Freeze-out conditions and pion spectrum in heavy-ion collisions

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(Received 21 June 1993; revised manuscript received 6 July 1994)

Pion multiplicity and energy spectrum in medium energy heavy-ion collisions are analyzed in a model with spherical expansion, resonance decay contribution, and chemical nonequilibrium effects. Fitting the data for central La+La collision at $E_{\text{lab}} = 1.35$ GeV/nucleon, we find that the physically reasonable freeze-out conditions require a strong collective motion to describe the pion spectrum and a positive value of pion chemical potential to fit the total pion multiplicity. The data do not lend to conclusive evidence on the presence of Δ resonance at the freeze-out.

PACS number(s): 13.75.−n, 25.75.+r

I. INTRODUCTION

One of the central problems in the study of high energy heavy-ion collisions is deducing the state of the system, i.e., the energy density, entropy, and temperature formed in the process, from the observed final particle properties. This can only be done by inference because the hot and dense matter produced initially cools and expands considerably before freeze-out, namely, when the particles fly toward the detectors without further interaction. The freeze-out stage of the system is most directly connected to the experimental data. If the freeze-out characteristics of the system can be determined from the observed properties, the evolution history of the system may then be reconstructed with the help of an appropriate model.

Pion production is a dominant feature in heavy-ion collisions [1]. In the present study, we will limit ourself to collisions with incident energy in the region of $E_{\text{lab}} = (1-2)$ GeV/nucleon, where measurements were done on both the pion multiplicity and spectrum. In this energy region a rather complete set of experimental information exists which contains the number of participating nucleons in the collision, energy per nucleon, and number of produced pions per participating nucleon. These data permit us to use energy and baryonic number conservation laws. They in turn place strong restrictions on the possible set of freeze-out parameters which would give rise to a good fit of the pion multiplicity and momentum spectrum.

Glendenning [2] showed that the thermodynamical model without collective motion overestimates the pion multiplicity per nucleon in the initial energy range $E_{\text{lab}} = (530-1800)$ MeV/nucleon and that the expansion stage of hadron matter should be taken into account. He demonstrated, on the other hand, that a hydrodynamical expansion up to physically reasonable values of the freeze-out baryonic density $\rho_b^f = (0.5 \pm 0.2)\rho_0$ ($\rho_0 \cong 0.16$ fm^{-3} is the normal nuclear matter density) leads to too small values for the pion multiplicity. As will be shown in this paper, this fact indicates the necessity of introducing nonzero values for the pion chemical potential, as was suggested in Ref. [3].

The enhancement of pion spectra at low transverse momenta, which was observed in CERN experiments at the bombarding energy $E_{\text{lab}}/A = 200$ GeV [4] and in AGS at $E_{\text{lab}}/A = 14.6$ GeV [5], also appears at medium initial energies $E_{\text{lab}}/A = (0.5-1.8)$ GeV [6,7]. Several different freeze-out characteristics have been proposed to account for this two-slope feature in the pion spectrum. Collective flow effects [8–10] and resonance decays [11–15] have been considered with some conflicting conclusions. Another possible physical origin of the low-$p_\perp$ pion enhancement, that pions are strongly out of chemical equilibrium, was suggested in Refs. [16,17] (see also Ref. [3]). It is supported by the hydrochemical model calculation [18] and the relativistic quantum molecular dynamics calculation [19]. Unfortunately, no attempt has been made to study these three possible effects, namely, the collective flow, resonance decays, and pion chemical nonequilibrium, at the same time, nor has a complete set of experimental observables been used.

It is hence of interest to pose the following question: What can be concluded from the experimental data on both the pion multiplicity and pion spectrum about the role and relative importance of the effects of collective flow, resonance decays, and pion chemical nonequilibrium at the freeze-out stage? As we will see, the simultaneous consideration of both the pion multiplicity and spectrum indeed helps us to establish unambiguously the presence of collective flow and nonvanishing pion chemical potential at the freeze-out.

In Sec. II, the formulas are given for the particle spectrum when the freeze-out stage of the system is characterized by the presence of collective motion, resonance particles and chemical nonequilibrium. Section III contains the results and discussions and we summarize in Sec. IV.

II. FORMULATION OF THE MODEL

In the present study, we will not try to formulate the model for the evolution of the fireball formed in

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the heavy-ion collision and find out the initial conditions for this evolution. Instead, our aim is just to construct physically reasonable freeze-out conditions, which include thermal and collective motion, resonance decays, and possible chemical nonequilibrium. In other words, we want to answer the following question: What can we learn from the experimental data on pion multiplicities and pion momentum spectrum about the role and relative importance of the above three physical effects at the freeze-out stage of the fireball formed in relativistic heavy ion collisions?

Assuming local thermal equilibrium, the momentum spectrum for hadron species i in the rest frame of the system element with volume $\triangle V^*$ at constant proper time is given as

$$\frac{dn_i}{d^3p^*} = \frac{\triangle V^*\ d_i}{(2\pi)^3} \frac{1}{\exp\left(\frac{E^*-\mu_i}{T}\right)\ +\ \eta_i} \equiv \frac{\triangle V^*\ d_i}{(2\pi)^3}\ f_i(E^*)\ , \tag{1}$$

where d_i, μ_i, and m_i are the degeneracy, chemical potential, and mass of hadron i, respectively. $E = (m_i^2 + \mathbf{p}^{*2})^{1/2}$, T is the temperature, $\eta_i = 1$ for fermions and -1 for bosons.

We are interested in the heavy-ion collisions at the initial bombarding energies $E_{\rm lab} = (1-2)$ GeV/nucleon. In particular, we shall fit the data [7] for central La+La collisions at $E_{\rm lab} = 1.35$ GeV/nucleon. Small values of the system temperature at these initial energies simplify the hadron chemical composition of the system. For example, at the temperatures considered here Brown *et al.* [13] found that the abundance of higher resonances is small and their effect is thus not expected to be large. We will hence take into account, apart from nucleons, only pions and Δ resonances. All heavier mesons and baryons do not play an important role in the formation of the pion spectrum and pion multiplicity and we leave them out. We have therefore $d_\pi = 3$, $d_N = 4$, and $d_\Delta = 16$ in Eq. (1). Small initial energy gives us one more simplification: simple spherical geometry of the final hadron state seems to be justified by the experimental data on particle spectra [7]. To begin with, we do not consider an electrical part μ_e of the chemical potentials and, therefore, do not differentiate the π^+, π^0, π^- isospin states of pion. The role of electrical chemical potential introduced in Ref. [3] will be discussed later.

The Δ finite decay width $\Gamma \cong 115$ MeV will be taken into account using the profile function [20]:

$$W(m)\ =\ \xi\ \theta(m-m_N-m_\pi)\ \frac{\Gamma/2}{(m-m_\Delta)^2\ +\ \Gamma^2/4}\ , \tag{2}$$

where ξ is the normalization constant

$$\xi^{-1}\ =\ \frac{\pi}{2}\ +\ \tan^{-1}\left(\frac{m_\Delta - m_N - m_\pi}{\Gamma/2}\right)\ .$$

All quantities involving the Δ particles will be integrated over the resonance mass m with the profile function in Eq. (2). For example, the particle number densities are

$$\rho_i(T,\mu_i)\ =\ \frac{d_i}{2\pi^2}\int_0^\infty p^2dp\ f_i\left(\sqrt{m_i^2+p^2}\right) \tag{3}$$

for $i = \pi, N$ and

$$\rho_\Delta(T,\mu_\Delta) = \frac{d_\Delta}{2\pi^2}\int_0^\infty dm\ W(m) \times \int_0^\infty p^2dp\ f_\Delta\left(\sqrt{m^2+p^2}\right) \tag{4}$$

for the Δ number density.

The "chemical reactions" in our $\pi N\Delta$ system are

$$\pi + N\ \leftrightarrow\ \Delta\ , \qquad N+N\ \leftrightarrow\ \Delta + N\ . \tag{5}$$

Assuming chemical equilibrium for both of these reactions, one obtains

$$\mu_\pi\ +\ \mu_N\ =\ \mu_\Delta,\quad \mu_N\ +\ \mu_N\ =\ \mu_\Delta\ +\ \mu_N\ , \tag{6}$$

and consequently $\mu_\pi = 0$. These conditions can be, however, violated because, either the second reaction in Eq. (5) ceases to be effective at small baryonic densities [2, 18], or the chemical nonequilibrium initial conditions for the expansion process takes place, as seen in Ref. [19]. In what follows we allow the possibility of nonzero values of μ_π, i.e., pions deviating from the chemical equilibrium. It will be shown that $\mu_\pi > 0$ is essential to explain the experimental pion multiplicity for physically reasonable freeze-out conditions. For Δ chemical potential we assume chemical equilibrium for the first reaction in Eq. (5), i.e., $\mu_\Delta = \mu_N + \mu_\pi$, and discuss other possibilities later.

Assuming the thermal particle spectrum of Eq. (1) in the rest frame of the system elements at constant proper time, we superimpose the spherical collective (hydrodynamical) motion in the form

$$v(r)\ =\ v_0\ \left(\frac{r}{R}\right)^n\ , \tag{7}$$

with linear ($n = 1$) or quadratic ($n = 2$) dependence of the collective velocity v on the radius r (R is the fireball radius and $0 \le r \le R$). Rewriting Eq. (1) in the relativistic-invariant form [$E = (m_i^2 + \mathbf{p}^2)^{1/2}$, $\mathbf{v} = v(r)\ \mathbf{r}/r$, $\gamma = (1-\mathbf{v}^2)^{-1/2}$]

$$E\frac{dn_i}{d^3p} = E^*\frac{dn_i}{d^3p^*} = \frac{\triangle V}{(2\pi)^3}\ (E-\mathbf{p}\cdot\mathbf{v})\ f_i\left[\gamma(E-\mathbf{p}\cdot\mathbf{v})\right]\ ,$$

then for the total thermal particle spectrum from the spherically expanding fireball with velocity function of Eq. (7) in the fireball center mass frame we obtain

$$E\frac{dN_i^{\rm th}}{d^3p}\ =\ \frac{d_i}{4\pi^2}\int_0^R r^2dr \times\int_{-1}^1 dx\ \frac{E-pvx}{\exp\left[\frac{\gamma(E-pvx)-\mu_i}{T}\right]\ +\ \eta_i}\ . \tag{8}$$

We call this spectrum, which includes also the collective motion, thermal, to distinguish it from an additional contribution (decay) to pion and nucleon spectra from the Δ decays.

The general formula for the particle spectrum in the hydrodynamical model looks like [21]

$$E\frac{dN_i}{d^3p} = \frac{d_i}{(2\pi)^3}\int_{\Sigma_f} d\sigma^\nu p_\nu\ f_i(u^\lambda p_\lambda) , \tag{9}$$

where $u^\nu = \gamma(1, \mathbf{v})$ is the four-velocity of the fluid element and $d\sigma^\nu$ is the four-vector normal to the freeze-out hypersurface Σ_f. Equation (1) in the rest frame of the fluid elements corresponds to the assumption of a constant "local-time freeze-out" (see Ref. [9(b)]). The freeze-out hypersurface is defined then from our velocity profile function (7) and Σ_f is a completely spacelike one. This is a typical ansatz for a treatment of the hydrodynamical evolution in heavy-ion collisions, see, e.g., the Bjorken one-dimensional scaling model [22]. A more rigorous approach requires finding the freeze-out hypersurface from the solution of the hydrodynamical equations with given initial conditions on the initial hypersurface Σ_i. Σ_f should be then closed to Σ_i, and in general Σ_f consists of both spacelike and timelike parts. To realize this procedure, however, one needs to know the form of Σ_i and the initial values of the hydrodynamical variables on it. They can be only obtained from the microscopic model consideration of the "formation stage" of the process of heavy-ion collisions before the local thermodynamical equilibrium is achieved. These initial hydrodynamic conditions are rather model dependent and, in fact, arbitrary at the moment. Besides, the particle emission through timelike parts of the freeze-out hypersurface influences the hydrodynomical evolution, and this influence should be included self-consistently into the hydrodynamical problem as additional "boundary conditions." We intend to treat this problem of the hydrodynamical approach in our future studies.

From the particle spectrum of Eq. (8) we calculate total particle numbers and energies as

$$\tilde{N}_i = \int d^3p \left(\frac{dN_i^{\rm th}}{d^3p}\right), \quad \tilde{E}_i = \int d^3p\, E \left(\frac{dN_i^{\rm th}}{d^3p}\right) . \tag{10}$$

For the Δ particle we should include the additional integration over m with probability $W(m)$.

To calculate the total pion momentum spectrum we add the Δ decay contribution to the thermal pion spectrum of Eq. (8). Each Δ with mass m ($m > m_N + m_\pi$) in the final state decays into a nucleon plus pion. In the Δ rest frame, the decay products are distributed isotropically and the pion spectrum is

$$E^*\frac{dn}{d^3p^*} = \frac{1}{4\pi p_\pi^*}\,\delta(E^* - E_\pi^*) ,$$

where the pion energy and momentum are defined by the decay kinematic as

$$E_\pi^* = \frac{m^2 - m_N^2 + m_\pi^2}{2m} , \quad p_\pi^* = (E_\pi^{*2} - m_\pi^2)^{1/2} .$$

The pion spectrum from the decays of all deltas are obtained by Lorentz transformation into the c.m. system of the expanding fireball (note that $Edn/d^3p = E^*dn/d^3p^*$), multiplication by the Δ spectrum (8) and integration over the Δ momentum and variable Δ mass:

$$E\frac{dN_\pi^{\rm dec}}{d^3p} = \int_0^\infty dm\ W(m)\ \int \frac{d^3p_\Delta}{E_\Delta}\left(E_\Delta\frac{dN_\Delta^{\rm th}}{d^3p_\Delta}\right) \times E\frac{dn}{d^3p} .$$

Representing E^* in terms of variables in the c.m. frame

$$E^* = \frac{E_\Delta E - \mathbf{p}_\Delta\cdot\mathbf{p}}{m} ,$$

we have succeeded in doing the angular integration over the directions of $\mathbf{p}_\Delta$. Finally we obtain

$$\frac{dN_\pi^{\rm dec}}{d^3p} = \frac{1}{2Ep}\int_0^\infty dm\ W(m)\ \frac{m}{p_\pi^*} \times \int_{E_-}^{E_+} dE_\Delta \left(E_\Delta\frac{dN_\Delta^{\rm th}}{d^3p_\Delta}\right) , \tag{11}$$

where

$$E_\mp = \frac{m}{m_\pi^2}(EE_\pi^* \mp pp_\pi^*)$$

are defined by the limiting cases of Δ decays when the pion momentum $\mathbf{p}^*$ is, respectively, parallel and antiparallel to the Δ momentum $\mathbf{p}_\Delta$. Similar expressions hold for the delta decay contribution to the nucleon spectrum.

III. RESULTS AND DISCUSSIONS

In this section we analyze the experimental data for La+La collisions at bombarding energy 1350 MeV/nucleon, which corresponds to 292 MeV kinetic energy per nucleon in the c.m. system. The experimental value for the pion number per participating nucleon is 0.18 [7]. For a fixed value of freeze-out baryonic density ρ_b^f, we obtain then the following system of equations:

$$\frac{\tilde{E}_N + \tilde{E}_\pi + \tilde{E}_\Delta}{\tilde{N}_N + \tilde{N}_\Delta} - m_N = 292 \text{ MeV} , \tag{12}$$

$$\frac{\tilde{N}_\pi + \tilde{N}_\Delta}{\tilde{N}_N + \tilde{N}_\Delta} = 0.18 , \tag{13}$$

$$\rho_b^f = \rho_N + \rho_\Delta , \tag{14}$$

where $\tilde{N}_i$ and $\tilde{E}_i$ are given by Eq. (10). In Eq. (13), the fact that each Δ will eventually decay into a nucleon plus pion after freeze-out has been taken into account. The total pion spectrum in our model is

$$\frac{dN_\pi}{d^3p} = \frac{dN_\pi^{\rm th}}{d^3p} + \frac{dN_\pi^{\rm dec}}{d^3p} . \tag{15}$$

The experimental data for the π^- spectrum are parametrized by two exponential functions [7], as shown in Fig. 1,

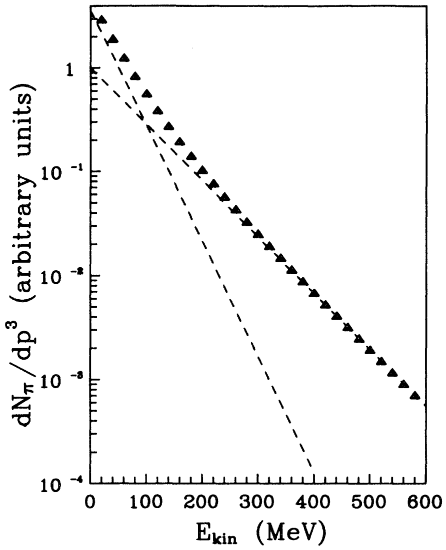

FIG. 1. Experimental pion spectrum of Ref. [7]. Dashed lines are the two exponential functions of Eq. (16).

$$\frac{dN_\pi}{d^3p} = A_1 \exp\left(-\frac{E_{\text{kin}}}{T_1}\right) + A_2 \exp\left(-\frac{E_{\text{kin}}}{T_2}\right) , \tag{16}$$

with $T_1 \cong 39$ MeV, $T_2 \cong 80$ MeV, and $A_1/A_2 \cong 3.65$.

We analyze the system (12)–(14) for three possible values of ρ_b^f : $0.3\rho_0$, $0.5\rho_0$, $0.7\rho_0$. At fixed ρ_b^f we solve the system of equations (12)–(14) numerically and find the sets of thermodynamical parameters T, μ_N, μ_π for a given velocity distribution (7) with given v_0. Then we can calculate pion spectrum (15) and compare it with the experimental one (16) to find the specific values of v_0 and the corresponding sets of the thermodynamical parameters which give the best fits to the experimental pion spectrum.

As the first step of our analysis we show the necessity of the collective motion to explain the experimental data. Without collective motion, i.e., $v_0 = 0$, the solutions of the system (12)–(14) are presented in Table I. One observes from Table I very large negative values for the pion chemical potentials, which should be in the system in order to fit the pion multiplicity data in Eq. (13). The chemical equilibrium ($\mu_\pi = 0$) thermodynamical model without collective motion greatly overestimates the pion number per nucleon (this is in agreement with Ref. [2]). However, admitting negative values of μ_π in the freeze-out parameters, as of Table I, in order to get the right total pion number we still have a completely wrong pion energy spectrum: $T \cong 130$ MeV in the thermodynamical system is much larger than the pion experimental slope parameters in Eq. (16). It becomes evident that pion slope parameters are the results of both thermal (with much smaller temperatures than those in Table I) and collective motion in the hadron gas system.

With the presence of the collective motion, the best possible choices of the freeze-out parameters [the solutions of Eqs. (12)–(14)] for different freeze-out baryonic densities are shown in Tables II and III. The comparison of the pion spectrum (15) with the quadratic velocity function to the experimental one (16) is shown in Figs. 2(a)–2(c). Similar fits are obtained with the linear velocity function.

TABLE I. Freeze-out parameters without collective motion [$v_0 = 0$ in Eq. (7)].

ρ_b^f/ρ_0	T (MeV)	μ_N (MeV)	μ_π (MeV)
0.3	128.16	471.69	−302.47
0.5	129.41	529.10	−264.52
0.7	130.29	567.31	−243.14

Representing total particle (pion and nucleon) spectra in the exponential form

$$\frac{dN}{d^3p} = \text{const} \times \exp\left(-\frac{E_{\text{kin}}}{T_{\text{eff}}}\right) , \tag{17}$$

we introduce an "effective temperature" $T_{\text{eff}} = T_{\text{eff}}(E_{\text{kin}})$ which defines the slope of the particle spectrum as a function of E_{kin}. The results for $T_{\text{eff}}^\pi(E_{\text{kin}})$ and $T_{\text{eff}}^N(E_{\text{kin}})$ are shown in Fig. 3 for the freeze-out parameters from Tables II and III at $\rho_b^f = 0.5\rho_0$. One observes that T_{eff}^N is essentially larger than T_{eff}^π. This is because the collective motion affects the heavier nucleons more than pions. We see also that T_{eff}^N, as a function of E_{kin}, exhibits different behaviors for linear ($n = 1$) and quadratic ($n = 2$) velocity functions.

From Figs. 2(a)–2(c) one can see that the experimental pion spectrum can be fitted with different values of the freeze-out baryonic density ρ_b^f. The relative importance of the Δ decay contribution increases with increasing ρ_b^f. The ratios of the total number of thermal pions ($\tilde{N}_\pi$) to the pion number from Δ decays ($\tilde{N}_\Delta$) are 1.27 at $0.3\rho_0$, 0.79 at $0.5\rho_0$, and 0.57 at $0.7\rho_0$. Figures 2(a)–2(c) also indicate that the Δ decay contributions to the pion spectrum enhancement at low E_{kin} become more important with increasing ρ_b^f.

In our consideration we use a Breit-Wigner form for the Δ mass distribution in Eq. (2) with an energy independent width Γ. A phenomenological parametrization of the Δ width as a function of the pion momentum in Ref. [12] gives the correct threshold behavior for the P-wave Δ resonance. It reduces the Δ decay contribution to the low energy part of the pion spectrum and, therefore, reduces the low transverse momentum enhancement. To fit the pion spectrum in this case we need to change slightly our freeze-out parameters and obtain larger values for the

TABLE II. Freeze-out parameters with the linear velocity function [$n = 1$ in Eq. (7)].

ρ_b^f/ρ_0	v_0	T (MeV)	μ_N (MeV)	μ_π (MeV)
0.3	0.635	53.81	819.52	40.00
0.5	0.625	56.39	838.11	44.90
0.7	0.620	57.52	853.73	48.66

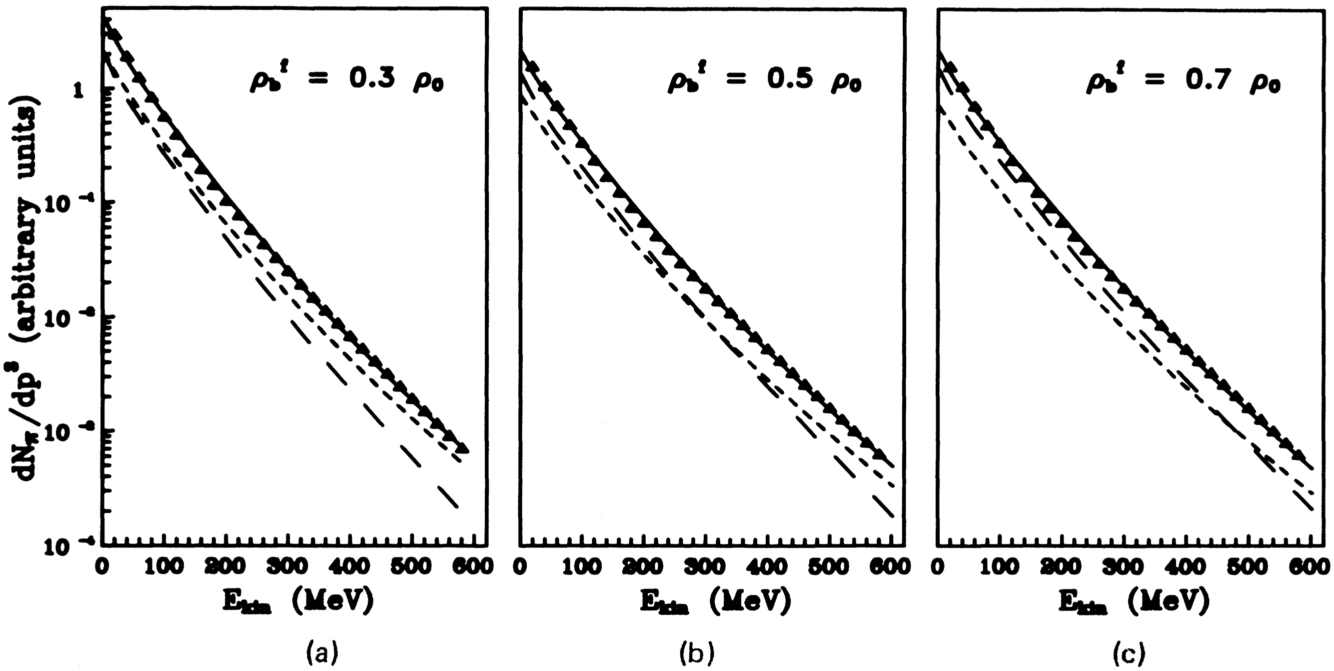

FIG. 2. Solid lines correspond to pion spectrum (15) for the freeze-out parameters from Table III. Short dashed lines are the thermal pion spectra (8) and long dashed lines are delta decay contributions (11) to pion spectra. The solid triangles represent the experimental data as parametrized by Eq. (16). In (a)–(c), $\rho_b^f = 0.3\rho_0, 0.5\rho_0$, and $0.7\rho_0$, respectively.

pion chemical potential.

It has been suggested in Ref. [23] that there should be no Δ decay contributions to the pion spectrum. Pions from the Δ decays thermalize by their subsequent elastic collisions with nucleons and other pions. The argument for this was that the pion experimental spectrum did not show the "delta decay peak." We find that the Δ collective motion and Δ finite width smear out the pion energy spectrum from Δ decays and, therefore, we do not see any "delta decay peak" in our calculations. On the other hand, there are also no straight indications from experimental data for the necessity of the Δ decay contributions to the pion spectrum. To check this in more detail we solve the system of equations (12)–(14) excluding Δ particles (we put $d_\Delta \equiv 0$). The results for the freeze-out parameters [the solutions of (12)–(14)] which fit the pion spectrum (16) are presented in Table IV. The pion spectrum ($dN_\pi^{\rm th}/d^3p$ in this case) for $\rho_b^f = 0.5\rho_0$ is shown in Fig. 4, where the agreement with data is excellent. We conclude, therefore, that the comparison of our model calculations with experimental data gives no definite indications of the presence of Δ particles at the freeze-out.

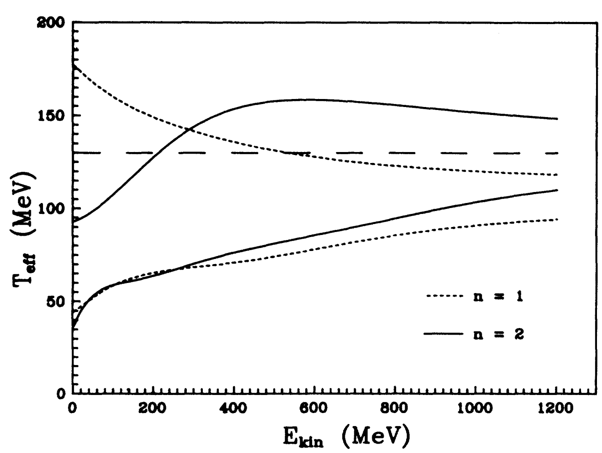

FIG. 3. Effective "temperatures" (spectrum slopes) for pion (lower lines) and nucleon (upper lines) spectra. The solid and dashed lines correspond to freeze-out parameters of Tables III and II, respectively, at $\rho_b^f = 0.5\rho_0$.

In the physical picture without Δ particles at the freeze-out we have essentially larger values of μ_π (compare the values of μ_π in Table IV with those in Tables II and III). Therefore, the effects of Bose statistics and the possibility of different shapes of π^- and π^+ spectra suggested in Ref. [3] should be studied. To do this we introduce the electrical chemical potential μ_e and define the chemical potentials of pions, protons, and neutrons

TABLE III. Freeze-out parameters with the quadratic velocity function [$n = 2$ in Eq. (7)].

ρ_b^f/ρ_0	v_0	T (MeV)	μ_N (MeV)	μ_π (MeV)
0.3	0.750	51.94	826.65	46.70
0.5	0.740	54.21	845.54	52.19
0.7	0.735	55.18	861.09	56.16

TABLE IV. Freeze-out parameters with quadratic velocity function [$n = 2$ in Eq. (7)] without Δ particles at the freeze-out.

ρ_b^f/ρ_0	v_0	T (MeV)	μ_N (MeV)	μ_π (MeV)
0.3	0.76	49.02	841.87	83.51
0.5	0.755	49.99	865.31	102.87
0.7	0.745	52.14	877.36	111.93

as

$$\mu_{\pi^+} = \mu_\pi + \mu_e, \ \mu_{\pi^0} = \mu_\pi, \ \mu_{\pi^-} = \mu_\pi - \mu_e,$$

$$\mu_p = \mu_N + \mu_e, \quad \mu_n = \mu_N. \tag{18}$$

We solve now the system (12)–(14) ($d_{\pi^+} = d_{\pi^0} = d_{\pi^-} = 1$, $d_p = d_n = 2$) excluding Δ particles ($d_\Delta \equiv 0$) and adding one more equation,

$$\frac{\tilde{N}_p + \tilde{N}_{\pi^+} - \tilde{N}_{\pi^-}}{\tilde{N}_p + \tilde{N}_n} = \frac{Q}{B} = \frac{57}{139}, \tag{19}$$

for the ratio of the total electric charge Q to the total baryonic number B (this ratio coincides with its initial value for La nuclei). Freeze-out parameters T, μ_N, μ_π, μ_e [the solutions of Eqs. (12)–(14),(19)] which give the best agreement with experimental π^- spectrum (16) are presented in Table V.

In Fig. 5 we show the π^- spectrum for the freeze-out parameters of Table V at $\rho_b^f = 0.5\rho_0$ (solid line) and compare it with its Boltzmann approximation, i.e., $\eta \equiv 0$ in Eq. (1) (dashed line). Figure 6 shows the Bose enhancement factor (see also Ref. [3]) which is the ratio of the Bose to Boltzmann pion spectra of Fig. 5. We observe that the essential part of the pion spectrum enhancement at low $E_{\rm kin}$ is now due to Bose-statistic effects.

Bose-statistic effects lead also to some difference in the shape of π^- and π^+ spectra because of the difference

TABLE V. Freeze-out parameters for the system of equations (12)–(14),(17) with quadratic velocity function [$n = 2$ in Eq. (7)] without Δ particles at the freeze-out.

ρ_b^f/ρ_0	v_0	T (MeV)	μ_N (MeV)	μ_π (MeV)	μ_e (MeV)
0.3	0.76	49.04	847.45	82.28	−11.90
0.5	0.75	51.22	867.23	98.53	−12.31
0.7	0.74	53.36	879.79	107.75	−12.65

between μ_{π^-} and μ_{π^+} [see Eq. (18)]. In the Boltzmann approximation we would obtain only the different π^- and π^+ multiplicities but the same shape of their spectra. The ratio of π^- to π^+ momentum spectra is shown in Fig. 7 for the freeze-out parameters from Table V at $\rho_b^f = 0.5\rho_0$. The asymptotic value of this ratio at large $E_{\rm kin}$ can be calculated within Boltzmann approximation and equals $\exp(-2\mu_e/T) \cong 1.62$. It is seen from Fig. 7 that this ratio increases at small $E_{\rm kin}$. As has been suggested in Ref. [3], we observe a larger enhancement at small kinetic energies for π^- than that for π^+.

IV. SUMMARY

In summary, we have set out to determine the freeze-out conditions of the fireball, formed in high energy heavy-ion collisions, from experimental data. For this purpose, we have formulated a model for the freeze-out state of the system which includes three physical ingredients: collective flow, resonance decays, and chemical nonequilibrium effects. These features of the freeze-out states are likely to be common to most of the heavy-ion collisions taking place in a wide range of initial energies. The model is used to analyze medium energy central collisions at $E_{\rm lab} = (1\text{–}2)$ GeV/nucleon for the following

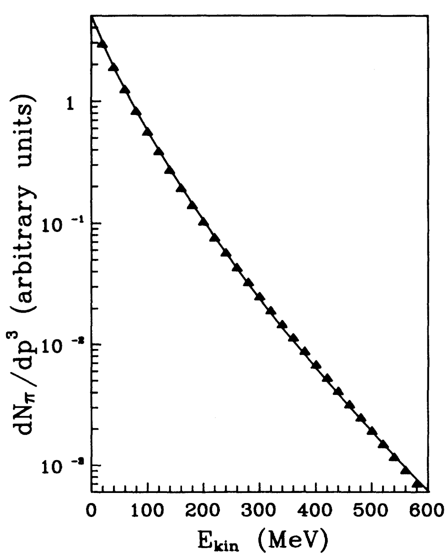

FIG. 4. The solid line is the pion spectrum (8) with the freeze-out parameters from Table IV at $\rho_b^f = 0.5\rho_0$.

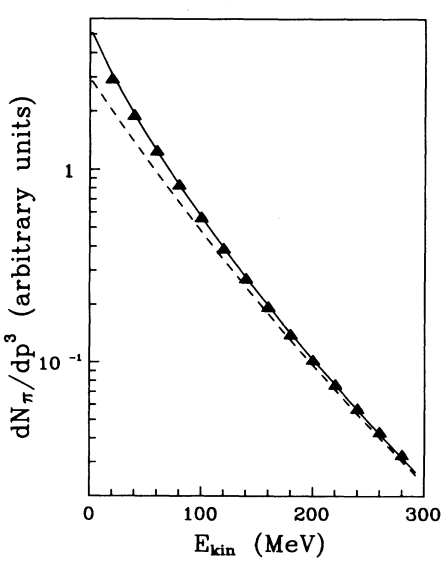

FIG. 5. The solid line is π^- spectrum (8) with the freeze-out parameters from Table V at $\rho_b^f = 0.5\rho_0$. The dashed line corresponds to the Boltzmann approximation, i.e., $\eta = 0$.

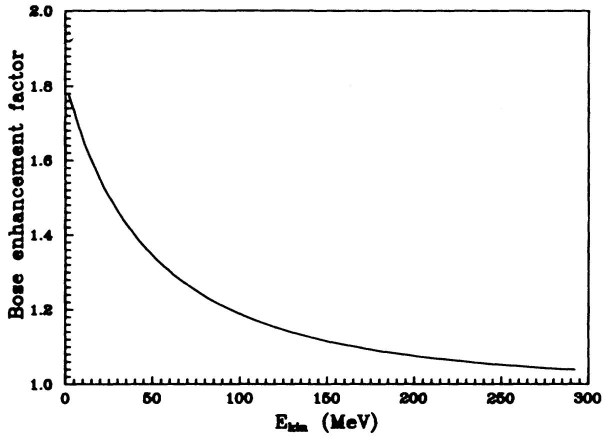

FIG. 6. The Bose enhancement factor for π^- spectrum.

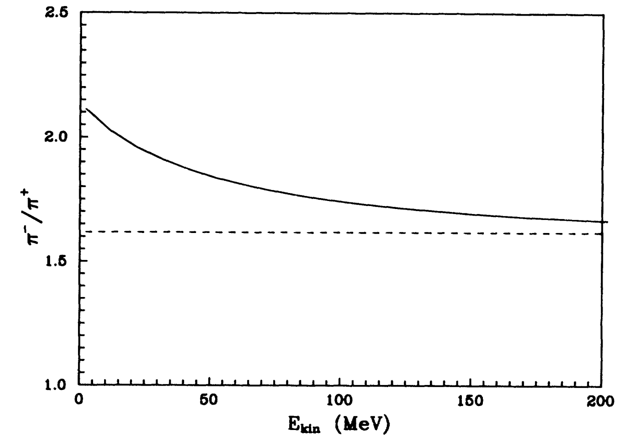

FIG. 7. The solid line is the ratio of π^- to π^+ momentum spectra for the freeze-out parameters from Table V at $\rho_b^f = 0.5\rho_0$. The dashed line corresponds to the asymptotic value of this ratio which coincides with its Boltzmann approximation.

reasons. First, at these energies the data exist for both the total pion numbers per participating nucleon and the pion momentum spectra. This enables us to determine essentially all the physically admissible sets of freeze-out parameters. Second, the freeze-out stage of the system at these initial energies is supposed to be relatively simple. We can restrict our consideration to just a $\pi N\Delta$ gas and neglect all other heavier mesons and baryons. Furthermore, the system can be approximated with a spherically symmetric geometry for the case of central collisions.

In the physically reasonable region of freeze-out baryonic density, $\rho_b^f = (0.5 \pm 0.2)\rho_0$, we find it impossible to explain the pion spectrum in La+La collisions at $E_{\rm lab} = 1.35$ GeV/nucleon [7] unless a strong collective flow is present. The presence of this collective flow further requires positive values of the pion chemical potential in order to obtain the correct pion multiplicity. It also leads to a strong difference between the pion and nucleon "effective temperatures," as shown in Fig. 3.

On the contribution of Δ decay to the pion spectrum, we do not find any definite indication from the data about its relative importance or even about its presence in the pion momentum spectrum. We find that it is still possible to give an excellent fit to the data in a model excluding the Δ's, as seen in Fig. 4. It requires instead a larger value of μ_π for the freeze-out stage. The enhancement in the pion spectrum at small energies is, in this case, connected with the Bose statistics. When the electrical chemical potential is included in the analysis, a difference in the shape of π^- and π^+ spectra at small energies develops, as was suggested in Ref. [3].

ACKNOWLEDGMENTS

M.I.G. is grateful to L. Bravina, K.A. Bugaev, St. Mrówczyński, R.A. Ritchie, and Yu. Shirmann for fruitful discussions. He would like to thank National Science Council, ROC for financial support, and the Department of Physics at the National Taiwan University for the warm hospitality. This work is supported in part by the National Science Council under Contract No. NSC 82-0208-M-002-012.

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