Induced parity nonconserving interaction and enhancement of two-nucleon parity nonconserving forces

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The two-nucleon parity nonconserving (PNC) interaction induced by the single-particle PNC weak potential and the two-nucleon residual strong interaction is considered. An approximate analytical formula for this induced PNC interaction (IPNCI) between proton and neutron is derived $[Q(\mathbf{r}\sigma_p \times \sigma_n)\delta(\mathbf{r}_p - \mathbf{r}_n)]$, and the interaction constant is estimated. As a result of coherent contributions from the nucleons to the PNC potential, IPNCI is an order of magnitude stronger ($\sim A^{1/3}$) than the residual weak two-nucleon interaction and has a different coordinate and isotopic structure (e.g., the strongest part of IPNCI does not contribute to the PNC mean field). IPNCI plays an important role in the formation of PNC effects, e.g., in neutron-nucleus reactions. In that case, it is a technical way to take into account the contribution of the distant (small) components of a compound state which dominates the result. The absence of such enhancement ($\sim A^{1/3}$) in the case of T- and P-odd interaction completes the picture.

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I. INTRODUCTION

The parity nonconserving (PNC) nucleon interaction in nuclei and PNC effects in neutron-nucleus reactions are the subject of current interest for both experimentalists and theorists [1–11]. The values of the PNC effects depend on the weak interaction matrix elements between compound states. Usually two sources of the PNC effects are discussed: a single-particle weak potential w which describes the interaction of a nucleon with a weak mean field of the nucleus and a residual two-particle weak interaction W. In principle, the matrix elements of w and W should be calculated with respect to the eigenstates of the strong interaction Hamiltonian. However, in practice some truncated basis set of states is used to describe physical states at excitation energies less than the gap between single-particle shells. For example, in the description of nuclear compound states and the P-odd admixtures in them [2,10], it is natural to include into the basis set only "principal" components, which have energies close to the energy of the compound state and dominate the normalization sum.

The number of such components is already about 10^6 in a compound state. However, it is still not enough since these components consist of the valence (incomplete) shell orbitals only (see, e.g., Ref. [12]) and do not contain opposite parity orbitals with the same angular momentum (these orbitals belong to different shells). Thus, the matrix element of the single-particle weak potential w between compound states is zero in the "principal component" approximation, since it can mix these opposite parity orbitals only [13,14]. To avoid this problem one should consider an admixture of the distant small components, which contain the necessary opposite parity orbitals from other shells. Any transfer of a particle from the valence shell to another one gives rise to an excitation energy $E_{sp} \sim 5, \ldots, 8$ MeV, which is much more than a typical matrix element of the residual strong interaction V_S. Therefore, one can easily admix a small component to compound states using perturbation theory in V_S:

$$|c\rangle = |c_0\rangle + \sum_\alpha \frac{\langle\alpha|V_S|c_0\rangle}{E_c - E_\alpha}|\alpha\rangle, \tag{1}$$

where $|c_0\rangle$ is the "principal part" of the compound state. Now we can calculate the matrix element of w between the opposite parity compound states $|s\rangle$ and $|p\rangle$:

$$\langle s|w|p\rangle = \sum_\alpha \frac{\langle s_0|w|\alpha\rangle\langle\alpha|V_S|p_0\rangle + \langle s_0|V_S|\alpha\rangle\langle\alpha|w|p_0\rangle}{E_0 - E_\alpha}. \tag{2}$$

The single-particle weak potential w can be written down in the form

$$\hat{w}(1) = \frac{Gg^W}{2\sqrt{2}m}\{(\sigma\mathbf{p})\rho + \rho(\sigma\mathbf{p})\} \simeq \frac{\xi}{m}\sigma\mathbf{p}, \tag{3}$$

$$\xi = \frac{Gg^W}{2\sqrt{2}}\rho_0, \qquad \rho_0 = \frac{2p_F^3}{3\pi^2},$$

where $G = 10^{-5}$ m^{-2} is the Fermi constant, m is the nucleon mass, $\mathbf{p}$ and σ are the nucleon momentum and doubled spin, ρ is the nuclear density, and $\rho \simeq \rho_0 = \text{const}$ inside the nucleus. We can use the relations

$$p = im[H, r], \qquad H|\alpha\rangle = E_\alpha|\alpha\rangle \tag{4}$$

to calculate sum over α in Eq. (2). Here H is a Hamiltonian of the system (here we neglect the spin-dependent part of H). Using the closure relation $\sum_\alpha |\alpha\rangle\langle\alpha| = 1$, we obtain

$$\langle s|w|p\rangle = i\langle s_0|\left[\sum_k \xi_k \sigma_k \mathbf{r}_k, V_S\right]|p_0\rangle, \quad (5)$$

and the sum is taken over nucleons [actually only the nucleons near the Fermi surface contribute to this sum (see Ref. [10] and below)].

If we introduce the effective interaction [the induced parity nonconserving interaction (IPNCI)]

$$V^{\rm IPNCI} = i\left[\sum_k \xi_k \sigma_k \mathbf{r}_k, V_S\right], \quad (6)$$

we need not refer to the small components, and we calculate the matrix elements of the IPNCI between the "principal" components of the compound states only [see Eq. (5)].

To derive formula (5) for IPNCI we used some approximations (constant nuclear density and spin-independent Hamiltonian H). When doing numerical calculations these approximations are not necessary. In our work [10] we have used a more accurate perturbation theory expression for the matrix elements of the IPNCI between the nuclear orbitals a, b, c, d:

$$V^{\rm IPNCI}_{abcd} = \sum_i \left[\frac{w_{ai}V_{S,ibcd}}{\epsilon_a - \epsilon_i} - \frac{V_{S,aicd}w_{ib}}{\epsilon_i - \epsilon_b} + \frac{w_{ci}V_{S,abid}}{\epsilon_c - \epsilon_i} - \frac{V_{S,abci}w_{id}}{\epsilon_i - \epsilon_d}\right]. \quad (7)$$

Using the approximation $\rho = \rho_0$ in Eqs. (3),(4), Eq. (7) can be reduced to Eq. (6) for the operator of the IPNCI.

The approximate analytical expression (6) is convenient to study coordinate, spin, and isospin structure and also the strength of the IPNCI. It will be shown that the IPNCI is an order of magnitude stronger than the residual two-particle weak interaction W. This amplification ($\sim A^{1/3}$) can be explained by a coherent contribution of all the nucleons to the PNC potential which induces the IPNCI. As a result, the IPNCI gives the dominating contribution to the matrix elements of the weak interaction between the compound states and determines the value of the PNC effects in nucleus-neutron reactions.

The natural question arises: we obtained the enhancement in the treatment of the residual strong interaction to first order of perturbation theory. Will this enhancement "survive" in "all-order" treatment? To answer this question we will present in the next section the derivation of the IPNCI which is not based on the perturbation theory treatment of the residual strong interaction.

II. DERIVATION OF IPNCI: UNITARY TRANSFORMATION

We start with the nuclear Hamiltonian H in the form

$$H = H_0 + V_S + W + F, \quad (8)$$

where the first term $H_0 = \mathbf{p}^2/2m + U_S(r)$ is the single-particle Hamiltonian of the nucleons with inclusion of the single-particle part of the strong interaction $U_S(r)$ (strong potential), V_S is the residual two-body strong interaction, F describes other possible interactions, e.g., coupling to electromagnetic field, anapole moment operator [7–9], etc. The operator $W = \hat{W}(1,2)$ is the two-body weak PNC interaction, [15–19]:

$$\hat{W}(1,2) = \frac{G}{\sqrt{2}}\frac{1}{2m}\left((g^W_{12}\sigma_1 - g^W_{21}\sigma_2)\cdot\{(\mathbf{p}_1 - \mathbf{p}_2)\delta(\mathbf{r}_1 - \mathbf{r}_2) + \delta(\mathbf{r}_1 - \mathbf{r}_2)(\mathbf{p}_1 - \mathbf{p}_2)\} + g'^W_{12}[\sigma_1 \times \sigma_2]\nabla_1\delta(\mathbf{r}_1 - \mathbf{r}_2)\right), \quad (9)$$

where $G = 10^{-5}$ m^{-2} is the Fermi constant, m is the nucleon mass, and $\mathbf{p}$ and σ are the nucleon momentum and its doubled spin, respectively[1] (hereafter, the notation $\mathbf{a} \times \mathbf{b}$ means exterior vector product). The nucleon dimensionless constants $g_{p,n}$ (see, e.g., Refs. [15–22]) are of the order of unity and may be chosen in such a way that only direct terms in (9) should be accounted for.

It is well known (see, e.g., [19,23]) that the main P-odd effects caused by the weak interaction $\hat{W}$ in (1) are usually due to the effective one-body P-odd interaction, or the "weak potential" $\hat{w}(1)$, acting on the nucleon 1, which arises from averaging $\hat{W}(1,2)$ over the states of the nucleon 2 [see Eq. (3) for $w \equiv w(1) = \langle W(1,2)\rangle$]. The weak potential constants g^W_p, g^W_n are given by $g^W_p = \frac{Z}{A}g^W_{pp} + \frac{N}{A}g^W_{pn}, g^W_n = \frac{Z}{A}g^W_{np} + \frac{N}{A}g^W_{nn}$ for the proton and neutron, respectively. (Now, the notation $\varepsilon \simeq 1.0 \times 10^{-8} g^W$ is widely used.) The coherent contribution from all the paired nucleons yields the nuclear density ρ in the expression (3).

As has been mentioned above, the coherent single-particle P-odd contribution (3) does not work effectively in mixing of the nearest excited nuclear states. Therefore, the P-odd effects in this energy region can be determined by the purely two-particle "residue," $:\hat{W}(1,2):$ of the weak interaction $\hat{W}(1,2)$, given by the difference

$$:\hat{W}(1,2): \equiv \hat{W}(1,2) - \langle\hat{W}(1,2)\rangle = \hat{W}(1,2) - \hat{w}(1), \quad (10)$$

which does not contain coherent summation, in contrast to (3).

As mentioned above, the purpose of this work is to show that the residual strong interaction V_S in the Hamiltonian (8) gives rise to appearance of an *effective P-odd two-particle interaction* (IPNCI) which turns out to be

[1] This weak Hamiltonian goes back to works by Feynman and Gell-Mann [18]; the constants g in it were the subject of numerous studies (see, e.g., [20,17], and references therein). We used the values of these constants from Refs. [16,19].

stronger than the initial one, $:\hat{W}(1,2):$. We show that IPNCI contains the enhancement of order $\sim A^{1/3}$ times, compared to the initial two-particle P-odd term, and, moreover, the additional enhancement can arise [24], if the residual parity conserving strong interaction contains momentum-dependent structures [25–27]. The latter is taken into account by the solving the equation for the effective field, which is equivalent to summation of the infinite sum of graphs, analogous to that considered in the theory of finite Fermi system (TFFS) [28,25].

We start with the case when the strong interaction V_S is "switched off." As is known from Refs. [29] and [21], in the simple model of a constant nuclear density $\rho \simeq \rho_0 = 2p_F^3/3\pi^2$ it is easy to find the result of the action of the perturbation $\hat{w}(1)$

$$\tilde{\psi} = \exp(-\hat{a})\psi^0 \simeq (1 - i\xi\sigma\mathbf{r})\psi^0, \qquad \hat{a} = i\xi\sigma\mathbf{r} \tag{11}$$

$$\xi = \frac{G}{\sqrt{2}} g^W \rho_0 = \varepsilon m, \qquad \xi = \xi_0 + \xi_\tau \tau_z$$

where ψ^0 is the unperturbed wave function, and $\tau_z = -1$ $(+1)$ is isospin projection for proton (neutron). To get this solution, one should also neglect spin-orbit interactions. Accordingly, the matrix elements of any operator O, including the Hamiltonian, can be calculated by using the unperturbed wave functions ψ^0 and the transformed operator $\tilde{O}$:

$$\langle\tilde{\psi}_a|O|\tilde{\psi}_b\rangle = \langle\psi_a^0|\tilde{O}|\psi_b^0\rangle = \langle\psi_a^0|e^{\hat{a}}Oe^{-\hat{a}}|\psi_b^0\rangle \simeq \langle\psi_a^0|O + [\hat{a}, O]|\psi_b^0\rangle,$$

where $e^{\hat{a}} \equiv e^{i\xi(\sigma\mathbf{r})}$ is the operator of the corresponding unitary transformation with the single-particle anti-Hermitian $\hat{a}$. Correct choice of the transformation yields compensation of the single-particle P-odd potential in the Hamiltonian $e^{\hat{a}}He^{-\hat{a}}$: $\hat{w} + [\hat{a}, H_0] = 0$. The effect of this potential is now included into the renormalized operators $\tilde{O}$ rather than the wave functions $\tilde{\psi}$.

Let us switch on the strong interaction V_S and seek now for an operator $e^{\hat{A}}$ with the renormalization resulting from V_S taken into account. (Eventually, as we will see below, the operator $\hat{A}$ differs from $\hat{a}$ mainly due to the renormalization of the weak interaction constant by the residual strong interaction V_S.) The transformed Hamiltonian looks like

$$\tilde{H} = e^{\hat{A}}He^{-\hat{A}} = H_0 + V_S + \hat{w} + :\hat{W}: + [\hat{A}, H_0] + F + [\hat{A}, F] + [\hat{A}, V_S], \tag{12}$$

where we have used the decomposition (10) and neglected all terms above the first order in the weak interaction. To obtain the effective two-particle P-odd interaction acting in the valence shells, we should find the operator $\hat{A}$ in such a way that the single-particle P-odd contribution in $e^{\hat{A}}He^{-\hat{A}}$ will be compensated. The last term in (12) is a two-body operator. We employ the same decomposition as in (10): $[\hat{A}, V_S] \equiv \langle[\hat{A}, V_S]\rangle + :[\hat{A}, V_S]:$, where the first single-particle term is the average over the paired nucleons, and the second one, $:[\hat{A}, V_S]:$, which yields zero under such averaging, is the effective induced two-particle interaction which we are seeking for

$$V_{\mathrm{IPNCI}} =: [\hat{A}, V_S] :, \quad \langle V_{\mathrm{IPNCI}}\rangle \equiv 0. \tag{13}$$

Thus, if we require the "compensation equation"

$$\hat{w} + [\hat{A}, H_0] + \langle[\hat{A}, V_S]\rangle = 0 \tag{14}$$

to be fulfilled, the transformed Hamiltonian takes the form

$$\tilde{H} = H_0 + V_S + :\hat{W}: + V_{\mathrm{IPNCI}} + F + [\hat{A}, F], \tag{15}$$

where no single-particle P-odd potential is present. Thus, there are three sources of the parity nonconservation in Eq. (15): (1) the commutator $[\hat{A}, F]$ which gives a direct contribution of the PNC potential $w(1)$ to the matrix elements of an external field F ($\langle\psi|F + [\hat{A}, F]|\psi'\rangle = \langle\tilde{\psi}|F|\tilde{\psi}'\rangle$); (2) the residual two-body weak interaction $:W:$; (3) V_{IPNCI}, which plays the same role as $:W:$, but is enhanced in comparison with $:W:$ (see below).

To solve the Eq. (14) and find the explicit form of the IPNCI we use the Landau-Migdal [25,28,30,26] parametrization of the strong interaction:

$$V(\mathbf{r}_1, \mathbf{r}_2) = C\delta(\mathbf{r}_1 - \mathbf{r}_2) \times [f + f'\tau_1\tau_2 + h\sigma_1\sigma_2 + h'\tau_1\tau_2\sigma_1\sigma_2], \tag{16}$$

where $C = \frac{\pi^2}{p_F m} = 300$ MeV fm^3 is the universal Migdal constant [28,25,26], and the strengths f, f', h, h' are in fact functions of r via density dependence: $f = f_{\mathrm{in}} - (f_{\mathrm{ex}} - f_{\mathrm{in}})[\rho(r) - \rho(0)]/\rho(0)$ (the same for f', h, h'). (Quantities subscripted by "in" and "ex" characterize interaction strengths in the depth of the nucleus and on its surface, respectively). This interaction goes backwards to Landau Fermi liquid theory [30]. With its parameter values listed below, it has been successfully used by many authors (see Ref. [25]) to quantitatively describe many properties of heavy nuclei. The conventional choice widely used for heavy nuclei is (see [28,25,26]): $f_{\mathrm{ex}} = -1.95$, $f_{\mathrm{in}} = -0.075$ $f'_{\mathrm{ex}} = 0.05$ $f'_{\mathrm{in}} = 0.675$, $h_{\mathrm{in}} = h_{\mathrm{ex}} = 0.575$, and $h'_{\mathrm{in}} = h'_{\mathrm{ex}} = 0.725$. It is easy to check that, in the same approximation of constant density as used above, the operator $\hat{A}$ is proportional to $\hat{a}$: $\hat{A} = i\tilde{\xi}(\sigma\mathbf{r})$. Evaluating the commutator in (13),(14), we obtain

$$[\hat{A}, V_S] = -2\tilde{\xi}_\tau C\delta(\mathbf{r}_1 - \mathbf{r}_2)\{(h' - h)(\tau_{2z} - \tau_{1z})\mathbf{r}_1 \cdot \sigma_2 \times \sigma_1 + (h' - f')(\tau_2 \times \tau_1)_z(\sigma_2 - \sigma_1, \mathbf{r}_1)\}, \tag{17}$$

$$\langle[\hat{A}, V_S]\rangle = 0, \quad V^{\mathrm{IPNCI}}(1,2) = [\hat{A}, V_S].$$

Since the last term in the compensation equation (14) is zero in this case, the operator $\hat{A}$ coincides with $\hat{a}$ and the values of the constants $\tilde{\xi}_\tau$ just coincide with their "bare" values ξ_τ (11) (i.e., without the strong interaction). The

first term in Eq. (17) induces transitions $pn \to pn$, while the second one $pn(np) \to np(pn)$. For contact interactions, the second term (which is in fact an exchange term in comparison with the first one) can be reduced to the first term using Fierz transformation (see, e.g., [31]). After this transformation,

$$\hat{V}^{\rm IPNCI}(1,2) = -\frac{1}{2}Q\delta(\mathbf{r}_1-\mathbf{r}_2)(\tau_{2z}-\tau_{1z})\mathbf{r}_1\sigma_2\times\sigma_1 \to Q\mathbf{r}_p(\sigma_p\times\sigma_n)\delta(\mathbf{r}_p-\mathbf{r}_n),$$

$$Q = -2(\xi_p-\xi_n)(f'-h)C = -\frac{4}{3}\frac{p_F^2}{m}\frac{G}{\sqrt{2}}(g_p^W-g_n^W)(f'-h). \tag{18}$$

We stress that this expression is valid within the nucleus only (recall that $Q \sim \xi \sim \rho$). When using this expression one has to assume the exchange term $pn \to np$ is excluded. However, the conventional choice of the Landau-Migdal interaction constants already corresponds to the same assumption. This means that the second term in the expression (17) for the IPNCI should be simply omitted (to avoid double counting) and the final expression for the IPNCI includes the *pn-pn* interaction only, i.e., the constant of the IPNCI is

$$Q = -2(\xi_p-\xi_n)(h'-h)C = \frac{4}{3}\frac{p_F^2}{m}\frac{G}{\sqrt{2}}(g_p^W-g_n^W)h_{pn}, \tag{19}$$

and $h_{pn} = h - h'$ is the constant of the residual strong proton-neutron spin-flip interaction. This problem with the definition of the IPNCI constant is due to the fact that the Landau-Migdal interaction is a phenomenological effective interaction rather than the *ab initio* strong interaction. For example, it can contain "fictitious" spin dependence coming from the Fierz transformation of the exchange term with the spin-independent interaction $C\delta(\mathbf{r}_1-\mathbf{r}_2)$. However, this "fictitious" spin dependence does not contribute to the IPNCI, since in the case of an initial spin-independent interaction the Fierz transformation gives $h'-h = f'-h = 0$. Therefore, only "real" spin dependence of the strong interaction (e.g., due to π-meson exchange) contributes to the IPNCI.

III. COMPARISON OF THE IPNCI WITH THE RESIDUAL TWO-BODY WEAK INTERACTION AND DISCUSSION

It is interesting to compare the IPNCI with the initial two-nucleon weak interaction $: \hat{W}(1,2) :$. The interaction (18,19) and the "bare" one, Eqs. (10),(8), have different isotopic and coordinate structure (momentum $\mathbf{p}$ or derivative ∇ instead of radius vector $\mathbf{r}$). Taking into account that $r \sim r_0A^{1/3}, p_Fr \sim p_Fr_0A^{1/3} \sim A^{1/3}$, we obtain

$$\frac{V^{\rm IPNCI}}{: W(1,2) :} \sim p_Fr \sim A^{1/3}. \tag{20}$$

For heavy nuclei where neutron-nucleus PNC effects were measured, the nucleon number $A \simeq 114, ..., 240$, and $r_0 = 1.15$ fm $\sim p_F^{-1}$ is internucleon distance. Thus, the IPNCI (18),(19) is an order of magnitude stronger than the initial weak interaction (10) acting within the valence shell. The numerical results for the matrix elements of $V_{\rm IPNCI}$ as compared to those of the initial interaction $: W :$ between valence shell states for Th-U region are presented in Table I ($\tilde{V}_{\rm IPNCI}$ takes into account the momentum-dependent component of the Landau-Migdal interaction; see below). In practical calculations, it is useful to treat $V_{\rm IPNCI}$ in the second quantization form using multipole expansion in the particle-hole channel: $V_{\rm IPNCI} = \frac{1}{2}\sum_J[(a^\dagger b)_J V^{{\rm IPNCI},J}_{abcd}(c^\dagger d)_J]_0$, where $(\cdots)_J$ means the coupling of nucleon creators $a^\dagger$ and destructors a to a given angular momentum J [12]. The values of the parameters g_{ik}^W and g_p^W, g_n^W were chosen according to [32], [10]. On the average, the enancement (14) is an order of magnitude.

We stress once more that the selection rules (change of parity and conservation of the angular momentum) forbid matrix elements of the single-particle weak potential between the valence orbitals presented in Table I, i.e., the IPNCI and the residual interaction $: W :$ are the only source of parity nonconservation in the compound states within the "principal component" approach. The equations expressing the root mean square matrix element between compound states in terms of matrix elements of

TABLE I. Absolute values of the matrix elements of $V_{\rm IPNCI}$ [Eqs. (17),(19)], $\tilde{V}_{\rm IPNCI}$ [Eqs. (17)–(19) with matrix elements w renormalized according to (22)], and $: W :$ [Eqs. (10),(9)] in eV for the Th-U region. a, b (c, d) denote the single-particle neutron (proton) upper states.

a	b	c	d	J	$\lvert V^{{\rm IPNCI},J}_{abcd}\rvert$	$\lvert \tilde{V}^{{\rm IPNCI},J}_{abcd}\rvert$	$\lvert : W :^J_{abcd}\rvert$
$2g\ 9/2$	$1j15/2$	$1h\ 9/2$	$1h\ 9/2$	3	0.067	0.082	0.009
$2g\ 9/2$	$1j15/2$	$1h\ 9/2$	$1h\ 9/2$	4	0.033	0.062	0.001
$2g\ 9/2$	$1j15/2$	$1h\ 9/2$	$1h\ 9/2$	5	0.035	0.048	0.012
$2g\ 9/2$	$1j15/2$	$1h\ 9/2$	$1h\ 9/2$	7	0.029	0.043	0.016
$2g\ 9/2$	$1j15/2$	$1h\ 9/2$	$1h\ 9/2$	8	0.043	0.082	0.001
$1i11/2$	$1j15/2$	$1h\ 9/2$	$1h\ 9/2$	3	0.144	0.184	0.007
$1i11/2$	$1j15/2$	$1h\ 9/2$	$1h\ 9/2$	5	0.130	0.165	0.016
$1i11/2$	$1j15/2$	$1h\ 9/2$	$1h\ 9/2$	7	0.131	0.166	0.032
$1i11/2$	$1j15/2$	$1h\ 9/2$	$1h\ 9/2$	9	0.172	0.218	0.027

the IPNCI and : W : (see Table I) are presented in Ref. [10].

Of course, the explicit form of IPNCI [Eqs. (18),(19)] based on the approximation (5) is semiquantitative. In particular, due to the smallness of the quantity $h-h'$, corrections to (18) may be considerable for particular matrix elements. Especially big corrections appear in the interference term (proportional to $g_p^W g_n^W$) in the calculation of the mean squared value of the weak matrix element between compound states. These matrix elements contain a sum of the products of the matrix elements between the nucleon orbitals [see Eq. (7) and Ref. [10] for the accurate formula]:

$$\overline{|\langle s|W|p\rangle|^2} \sim \sum V_{abcd}^{\rm IPNCI} V_{cdab}^{\rm IPNCI} \sim \sum V_{S,ibcd} V_{S,ajcd} w_{ai} w_{jb} + \cdots .$$

The coefficients before $(g_p^W)^2$ and $(g_n^W)^2$ in this sum are positive, and the result is stable. On the other hand, the coefficients before the interference term ($\sim g_p^W g_n^W$) are not positively defined and this coefficient tends to decrease after the summations [in comparison with the ones before $(g_p^W)^2$ and $(g_n^W)^2$]. Therefore, the result for the mean squared matrix element is proportional to $|g_p^W|^2+|g_n^W|^2$ with a small coefficient before $g_p^W g_n^W$ rather than to $(g_p^W - g_n^W)^2$ [as it could follow from the approximate formula (18) for the IPNCI].

The numerical calculation of the root mean square matrix elements between compound states has shown that the contribution of the IPNCI [Eqs. (18)–(19)] is about $7, ..., 12$ times bigger than the contribution of the initial weak interaction W [Eq. (10)], confirming the estimate (20).

As was mentioned above, the results (13)–(19) can be obtained using perturbation theory considerations [see Eqs. (1)–(7)]. Formally, the result in Eq. (6) is obtained in the first order in residual strong interaction V_S. However, iterations ($w_1 \to V_{\rm IPNCI} \to w_1 + \delta w_1 \to V_{\rm IPNCI} + \delta V_{\rm IPNCI} \to \cdots$) of the contribution of the velocity-independent part of the interaction V_S do not change the result, since $V_{\rm IPNCI}$ does not contribute to the weak potential ($\langle V_{\rm IPNCI}\rangle_{\rm core} = \langle [A, V_S]\rangle_{\rm core} = 0$; see Eq. (7)). This explains why "all-orders" results (13)–(19) coincide with the first-order result (6): the self-consistent random-phase-approximation-like chain is terminated after the first iteration. The situation changes if one takes into account the momentum-dependent corrections to the Landau-Migdal interaction given by [25,26]. In this case, the summation of the series produces an additional enhancement factor ~ 1.5.

IV. CONTRIBUTION OF THE VELOCITY-DEPENDENT RESIDUAL STRONG INTERACTION TO THE RENORMALIZATION OF THE WEAK POTENTIAL AND THE IPNCI

Let us consider now these momentum-dependent corrections V_1 to the Landau-Migdal interaction (16), given by

$$V_1 = \frac{1}{4} C p_F^{-2} (h_1 + h_1' \tau_1 \tau_2)(\sigma_1 \sigma_2) \times [\mathbf{p}_1 \mathbf{p}_2 \delta(\mathbf{r}_1 - \mathbf{r}_2) + \mathbf{p}_1 \delta(\mathbf{r}_1 - \mathbf{r}_2)\mathbf{p}_2 + \mathbf{p}_2 \delta(\mathbf{r}_1 - \mathbf{r}_2)\mathbf{p}_1 + \delta(\mathbf{r}_1 - \mathbf{r}_2)\mathbf{p}_1 \mathbf{p}_2]. \quad (21)$$

This form originates from the π-meson exchange contribution to the nucleon-nucleon interaction [25], [26]. Its constants are known to be $h_1 = -0.5$, $h_1' = -0.26$ (Ref. [26]). Note that we keep in (21) only those $\mathbf{p}$-dependent corrections which yield nonzero contributions to the P-odd field renormalization (see below). To the lowest powers of $\mathbf{p}$, these terms should be $\sim \sigma_1 \mathbf{p}_1 \sigma_2 \mathbf{p}_2$. Spin-independent velocity contributions to (21) responsible, e.g., for the effective mass renormalization, are therefore irrelevant and the effects caused by them (e.g., effective mass renormalization) are assumed to be taken into account in definition of the constants C, m, and h_i.

It is easy to see that in this case the operator $\hat{A}$ should be of the same form as $\hat{a}$, but with its constants renormalized. The inclusion of the additional term V_1 (21) ($V_S = V + V_1$) gives the "compensation equation" (14) for the effective single-particle field in the form

$$\hat{w}(1) + \left[i \sum_{a=1,2} \xi_a \sigma_a \mathbf{r}_a, \frac{\mathbf{p}^2}{2m} \right] + K\{(\sigma \mathbf{p}), \rho\} = 0,$$

where $K = -\frac{C}{2p_F^2}[\frac{Z}{A}(h_1 \pm h_1')\xi_p + \frac{N}{A}(h_1 \mp h_1')\xi_n]$ with upper (lower) signs for proton (neutron), respectively (see Ref. [24]). In the constant density approximation, all terms in this equation have the same operator structure and its solution is equivalent to the renormalization of the constants ξ in (11), obtained by replacement of "bare" weak constant $g_{p,n}^W$ by their renormalized values $\tilde{g}_{p,n}^W$:

$$\tilde{g}_p^W = \frac{1}{D}\left(g_p^W \left[1 + \frac{2N}{3A}(h_1 + h_1') \right] - \frac{2N}{3A} g_n^W (h_1 - h_1') \right),$$
$$\tilde{g}_n^W = \frac{1}{D}\left(g_n^W \left[1 + \frac{2Z}{3A}(h_1 + h_1') \right] - \frac{2Z}{3A} g_p^W (h_1 - h_1') \right), \quad (22)$$

with $D = [1+\frac{2N}{3A}(h_1+h_1')][1+\frac{2Z}{3A}(h_1+h_1')] - \frac{4NZ}{9A^2}(h_1-h_1')^2$ (firstly, this result has been obtained in our work [24]). Thus, with the account for V_1, the IPNCI takes the form

$$V_{\rm IPNCI} = \tilde{V}_{\rm IPNCI} + V_{\rm IPNCI}^{\rm vel} = 2(\tilde{\xi}_n - \tilde{\xi}_p)(h' - h) C (\sigma_p [\sigma_n \times \mathbf{r}]) \delta(\mathbf{r}_p - \mathbf{r}_n) + V_{\rm IPNCI}^{\rm vel}, \quad (23)$$

where the first term has the form of (18) but with the renormalized constants $\tilde{\xi}_p$, $\tilde{\xi}_n$, which yields an additional enhancement at the negative values of h_1, h_1' ($\tilde{\xi} \sim 1.5\xi$ for $h_1 = -0.5$, $h_1' = -0.26$; see Table I). Note that at present there is an uncertainty in the values of h_1, h_1'. In Ref. [24], we carried out one more calculation of the weak potential renormalization basing on the underlying $(\pi + \rho)$-exchange strong interaction, which also produces a tensor contribution to V_1. These calculations give even more substantial enhancement of the weak potential constants $g^{\tilde{W}}, \tilde{\xi}$.

The second term contains velocity-dependent corrections:

$$V^{\rm vel}_{\rm IPNCI} = :[A,V_1]:$$
$$= -\frac{C}{4p_F^2}\Big[:\Big((\tilde\xi_n+\tilde\xi_p)(h_1+h_1'\tau_1\tau_2)+\frac{1}{2}(\tilde\xi_n-\tilde\xi_p)(h_1+h_1')(\tau_{1z}+\tau_{2z})\Big)\{(\sigma_1\mathbf{p}_1)+(\sigma_2\mathbf{p}_2),\delta(\mathbf{r}_1-\mathbf{r}_2)\}$$
$$+\frac{1}{2}(\tilde\xi_n-\tilde\xi_p)(h_1-h_1')(\tau_{2z}-\tau_{1z})\{(\sigma_1\mathbf{p}_1)-(\sigma_2\mathbf{p}_2),\delta(\mathbf{r}_1-\mathbf{r}_2)\}:$$
$$+(\tilde\xi_n-\tilde\xi_p)(h_1-h_1')(\tau_{2z}-\tau_{1z})\{\mathbf{p}_1,\{\mathbf{p}_2,(\sigma_1[\sigma_2\mathbf{r}])\delta(\mathbf{r}_1-\mathbf{r}_2)\}\}\Big],$$

where $\frac{C}{4p_F^2}\xi=\frac{1}{3}\frac{Gg}{2\sqrt{2}m}$, and $V^{\rm vel}_{\rm IPNCI}$ (except the last term) has no enhancement in comparison with the two-body weak interaction (10). Thus, it is considerably smaller than the first term in Eq. (23). The last term in $V^{\rm vel}_{\rm IPNCI}$ is in fact the momentum-dependent correction to IPNCI [Eqs. (18),(23)].

V. CONTRIBUTION OF THE IPNCI TO THE REGULAR PNC EFFECTS

In principle, the IPNCI can also give some regular PNC effect in the neutron capture, besides the main "random" one [10]. Consider the neutron capture into a compound state of negative parity (p-wave compound resonance for the positive parity target nucleus). The strong residual interaction can capture the neutron in the p-wave only. The IPNCI [Eqs. (18),(19),(23)] can capture the s-wave neutron. The slow neutron wave function $\exp(i\mathbf{kr})\chi\simeq[1+i(\mathbf{kr})]\chi$ (χ is the spinor) contains both s-wave and p-wave parts which are connected by the relation $\psi_{p1/2}=\frac{ik}{3}(\sigma\mathbf{r})\psi_s$. It is clear that the IPNCI contribution to the s-wave neutron capture matrix element proportional to $Q(\mathbf{r}\sigma_p\times\sigma_n)\psi_s$ [see Eq. (18)] is similar to the spin-dependent part of the $p_{1/2}$-wave strong contribution $(\sigma_p\sigma_n)\psi_{p1/2}\to\frac{k}{3}(\mathbf{r}\sigma_p\times\sigma_n)\psi_s+\cdots$ (i.e., $V_{\rm IPNCI}\psi_s\sim V_S\psi_p$). The similarity of these two fields means that s-wave and p-wave neutrons can excite the same state of the nucleus, and there is a coherent contribution to the PNC effects (which is proportional to the doubled ratio of the s-wave to $p_{1/2}$-wave capture amplitudes):

$$P\sim 2\frac{T_s}{T_{p1/2}}\sim 6\frac{\tilde\xi}{k}=\frac{1.3\times10^{-3}\tilde g_n^W}{\sqrt{E}}.$$

Here, E is neutron energy in eV. This value is comparable to the valence contribution estimates [2,4,23], if $\tilde g_n^W\sim g_n^W\sim 1$ (small renormalization of P-odd field) and it is too small in comparison to the observed regular effect in neutron capture by ^{232}Th ($P\simeq 0.3/\sqrt{E}$).

VI. INDUCED PARITY AND TIME INVARIANCE VIOLATING INTERACTION

It is interesting to compare the IPNCI with a similar parity and time invariance violating interaction (IPTI) which is induced by the strong interaction V_S and P,T-odd nuclear potential [instead of P-odd potential (3)]:

$$W_{PT}=\eta_{PT}\frac{G}{2\sqrt{2}m}(\sigma\nabla)\rho\simeq-\lambda(\sigma\nabla)U,$$

with $\lambda=\eta_{PT}\frac{G}{2\sqrt{2}m}\frac{\rho(0)}{|U(0)|}=2\times10^{-8}\eta$ fm. The shapes of the strong potential U and that of the nuclear density are assumed to be similar. The wave function perturbed by this interaction (see Ref. [33]) can be written as $\tilde\psi=\exp(-\hat A_{PT})\psi\simeq(1-\hat A_{PT})\psi$, $\hat A_{PT}=\lambda(\sigma\nabla)$. Calculations similar to those we have done for the IPNCI give the following result:

$$V^{IPTI}=[\hat A_{PT},V_s]$$
$$\sim C\lambda[(\sigma\nabla),\delta(\mathbf{r}_1-\mathbf{r}_2)]$$
$$-iC\lambda[\sigma_1\times\sigma_2\{\nabla_1,\delta(\mathbf{r}_1-\mathbf{r}_2)\}]$$

with $C\lambda=C\frac{\rho}{|U|}\frac{G}{2\sqrt{2}m}\eta_{PT}\simeq\frac{G}{2\sqrt{2}m}\eta$. We see that the structure and strength of IPTI is similar to those of the initial two-body P,T-odd interaction (see, e.g., [19]). Thus, in the case of P,T-odd interaction there is no $A^{1/3}$ enhancement.

VII. CONCLUSION

Let us stress, in conclusion, that we considered here the IPNCI term in the Hamiltonian caused by the change of the residual strong interaction by the coherent PNC field. An explicit expression for the IPNCI is obtained. It is shown that the IPNCI is $\sim A^{1/3}$ times stronger than residual two-nucleon weak interaction. This enhancement is due to coherent contributions of all nucleons to the weak nucleon-nucleus potential. (In the initial weak interaction, only the two external nucleons interact, while in the IPNCI, contributions of all the nucleons into the weak potential are accumulated.) Of course, the P-even strong interaction V_S remains unchanged in the total Hamiltonian [see Eq. (15)], and its main effect, i.e., mixing of configurations in true eigenstates, remains to be a separate problem.

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