ρ, ω, and ϕ meson-nucleon scattering lengths from QCD sum rules

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(Received 17 October 1994)

The QCD sum rule method is applied to derive a formula for the ρ, ω, and ϕ meson-nucleon spin-isospin–averaged scattering lengths $a_{\rho,\omega,\phi}$. We found that the crucial matrix elements are $\langle \bar{q}\gamma_\mu D_\nu q\rangle_N$ ($q = u, d$) (twist-2 nucleon matrix element) for $a_{\rho,\omega}$ and $m_s\langle\bar{s}s\rangle_N$ for a_ϕ, and obtained $a_\rho = 0.14 \pm 0.07$ fm, $a_\omega = 0.11 \pm 0.06$ fm, and $a_\phi = 0.035 \pm 0.020$ fm. These small numbers originate from a common factor $1/(m_N + m_{\rho,\omega,\phi})$. Our result suggests a slight increase (< 60 MeV for ρ and ω, and < 15 MeV for ϕ) of the effective mass of these vector mesons in nuclear matter (in the *dilute* nucleon-gas approximation). The origin of the discrepancy with Hatsuda-Lee was clarified.

PACS number(s): 13.75.−n, 12.38.Lg, 11.55.Hx, 24.85.+p

The operator-product expansion (OPE) provides us with a convenient tool to decompose a variety of correlation functions into the perturbatively calculable c-number coefficients and the nonperturbative matrix elements. In its application to the QCD sum rules (QSR's) [1, 2], the OPE supplies an expression for the resonance parameters in terms of the vacuum condensates representing the nonperturbative dynamics in the correlators. In the application to deep inelastic scattering (DIS) [3], the OPE isolates the quark-gluon distribution functions of the target from the short-distance cross sections. In this paper, we investigate the vector meson ρ, ω, ϕ-nucleon scattering lengths utilizing these two aspects of the OPE. These scattering lengths can be measured through the photoproduction of these vector mesons. Furthermore, they determine the mass shift of the vector mesons in the dilute nuclear medium. This will be discussed in the last part of this paper. A similar idea was recently presented for the nucleon-nucleon scattering length in Ref. [4].

We start our discussion with the forward scattering amplitude of the vector current J_μ^V ($V = \rho, \omega, \phi$) off the nucleon target with four momentum $p = (p^0, \boldsymbol{p})$ and polarization s:

$$T_{\mu\nu}(\omega, \boldsymbol{q}) = i \int d^4x\, e^{iqx} \langle ps| T\left(J_\mu^V(x) J_\nu^V(0)\right)|ps\rangle, \qquad (1)$$

where $q{=}(\omega, \boldsymbol{q})$ is the four-momentum carried by J_μ^V and the nucleon state is normalized covariantly as $\langle p|p'\rangle = (2\pi)^3 2p^0 \delta(\boldsymbol{p} - \boldsymbol{p}')$. We set $p = (m_N, \boldsymbol{0})$ throughout this work and suppress the explicit dependence on p and s. The vector current J_μ^V is defined as $J_\mu^{\rho,\omega}(x) = \frac{1}{2}[\bar{u}\gamma_\mu u(x) \mp \bar{d}\gamma_\mu d(x)]$, $J_\mu^\phi(x) = \bar{s}\gamma_\mu s(x)$. Near the pole position of the vector meson, $T_{\mu\nu}$ can be associated with the T matrix for the forward V-N helicity amplitude, $\mathcal{T}_{hH,h'H'}(\omega, \boldsymbol{q})$ [$h(h')$ and $H(H')$ are the helicities of the initial (final) vector meson and the initial (final) nucleon, respectively, and they take the values of $h, h' = \pm 1, 0$ and $H, H' = \pm 1/2$] as

$$\epsilon_\mu^{(h)}(q) T_{\mu\nu}(\omega, \boldsymbol{q}) \epsilon_\nu^{(h')*}(q) \simeq -\frac{f_V^2 m_V^4}{(q^2 - m_V^2 + i\eta)^2} \times \mathcal{T}_{hH,h'H'}(\omega, \boldsymbol{q}), \qquad (2)$$

where we introduced the coupling f_V and the mass m_V of the vector meson V by the relation $\langle 0|J_\mu^V|V^{(h)}(q)\rangle = f_V m_V^2 \epsilon_\mu^{(h)}(q)$ with the polarization vector $\epsilon_\mu^{(h)}(q)$ normalized as $\sum_{\rm pol} \epsilon_\mu^{(h)}(q)\epsilon_\nu^{(h)}(q) = -g_{\mu\nu} + q_\mu q_\nu/q^2$. As is well known in DIS, $T_{\mu\nu}$ can be decomposed into the four scalar components respecting the current conservation and the invariance under parity and time reversal. (Two of them correspond to the spin-averaged structure functions W_1 and W_2, and the other two to the spin-dependent ones G_1 and G_2.) Correspondingly, there are four independent helicity amplitudes for the vector-current–nucleon scattering, $\mathcal{T}_{1\frac{1}{2},1\frac{1}{2}}$, $\mathcal{T}_{1\frac{-1}{2},1\frac{-1}{2}}$, $\mathcal{T}_{0\frac{1}{2},0\frac{1}{2}}$, $\mathcal{T}_{1\frac{-1}{2},0\frac{1}{2}}$, all the rest being obtained by time reversal and parity from these four. Since information on G_1 and G_2 is still lacking, we shall focus on the combination $T = T_1 + [1 - (pq)^2/m_N^2 q^2]T_2$ ($\mathrm{Im}\, T_i \sim W_i$, $i = 1, 2$), which projects the V-N spin-averaged T matrix, $\mathcal{T}(\omega, \boldsymbol{q})$. In the low-energy limit ($\boldsymbol{q} \to \boldsymbol{0}$), $\mathcal{T}$ is reduced to the V-N spin-averaged scattering length $a_V = (1/3)(a_{1/2} + 2a_{3/2})$ ($a_{1/2}$ and $a_{3/2}$ are the scattering lengths in the spin-1/2 and -3/2 channels, respectively) as $\mathcal{T}(m_V, \boldsymbol{0}) = 24\pi(m_N + m_V)a_V$ [5]. A useful quantity for the dispersion analysis is the retarded correlation function defined as

$$T_{\mu\nu}^R(\omega, \boldsymbol{q}) = i \int d^4x\, e^{iqx} \theta(x^0) \langle ps| \left[J_\mu^V(x), J_\nu^V(0)\right]|ps\rangle = \frac{1}{\pi}\int_{-\infty}^{\infty} du \frac{\mathrm{Im}\, T_{\mu\nu}^R(u, \boldsymbol{q})}{u - \omega - i\eta}, \qquad (3)$$

which is analytic in the upper half ω plane with a fixed $\boldsymbol{q}$. Noting the crossing symmetry, the V-N scattering contribution to the spin-averaged spectral function at $\boldsymbol{q} = \boldsymbol{0}$ can be written as

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$$\frac{1}{\pi}\mathrm{Im}\left[T^R(\omega,\mathbf{0})\right] = \theta(\omega)\frac{1}{\pi}\mathrm{Im}\left[T(\omega,\mathbf{0})\right] - \theta(-\omega)\frac{1}{\pi}\mathrm{Im}\left[T(\omega,\mathbf{0})\right]$$
$$= -24\pi f_V^2 m_V^4 (m_N + m_V) a_V \left[\theta(\omega)\delta'(\omega^2 - m_V^2) - \theta(-\omega)\delta'(\omega^2 - m_V^2)\right] + (\mathrm{SP}), \quad (4)$$

where $\delta'(x)$ is the first derivative of the δ function (double pole term) and SP denotes the simple pole term representing the off-shell energy dependence of the T matrix. Equation (4) can also be derived starting from the spectral representation. Using this form of the spectral function in Eq. (3), and noting that the retarded correlation function $T^R_{\mu\nu}$ becomes identical to the causal correlation function $T_{\mu\nu}$ in the deep Euclidean region $\omega^2 = -Q^2 \to -\infty$, one gets

$$T(\omega^2 = -Q^2) = -24\pi f_V^2 m_V^4 (m_N + m_V) a_V \frac{1}{(m_V^2 + Q^2)^2} + \mathcal{R}_{\mathrm{OS}}(Q^2) + \mathcal{R}_c(Q^2), \quad (5)$$

where we have used the fact that T becomes a function of ω^2 in this limit. In Eq. (5), we assumed that the spectral function is saturated by the V-N scattering (with its off-shell effect) and the "continuum" contribution [6]: $\mathcal{R}_{\mathrm{OS}}(Q^2)$ denotes the simple pole term corresponding to the off-shell part of the V-N T matrix [$\sim 1/(m_V^2 + Q^2)$] and $\mathcal{R}_c(Q^2)$ stands for the "continuum" contribution with its threshold S_0' [$\sim 1/(S_0' + Q^2)$] [7]. The sum of the residues of $\mathcal{R}_{\mathrm{OS}}(Q^2)$ and $\mathcal{R}_c(Q^2)$ is constrained by the $1/Q^2$ term in the OPE side of the correlator. [See Eq. (6) below.]

We now proceed to the OPE side of $T(\omega^2 = -Q^2)$ [left-hand side (lhs) of Eq. (5)]. Unlike in DIS, our OPE is the short-distance expansion and hence all the operators with the same dimension contribute in the same order with respect to $1/Q^2$ ($= -1/\omega^2$ at $\mathbf{q} = \mathbf{0}$). The complete OPE expression for $T(Q^2)$ including the operators up to dimension=6 is given in Ref. [8] in the context of the finite-temperature QSR's. For the ρ and ω mesons, it reads from Eq. (2.13) of [8] as ($-$ for ρ and $+$ for ω)

$$T^{\rho,\omega}(Q^2) = \frac{1}{4}\left[-\frac{2m_q}{Q^2}\langle\bar{u}u + \bar{d}d\rangle_N - \frac{1}{6Q^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_N + \frac{2\pi\alpha_s}{Q^4}\left(\left\langle\mathcal{Q}_5^{\mp} + \frac{2}{9}\mathcal{Q}^{+}\right\rangle_N\right)\right]$$
$$-\frac{m_N^2}{2Q^2}A_2^{u+d} + \frac{5m_N^4}{6Q^4}A_4^{u+d} - \frac{m_N^2}{2Q^4}\left(B_{1\mp} + \frac{1}{4}B_2 + \frac{5}{8}B_3\right), \quad (6)$$

where $\langle\cdot\rangle_N$ denotes the spin-averaged nucleon matrix element, and $\mathcal{Q}_5^{\mp}$ and $\mathcal{Q}^{+}$ are the scalar fourquark operators familiar in the QSR's for the ρ and ω mesons: $\mathcal{Q}_5^{\mp} = (\bar{u}\gamma_\mu\gamma_5\lambda^a u \mp \bar{d}\gamma_\mu\gamma_5\lambda^a d)^2$ and $\mathcal{Q}^{+} = (\bar{u}\gamma_\mu\lambda^a u + \bar{d}\gamma_\mu\lambda^a d)\sum_q^{u,d,s}\bar{q}\gamma_\mu\lambda^a q$. In Eq. (6), $A_n^{u+d} \equiv A_n^u + A_n^d$ ($n = 2, 4$) are related to the twist-2 operators and are given as the nth moment of the parton distribution function ($q = u, d, s$): $\langle\mathcal{ST}(\bar{q}\gamma_{\mu_1}D_{\mu_2}\cdots D_{\mu_n}q(\mu))\rangle_N = (-i)^{n-1}A_n^q(\mu)(p_{\mu_1}\cdots p_{\mu_n} - \text{traces})$ and $A_n^q(\mu) = 2\int_0^1 dx\, x^{n-1}[q(x,\mu) + (-1)^n\bar{q}(x,\mu)]$ with the renormalization scale μ. B_i ($i = 1\mp, 2, 3$) are associated with the twist-4 matrix elements as $\langle\mathcal{O}^i_{\mu\nu}(\mu)\rangle_N = (p_\mu p_\nu - m_N^2 g_{\mu\nu}/4)B_i(\mu)$ with $\mathcal{O}^{1\mp}_{\mu\nu} = (g^2/4)\mathcal{ST}[(\bar{u}\gamma_\mu\gamma_5\lambda^a u \mp \bar{d}\gamma_\mu\gamma_5\lambda^a d)(\mu \to \nu)]$, $\mathcal{O}^2_{\mu\nu} = (g^2/4)\mathcal{ST}[(\bar{u}\gamma_\mu\lambda^a u + \bar{d}\gamma_\mu\lambda^a d) \times \sum_q^{u,d,s}\bar{q}\gamma_\nu\lambda^a q]$, and $\mathcal{O}^3_{\mu\nu} = ig\mathcal{ST}[\bar{u}\{D_\mu,{}^* G_{\nu\lambda}\}\gamma^\lambda\gamma_5 u + (u \to d)]$, where the color matrix λ^a is normalized as $\mathrm{Tr}(\lambda^a\lambda^b) = 2\delta^{ab}$ and the symbol $\mathcal{ST}$ makes the operators symmetric and traceless with respect to the Lorentz indices.

To get an expression for the V-N scattering length, we first make a Borel transform of Eqs. (5) and (6) with respect to Q^2, and then eliminate the unknown parameter which determines the ratio between the coefficient of $\mathcal{R}_{\mathrm{OS}}(Q^2)$ and $\mathcal{R}_c(Q^2)$, using the sum rule obtained by taking the derivative with respect to the Borel mass M^2. We also eliminate the unknown coupling constant f_V by taking the ratio between the obtained sum rule and the QSR expression for the vector current correlators in the vacuum. We thus get an expression for the spin-averaged scattering length a_V as

$$a_{\rho,\omega} = \frac{\pi M^2}{3m_{\rho,\omega}^2(m_N + m_{\rho,\omega})}\frac{r\beta/(\alpha M^2) + t\gamma/(\alpha M^4)}{(1 + \frac{\alpha_s}{\pi})(1 - e^{-S_0/M^2}) + b/M^4 - c/M^6}, \quad (7)$$

with

$$r = m_N\Sigma_{\pi N} - \frac{2}{27}m_0^2 + \frac{m_N^2}{2}A_2^{u+d},$$
$$t = -\frac{112\pi\alpha_s}{81}\left(\langle\bar{u}u\rangle\langle\bar{u}u\rangle_N + \langle\bar{d}d\rangle\langle\bar{d}d\rangle_N\right) - \frac{5}{6}m_N^4 A_4^{u+d} + \frac{m_N^2}{2}\left(B_{1\mp} + \frac{1}{4}B_2 + \frac{5}{8}B_3\right),$$

$$b = 4\pi^2 m_q\langle\bar{u}u + \bar{d}d\rangle + \frac{\pi^2}{3}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle,$$
$$c = \frac{448\pi^3\alpha_s}{81}\langle\bar{u}u\rangle^2,$$

where $\langle\cdot\rangle$ denotes the vacuum condensate and S_0 is the continuum threshold in the vacuum sum rule. The factors α, β, and γ appeared through the process of eliminating the parameter which determines the ratio between the residues of $\mathcal{R}_{\rm OS}(Q^2)$ and $\mathcal{R}_c(Q^2)$, and they are defined as $\alpha = 1 - e^{-(S_0'-m^2)/M^2}(1+\frac{S_0'-m^2}{M^2})$, $\beta = \frac{m^2}{M^2} + \frac{S_0'-m^2}{M^2}e^{-S_0'/M^2} - \frac{S_0'}{M^2}e^{-(S_0'-m^2)/M^2}$, and $\gamma = 1 + \frac{m^2}{M^2} - (1+\frac{S_0'}{M^2})e^{-(S_0'-m^2)/M^2}$ with $m = m_{\rho,\omega}$. If we ignore $\mathcal{R}_c(Q^2)$ from the beginning, the corresponding formula is obtained by the replacement $\alpha \to 1$, $\beta \to 1 - e^{-m^2/M^2}$, $\gamma \to 1$. In Eq. (7), we have used the following relations for the matrix elements as has been used in the study of QCD sum rules in nuclear matter [9, 10]: (i) πN σ-term $\Sigma_{\pi N}$ is introduced through the relation $m_q\langle \bar{u}u + \bar{d}d\rangle_N = 2m_N\Sigma_{\pi N}$. (ii) The nucleon mass in the chiral limit, m_0, is introduced in favor of $\langle\frac{\alpha_s}{\pi}G^2\rangle_N$ through the QCD trace anomaly: $\langle\frac{\alpha_s}{\pi}G^2\rangle_N = -(16/9)m_0^2$. (iii) Factorization is assumed for the vacuum four-quark condensates $\langle\mathcal{Q}_5^{\mp}\rangle$ and $\langle\mathcal{Q}^+\rangle$, as is usually adopted in the literature [1, 2]. (iv) Factorization is also employed to estimate the nucleon matrix elements of the scalar four-quark operators $\langle\mathcal{Q}_5^{\mp}\rangle_N$ and $\langle\mathcal{Q}^+\rangle_N$ after making the Fierz transform [10], i.e., $\langle(\bar{q}\Gamma\lambda q)^2\rangle_N \to \langle(\bar{q}q)^2\rangle_N \simeq 2\langle\bar{q}q\rangle\langle\bar{q}q\rangle_N$.

By repeating the same steps as above for J_μ^ϕ, one gets the spin-averaged ϕ-N scattering length as

$$a_\phi = \frac{\pi M^2}{3m_\phi^2(m_N+m_\phi)}\frac{r_s\beta/(\alpha M^2) + t_s\gamma/(\alpha M^4)}{(1+\frac{\alpha_s}{\pi})(1-e^{-S_0/M^2}) - 6m_s^2/M^2 + b_s/M^4 - c_s/M^6}, \tag{8}$$

with

$$r_s = m_s\langle\bar{s}s\rangle_N - \frac{2}{27}m_0^2 + m_N^2 A_2^s,$$

$$t_s = -\frac{224\pi\alpha_s}{81}\langle\bar{s}s\rangle\langle\bar{s}s\rangle_N - \frac{5}{3}m_N^4 A_4^s + m_N^2\left(B_1^s + \frac{1}{4}B_2^s + \frac{5}{8}B_3^s\right),$$

$$b_s = 8\pi^2 m_s\langle\bar{s}s\rangle + \frac{\pi^2}{3}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle,$$

$$c_s = \frac{448\pi^3\alpha_s}{81}\langle\bar{s}s\rangle^2,$$

where the strange twist-4 matrix elements B_i^s (i = 1–3) are defined similarly to the case of the ρ and ω mesons. For the vacuum condensates and the quark masses in Eqs. (7) and (8), we use the standard values at the renormalization scale μ = 1 GeV [2]: $\alpha_s = 0.36$, $m_q = 7$ MeV, $m_s = 110$ MeV, $\langle\bar{u}u\rangle = \langle\bar{d}d\rangle = (-0.28\,\mathrm{GeV})^3$, and $\langle\bar{s}s\rangle = 0.8\langle\bar{u}u\rangle$. With these vacuum condensates and the continuum threshold $S_0 = 1.75$ GeV^2 for ρ, ω and $S_0 = 2.0$ GeV^2 for ϕ, the experimental values for $m_{\rho,\omega,\phi}$ are well reproduced. We thus fixed S_0 at these values and use $m_{\rho,\omega} = 770$ MeV and $m_\phi = 1020$ MeV in Eqs. (7) and (8). As a measure of the strangeness content of the nucleon, we introduce the parameter $y = 2\langle\bar{s}s\rangle_N/(\langle\bar{u}u\rangle_N + \langle\bar{d}d\rangle_N)$ and write $\langle\bar{s}s\rangle_N = ym_N\Sigma_{\pi N}/m_q$. For the nucleon matrix elements we use $\Sigma_{\pi N} = 45 \pm 7$ MeV, $y = 0.2$, and $m_0 = 830$ MeV obtained by chiral perturbation theory [11]. Since we ignored the twist-2 gluon operators in Eqs. (7) and (8), we consistently use the leading order (LO) parton distribution functions of Glück, Reya, and Vogt [12] to determine A_i^{u+d} and A_i^s ($i = 2, 4$). It gives $A_2^{u+d} = 0.90$, $A_4^{u+d} = 0.12$, $A_2^s = 0.05$, and $A_4^s = 0.002$ at $\mu^2 = 1$ GeV^2. For the twist-4 matrix elements B_i and B_i^s, we use our recent result [13] extracted from the newest DIS data at CERN and SLAC. It is based on the SU(2) flavor symmetry [i.e., $B_i^s = 0$ (i = 1–3)] and a mild assumption on the matrix elements invoked by the flavor structure of the twist-4 operators. Both for the proton and the neutron, it gives $B_{1\mp} + B_2/4 + 5B_3/8 = -0.24 \pm 0.15$ (-0.41 ± 0.23) GeV^2 for the ρ (ω) meson at $\mu^2 = 5$ GeV^2 [14]. [Note that our $\rho^0 - N$ (N can be either proton or neutron) scattering length corresponds to the isospin-spin averaged one.]

Using these numbers for the matrix elements, the Borel curves for the ρ, ω, ϕ-nucleon scattering lengths $a_{\rho,\omega,\phi}$ [Eqs. (7) and (8)] are shown in Fig. 1. We determined the values of S_0' in order to minimize the slope of the curves at $0.8 < M^2 < 1.3$ GeV^2. They are 3.32 GeV^2 for ρ, 3.29 GeV^2 for ω, and 4.40 GeV^2 for ϕ. With the above parameters, r in Eq. (7) reads $r = 0.04 - 0.05 + 0.40 = 0.39$ GeV^2 from the first to the third terms. Thus r is totally dominated by the twist-2 nucleon matrix element A_2^{u+d} and the canceling contribution from the first and the second terms makes the ambiguity in $\Sigma_{\pi N}$ and m_0 less important. The t term in Eq. (7) reads $t = 0.42 - 0.08 - 0.11 \pm 0.07\,(-0.18 \pm 0.10) = 0.23 \pm 0.07\,(0.16 \pm 0.10)$ GeV^4 for ρ (ω), which shows the contribution from the twist-4 matrix elements is sizable. To get an insight into the sensitivity of the results to the variation of t, we also showed $a_{\rho,\omega}$ without the twist-4 matrix elements in t with $S_0' = 3.35$ GeV^2. One sees that the inclusion of B_i reduces the $a_{\rho,\omega}$ by about 20% (30%) for ρ (ω). With

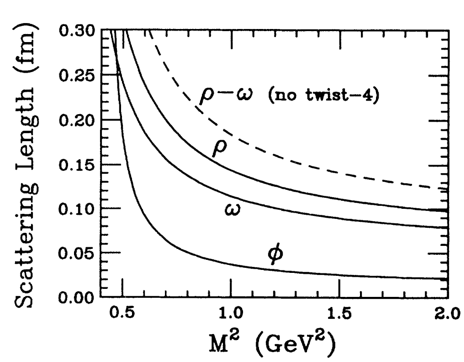

FIG. 1. The Borel curves for the ρ, ω, ϕ-nucleon scattering lengths. The dashed line denotes the one for ρ and ω without the twist-4 matrix elements in Eq. (7).

the uncertainty of B_i in mind, we assign error bars as $a_\rho = 0.14 \pm 0.07$ fm and $a_\omega = 0.11 \pm 0.06$ fm, taking the values for $a_{\rho,\omega,\phi}$ around $M^2 = 1$ GeV^2. For the case of a_ϕ, the value of $m_s\langle\bar{s}s\rangle_N$ governs the whole result because of large m_s, i.e., $r_s = 0.13 - 0.05 + 0.04 = 0.12$ GeV^2 and $t_s = 0.066 - 0.003 + (\text{twist} - 4 \equiv 0) = 0.063$ GeV^4 from the first to the third terms in r_s and t_s. Due to the uncertainty in $m_s\langle\bar{s}s\rangle_N$, we read, from Fig. 1, $a_\phi = 0.035 \pm 0.020$ fm. Some phenomenological analyses on the nucleon form factor [15] and the nuclear force [16] suggest quite a large Okubo-Zweig-Iizuka (OZI-) violating ϕNN coupling constant $g_{\phi NN}/g_{\omega NN} \sim 0.4$. Equation (8) supplies a neat expression for the $\phi N \to \phi N$ interaction strength in terms of the strangeness content of the nucleon, showing the importance of $m_s\langle\bar{s}s\rangle_N$ rather than $\langle\bar{s}\gamma_\mu D_\nu s\rangle_N$.

If we calculate the scattering lengths without $\mathcal{R}_c(Q^2)$ in Eq. (5), we get even smaller numbers for the scattering lengths: $a_\rho \sim 0.1$ fm, $a_\omega \sim 0.08$ fm, and $a_\phi \sim 0.01$ fm around $M = 1$ GeV. This way, the actual numbers for $a_{\rho,\omega,\phi}$ depend on the assumption made in the spectral function, although their typical order of magnitude does not change.

One might be surprised by the smallness of these scattering lengths compared with a typical hadronic size (~ 1 fm). From Eqs. (7) and (8), one sees $a_V \sim 1/(m_N + m_V)$, since r and r_s are dominated by the third and the first terms, respectively. If one applies the present method to the axial vector correlator, one can easily get the pion-nucleon scattering length in the isospin symmetric channel as $a_{\pi N} \propto m_N \Sigma_{\pi N}/f_\pi^2(m_N + m_\pi)$, which is the same result as that of the current algebra [17]. (In the chiral limit, $a_{\pi N} = 0$, since $\Sigma_{\pi N} = 0$.) Therefore it is interesting to observe that our method of deriving the vector-meson–nucleon scattering length is a generalization of the current algebra technique for the pion-nucleon scattering length. For the vector meson case, the common factor $1/(m_N + m_V)$ makes a_V small. We believe this smallness of the V-N scattering lengths somehow sketches the real situation, although the actual numbers for a_V are not trustable because of the simplified form for the spectral function in our calculation as was noted before. A model calculation of the ρ-N scattering amplitude based on an effective hadronic Lagrangian suggests a similar small number for a_ρ [18].

Let us finally discuss the mass shift of the vector mesons in the nuclear medium using the result for the scattering lengths here. In the *dilute* nucleon-gas approximation, the V-current correlator in the nuclear medium can be written as

$$\Pi_{\mu\nu}^{\mathrm{NM}}(\omega, \boldsymbol{q}) = i\int d^4x\, e^{iq\cdot x}\langle T\left(J_\mu^V(x)J_\nu^V(0)\right)\rangle + \sum_{\mathrm{pol}}\int^{p_f}\frac{d^3p}{(2\pi)^3 2p^0}T_{\mu\nu}(\omega, \boldsymbol{q}). \tag{9}$$

By ignoring the Fermi motion of the nucleon, $\Pi_{\mu\nu}^{\mathrm{NM}}(\omega, \boldsymbol{q} = \boldsymbol{0})$ can be approximated near the pole position as

$$\begin{aligned}\Pi_{\mu\nu}^{\mathrm{NM}}(\omega, \boldsymbol{0}) &\simeq f_V^2 m_V^4\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{\omega^2}\right)\left(\frac{1 + O(\rho_N)}{\omega^2 - m_V^2} + \Delta m_V^2\frac{1}{(\omega^2 - m_V^2)^2}\right)\\ &\sim \frac{1 + O(\rho_N)}{\omega^2 - m_V^2 - \Delta m_V^2} + O(\rho_N^2),\end{aligned} \tag{10}$$

where $\Delta m_V^2 = 12\pi a_V \rho_N(m_N + m_V)/m_N$ with the nucleon density ρ_N. From this relation, Δm_V^2 can be viewed as a shift of m_V^2 in the nuclear medium [19]. Our values for $a_{\rho,\omega,\phi}$ suggest that the effective mass for the vector mesons increases by about 27–57 MeV for ρ, 20–48 MeV for ω, and 5–13 MeV for ϕ at the nuclear matter density $\rho_N = 0.17$ fm^{-3} [20]. (Note that the validity of the mass shift discussed here hinges on the assumption that the off-shell energy dependence and the momentum dependence of the V-N scattering amplitude is weak within the range of the nucleon's Fermi momentum.)

The authors of Ref. [10] applied the QSR method to study mass shifts of the ρ, ω, and ϕ mesons in the nuclear medium. Although their approximation in the OPE side of the correlation functions is essentially the same as ours, Eq. (9), they found a serious *decrease* of these vector meson masses in nuclear matter. Here we clarify the origin of this discrepancy. In the recent literature of the QSR method in the nuclear medium for baryons and mesons [9, 10], the common starting point is that the density dependence of correlation functions is ascribed to the density-dependent condensates:

$$\Pi^{\mathrm{NM}}(q, \rho_N) = \sum_i C_i(q, \mu)\langle\mathcal{O}_i(\mu)\rangle_{\rho_N}, \tag{11}$$

where $C_i(q, \mu)$ and $\mathcal{O}_i(\mu)$ are the Wilson coefficient and a local operator, respectively, and we suppressed all the spinor and Lorentz indices. In the dilute nuclear medium, $\langle\mathcal{O}_i(\mu)\rangle_{\rho_N}$ has been approximated as

$$\begin{aligned}\langle\mathcal{O}_i(\mu)\rangle_{\rho_N} &= \langle\mathcal{O}_i(\mu)\rangle + \sum_{\mathrm{pol}}\int^{p_f}\frac{d^3p}{(2\pi)^3 2p^0}\langle ps|\mathcal{O}_i(\mu)|ps\rangle\\ &= \langle\mathcal{O}_i(\mu)\rangle + \frac{\rho_N}{2m_N}\langle\mathcal{O}_i(\mu)\rangle_N + o(\rho_N).\end{aligned} \tag{12}$$

As is easily seen by inserting Eq. (12) into Eq. (11), the approximation for the condensate, Eq. (11), is equivalent to the approximation, Eq. (9), for the correlator. Therefore the density-dependent part of the correlator has to be analyzed from the point of view of the forward current-nucleon scattering amplitude as was done in this paper. In this approximation, what is relevant for the mass shift is the double pole structure at the pole position appearing in the forward amplitude.

To understand the difference between our result and the one in [10], it is convenient to recall the QSR's for the vector meson in the vacuum. In the vacuum, the vector current correlator can be written as $\Pi_{\mu\nu}(q) = (q_\mu q_\nu - g_{\mu\nu}q^2)\Pi(q^2)$. As long as one uses a spectral function with a single narrow resonance and the continuum, both $\Pi(q^2)$ and $q^2\Pi(q^2)$ can be used for the QSR analysis. The formulas for the vector meson mass in the Borel sum rule obtained by using these two sum rules are, respectively,

$$\frac{m_V^2}{M^2} = \frac{(1+\frac{\alpha_s}{\pi})[1-(1+S_0/M^2)e^{-S_0/M^2}] - D_4/M^4 + D_6/M^6}{(1+\frac{\alpha_s}{\pi})[1-e^{-S_0/M^2}] + D_4/M^4 - D_6/2M^6}, \tag{13}$$

$$\frac{m_V^2}{M^2} = \frac{2(1+\frac{\alpha_s}{\pi})[1-(1+S_0/M^2+S_0^2/2M^4)e^{-S_0/M^2}] - D_6/M^6}{(1+\frac{\alpha_s}{\pi})[1-(1+S_0/M^2)e^{-S_0/M^2}] - D_4/M^4 + D_6/M^6}, \tag{14}$$

where D_4 and D_6 are the sums of the dim=4 and dim=6 condensates, respectively. Both sum rules give the right amount for m_V. However, one should note that the sign of the power correction due to the condensates is opposite between the two sum rules. Thus the shift of the condensates due to medium effects [the second term of Eq. (9) or Eq. (12)] is expected to cause opposite physical effects depending on which sum rule one uses. The authors of [10] analyzed $\Pi^{\rm NM}(\omega,\mathbf{0}) = \Pi^{\rm NM\mu}_{\mu}(\omega,\mathbf{0})/(-3\omega^2)$, with a simple pole ansatz for the vector meson (together with a scattering term) in the spectral function, which picks up the same effect of the shift of the condensate as Eq. (13). On the other hand, if we recognize that the second term in Eq. (9) is associated with the V-N forward amplitude through Eq. (2), we can easily see that it is $\omega^2\Pi^{\rm NM}(\omega,\mathbf{0})$ which has to be analyzed with a simple pole ansatz in the order $O(\rho_N)$ as is shown in Eq. (10). In this case, the vector meson mass receives the effect of the change of the condensate as is expected from the formula Eq. (14). In $\Pi^{\rm NM}(\omega,\mathbf{0})$, the density-dependent part appears as a form of $(\rho_N/2m_N)T(\omega,\mathbf{0})/\omega^2$, which brings a factor $-\Delta m_V^2(m_V^2/Q^2)$ $(Q^2=-\omega^2>0)$ instead of Δm_V^2 in the first line of Eq. (10). In this case, due to the additional factor $1/Q^2$, the double pole term can not be incorporated into the mass shift. Thus the use of Eq. (13) in the nuclear medium is simply wrong. Therefore an inadequate form of the spectral function in [10] led to a fictitious "negative" mass shift. Since the second term in Eq. (9) has a unique relation with the V-N T matrix around the pole position as is shown in Eq. (2), we believe that the mass shift of the vector mesons in the nuclear medium in the context of the QSR's should be understood as presented in this work.

Finally, we wish to make a comment on the speculation on the in-medium behavior of the hadron masses existing in the literature. From the finite energy sum rule analysis, one gets the ρ meson mass as $m_\rho \propto |\langle\bar\psi\psi\rangle|^{1/3}$ *in the vacuum.* Since $|\langle\bar\psi\psi\rangle|$ decreases in the nuclear medium according to the formula Eq. (12), one might naively expect m_ρ would also decrease [21]. We have illustrated, however, that a consistent organization of the QCD sum rule does not predict such a behavior. It would rather support (within our crude approximation) another naive expectation that a tendency of ρ-A_1 degeneracy might occur in the nuclear medium, since the application of our method to the A_1 meson gives decreasing m_{A_1}. We also remind the reader that (1) $|\langle\bar\psi\psi\rangle_T|$ decreases as the temperature (T) goes up, while all hadron masses stay constant in the order $O(T^2)$ [22], (2) a consistent organization of the sum rule at finite temperature certainly gives the same behavior [19] unlike the above naive expectation, and (3) this is because the pion-hadron scattering length is zero in the chiral limit.

In conclusion, we have derived the ρ,ω,ϕ -nucleon spin-isospin averaged scattering lengths $a_{\rho,\omega,\phi}$ from QCD sum rules. We obtained very small positive numbers (corresponding to the repulsive interaction) for $a_{\rho,\omega,\phi}$, $a_{\rho,\omega,\phi}\sim 1/(m_N+m_{\rho,\omega,\phi})$, although the actual numbers depend on the factorization assumption for the nucleon matrix element of the scalar four-quark operator as well as the simplified form for the spectral function. This result suggests a slight increase of these vector mesons in the nuclear medium, which is contradictory to the previous result by Hatsuda and Lee [10]. We have clarified the origin of this discrepancy and pointed out the problem of the analysis in Ref. [10]. Details of the calculation will be published elsewhere.

I would like to thank T. Hatsuda, S. H. Lee, P. Danielewicz, F. Lenz, O. Morimatsu, E. V. Shuryak, and K. Yazaki for many valuable discussions and comments. This work is supported in part by the U.S. National Science Foundation under Grant No. PHY-9017077.

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