. Puri and R. K. Gupta, Int. J. Mod. Phys. E **1**, 269 (1992).
[18] H. Ngô and C. Ngô, Nucl. Phys. **A348**, 140 (1980).
[19] Fl. Stancu and D. M. Brink, Nucl. Phys. **A270**, 236 (1976); D. M. Brink and Fl. Stancu, *ibid.* **A299**, 321 (1978).
[20] B. Behera, K. C. Panda, and R. K. Sathpathy, Phys. Rev. C **20**, 683 (1979).
[21] D. Vautherin and D. M. Brink, Phys. Rev. C **5**, 626 (1972).
[22] T. H. R. Skyrme, Nucl. Phys. **9**, 615 (1959).
[23] M. Beiner, H. Flocard, N. Van Giai, and P. Quentin, Nucl. Phys. **A238**, 29 (1975).
[24] H. Gräf, Nucl. Phys. **A343**, 91 (1980), and earlier reference therein.