Comparison between nonlocal effects and coupled channels in a simple nuclear fusion model

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It is discussed how the introduction of a nonlocal potential in the relative coordinate of a nuclear collision described by a simple model Hamiltonian can also account for an enhancement in the nuclear fusion cross section. The interplay between this effect and the channel coupling interaction is studied in this simple model and their different contributions to the fusion cross sections are analyzed.

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I. INTRODUCTION

In the past years much effort has been devoted to the understanding of the fusion cross section in heavy-ion nuclear collisions at energies around and below the barrier [1]. One of the most interesting features that appeared in connection with the measured cross sections was that the prevailing theoretical model of one-dimensional barrier tunneling failed to describe the process [2]. This failure of that simple model has motivated the study of the introduction of the effects coming from the presence of other degrees of freedom, besides that associated with the relative motion of the colliding nuclei, in the calculation of the nuclear fusion cross section. In this connection the importance of considering the colliding nuclei as complex systems with intrinsic structures to which the relative motion must be coupled was soon recognized; this will allow for the nuclear systems to vibrate and exchange particles, for instance, before they fuse. Therefore, in this kind of approach [3–5] the enhancement of the nuclear fusion cross section can be accounted for by considering the change caused by that coupling of the relative motion to the intrinsic structure in the total barrier transmission function.

In a recent paper [6] the authors have drawn attention to still another quantum effect which can also account for an enhancement in the heavy-ion nuclear fusion cross section. By considering the colliding nuclei as complex fermionic structures it is possible to introduce, in a phenomenological way, nonlocal terms of a short range exchange nature which are essential in mean field descriptions of such kinds of systems. Those nonlocal effects are simulated by a Perey-Buck-like nonlocal short range potential [7,8] which, together with the remaining terms of a nuclear Hamiltonian, describes the colliding system. Furthermore, for simplicity, that nonlocal term is treated in an adiabatic approximation and it is shown that, to this order, the effective reduced mass which embodies all the exchange nonlocal effects can account for an enhancement in the transmission factor through the barrier [6,9]. Such an effect has been studied in some cases and the nonlocality range parameter was fixed by fitting the fusion cross section of a system for which the inclusion of all known channels was not able to account for the experimental data [6] . The fitted value, $b = 0.94$ fm, is in agreement with the expected range for the exchange terms, as is well known from mean field calculations [10,11], e.g., the range of nuclear forces, and it is also close to the result obtained by Perey and Buck ($b = 0.85$ fm) in their analysis of nucleon-nucleus scattering [7]. The importance of considering this effect in the fusion calculations was also stressed in that work since nonlocal exchange terms will be always present in the description of collision processes, thus giving a background contribution over which other effects may also appear. In this sense, it was proposed that the starting point for the description of fusion processes in this kind of simple model is not just the one-dimensional barrier transmission as described for instance by Wong's formula [12], but that modified by the inclusion of exchange nonlocal effects.

In the present paper we intend to discuss the interplay between exchange nonlocal effects, introduced in the same phenomenological way as we did before, without discussing the results of some other treatment of nonlocal effects in heavy ions [13,14], and channel couplings in order to study how they compare in the fusion process. To this end we will adopt a schematic model which is able to exhibit the main features of the problem we want to describe, and that has been already discussed in its principal aspects in the literature [4]. This model describes the coupling of the relative motion to a selected intrinsic degree of freedom through a Hamiltonian which allows for a complete diagonalization of the coupling term in the channels space and, thus, adapting it by introducing nonlocal effects in the equivalent Schrödinger equation, we are in a position to discuss the interplay between those effects.

In Sec. II we present the coupled-channels model Hamiltonian which also embodies the nonlocal interaction term and we show that the nonlocal contribution to the final Schrödinger equation is only present in the effective reduced mass of the colliding system. This allows us to use the same method and approximations described by Dasso *et al.* [4] to decouple the equations. When one studies the solutions of the new equations one finds that the main contribution of the nonlocal effects to the transmission factor stems from a modification of the barrier curvature, in contrast to the channel coupling effects whose dominant result consists in altering the barrier height. In Sec. III we discuss the particular case of a two-channel model in order to explicitly and quantitatively compare the contributions of the two effects to the transmission factor and to estimate their roles in nuclear fusion cross sections. Finally in Sec. IV we present our conclusions.

II. COUPLED-CHANNELS AND NONLOCAL EFFECTS

Let us consider a model Hamiltonian, associated to a nuclear system consisting of two colliding nuclei, which is expressed in terms of global collective coordinates (qp) referring to the relative motion and a coordinate ξ characterizing an intrinsic degree of freedom of the nuclei that is coupled to the relative motion. In its general form, such a Hamiltonian can be obtained through the Weyl-Wigner transformation [15] of a nonlocal kernel and is written as

$$H(q,p,\xi) = \delta_{k,\xi} \int e^{\frac{i}{\hbar}pv} H(q,v,k)dv, \tag{1}$$

where

$$H(q,v,k) = \langle q - v/2, k \mid \hat{H} \mid q + v/2, k\rangle \ . \tag{2}$$

This approach of treating the proposed model Hamiltonian is particularly convenient because we want to consider an schematic model for $H(q,v,k)$ which embodies, besides the terms taking into account the coupling of the relative motion to the previously chosen intrinsic degree of freedom, the nonlocal effects stemming from a Perey-Buck-like nucleus-nucleus interacting potential [7]. This term is introduced so as to simulate nonlocal effects originating from quantum correlations, e.g., mainly the nonlocality due to exchange effects [8]. We then write $H(q,v,k)$ in an explicit form as

$$H(q,v,k) = -\frac{\hbar^2}{2\mu}\delta''(v) + V_{\rm NL}(q,v) + V_L(q,v)\delta(v) + V_{cpl}(q,v,k)\delta(v) + H_0(k)\delta(v) \ . \tag{3}$$

Here μ is the reduced mass of the relative motion, $V_{\rm NL}$ is the nonlocal potential written as a Perey-Buck-like interaction [7],

$$V_{\rm NL}(q,v) = \frac{1}{b\sqrt{\pi}} V(q) \exp\left(-\frac{v^2}{b^2}\right), \tag{4}$$

where b, measuring the nonlocal range, will be considered a free parameter, V_L is a local potential, and $H_0(k)$, the intrinsic Hamiltonian of the system, is associated with the eigenvalue problem

$$\hat{H}_0 \mid k\rangle = \epsilon_k \mid k\rangle, \tag{5}$$

with eigenvectors $\mid k\rangle$ characterizing the spectrum of the selected intrinsic degree of freedom. The term $V_{cpl}(q,v,k)$ represents the interaction coupling the relative motion and the intrinsic degree of freedom and will be treated below in a similar fashion as that discussed by Dasso *et al.* [4]. As has been already shown [9], it is possible to write the Weyl-Wigner mapped expression for that Hamiltonian as

$$H(q,p,\xi) = \frac{p^2}{2\mu} + \sum_{n=0}^{\infty} \frac{(-1)^n}{\hbar^n} p^n V^{(n)}(q) + V_L(q) + V_{cpl}(q,\xi) + H_0(\xi) \ , \tag{6}$$

where $V^{(n)}(q)$ is the nth moment of the nonlocal potential with respect to v.

In the power series term, only even values of n will be present in our expression; otherwise the Hamiltonian would be dissipative, and furthermore we will consider that the first two terms already give the dominant contribution to the model Hamiltonian. This assumption corresponds to an adiabatic approximation and is valid for $pb/\hbar < 1$. Thus, up to $n = 2$, we have

$$H(q,p,\xi) \simeq \frac{p^2}{2\mu(q;b)} + V^{(0)}(q + V_L(q)) + V_{cpl}(q,\xi) + H_0(\xi) \ , \tag{7}$$

where

$$\mu(q;b) = \mu \Big/ \left(1 - \frac{\mu}{\hbar^2} V^{(2)}(q)\right) = \mu \Big/ \left(1 - \frac{\mu b^2}{2\hbar^2} V^{(0)}(q)\right) \tag{8}$$

is an effective reduced mass, similar to that of Frahn and Lemmer [16] which depends now on the nonlocality range. Hereafter $V^{(0)}(q)$, the zeroth moment of the nonlocal potential, will be identified as a standard nucleus-nucleus attractive potential $V_N(q)$ and $V_L(q)$ as the Coulomb interaction $V_C(q)$. It is worth noting that $\mu(q;b) \to \mu$ for $V^{(2)}(q) \to 0$ or $b \to 0$, this being the local limit [9,6] which also defines the asymptotic behavior of the mass $(q \to \infty)$. From Eq. (8) we see also that this behavior of the effective reduced mass indicates that the considered quantum exchange effects between the two colliding nuclei will be relevant only around the nonlocality range.

The extraction of the operator $\hat{H}$ corresponding to $H(q,p,\xi)$, Eq. (7), follows the standard technique of the Weyl-Wigner quantum phase space formalism [17], leading to

$$\hat{H}(\hat{q},\hat{p},\hat{\xi}) = \frac{1}{4}\left\{\frac{1}{2\mu(\hat{q};b)}\hat{p}^2 + \hat{p}\frac{1}{\mu(\hat{q};b)}\hat{p} + \hat{p}^2\frac{1}{2\mu(\hat{q};b)}\right\} + V(\hat{q}) + V_{cpl}(\hat{q},\hat{\xi}) + H_0(\hat{\xi}) \ , \tag{9}$$

where

$$V(\hat{q}) = V_N(\hat{q}) + V_C(\hat{q}) \ . \tag{10}$$

This Hamiltonian operator gives rise to a one-dimensional Schrödinger equation embodying now the nonlocal effects

$$\left\{\frac{-\hbar^2}{2\mu(x;b)}\frac{d^2}{dx^2}-\frac{\hbar^2}{2}\left[\frac{d}{dx}\frac{1}{\mu(x;b)}\right]\frac{d}{dx}-\frac{\hbar^2}{8}\left[\frac{d^2}{dx^2}\frac{1}{\mu(x;b)}\right]\right\}\Psi(x)=\{E-[V(x)+V_{cpl}(x,\xi)+H_0(\xi)]\}\Psi(x)\ . \quad (11)$$

Hereafter we will denote

$$H_{\rm NL}(x;b)=-\frac{\hbar^2}{2}\left[\frac{d}{dx}\frac{1}{\mu(x;b)}\right]\frac{d}{dx}-\frac{\hbar^2}{8}\left[\frac{d^2}{dx^2}\frac{1}{\mu(x;b)}\right]\ . \quad (12)$$

To solve Eq. (11) one assumes the total wave functions to be expanded as

$$\Psi(x)=\sum_\alpha u_\alpha(x)\mid\alpha\rangle\ , \quad (13)$$

where α refers to the intrinsic states, so that one ends up with the set of coupled equations

$$\left[-\frac{\hbar^2}{2\mu(x;b)}\frac{d^2}{dx^2}+H_{\rm NL}(x;b)+V(x)-E\right]u_\alpha(x) = -\sum_\beta\left[\epsilon_\alpha\delta_{\alpha\beta}+\langle\alpha\mid V_{cpl}(x,\xi)\mid\beta\rangle\right]u_\beta(x). \quad (14)$$

The right-hand side (rhs) of this equation contains the terms coupling the relative motion and the selected intrinsic degree of freedom as usual, while the lhs, bearing the information concerning the relative motion, will account for the transmission factor through the barrier described by the total model potential including the nonlocal and the coupling effects coming from the diagonalization. It is then clear that this transmission factor will be modified also by the presence of the effective reduced mass and $H_{\rm NL}(x;b)$ terms which take into account the nonlocal effects, up to p^2 terms. Furthermore, it has been already verified that this inclusion can give rise to an enhancement of the nuclear fusion cross section [6].

Now, we want to find the solutions of that set of coupled equations so that they have the asymptotic behavior (for short range nonlocal potentials)

$$u_\alpha(x)\to\begin{cases}\delta_{\alpha0}e^{-ik_\alpha x}+r_\alpha e^{ik_\alpha x} & x\to\infty,\\ t_\alpha e^{-ik_\alpha x} & x\to-\infty,\end{cases} \quad (15)$$

where $\hbar^2k_\alpha^2/2\mu=E-\epsilon_\alpha$ and the colliding nuclei are considered to be in their ground states. Furthermore, we will also use the simplifying assumption that $V_{cpl}(x,\xi)$ factorizes into a product of two terms, one describing the coupling potential for the relative motion and the other for the intrinsic degree of freedom, respectively, and we will additionally consider the potential associated with the relative motion to be represented by its value at the barrier position, F, for all channels α [4]. Thus we see that, under these hypotheses, the equations can be decoupled and that this can be accomplished by diagonalizing the matrix

$$M_{\alpha\beta}\equiv\langle\alpha\mid H_0(\xi)+V_{cpl}(x,\xi)\mid\beta\rangle=\epsilon_\alpha\delta_{\alpha\beta}+FV_{\alpha\beta}, \quad (16)$$

which gives the eigenvalues λ_β of $H_0(\xi)+V_{cpl}(x,\xi)$. The new uncoupled Schrödinger equations with nonlocal effects are

$$\left[-\frac{\hbar^2}{2\mu(x;b)}\frac{d^2}{dx^2}+H_{\rm NL}(x;b)+V(x)+\lambda_\beta-E\right]v_\beta(x) =0\ , \quad (17)$$

being the new solution $v_\beta(x)$ related to the $u_\alpha(x)$ by the matrix which diagonalizes the matrix $M_{\alpha\beta}$.

Since we are concerned with the asymptotic behavior of the solution of Eq. (17), we are allowed to also assume, for large incident energies compared to the intrinsic energies and coupling strengths, Eq. (15) to be valid with $\hbar^2k^2/2\mu=E$. The transmission factor is then written as a sum of the contributions coming from all channels β, each with a weight associated with the overlap of the initial state with the corresponding eigenstates of the matrix M. The total transmission coefficient associated with Eq. (17) is then written in the form

$$T=\sum_\beta\mid\langle0\mid\beta\rangle\mid^2\mid t_\beta\mid^2 =\sum_\beta\mid\langle0\mid\beta\rangle\mid^2 T(E,V(x)+\lambda_\beta;b)\ , \quad (18)$$

and from this expression it is worth noting that the nonlocal effects present in the relative motion give a contribution to the transmission coefficient of a different character as that coming from λ_β, which is associated to the coupled-channels effects allowed by the model. The effect of the channel coupling corresponds, as usual [4], to replacing the barrier $V(x)$ by a family of barriers $V(x)+\lambda_\beta$, being the total transmission factor given by the sum over the transmission coefficients calculated for each barrier in the family weighted by the overlap factors $\mid\langle0\mid\beta\rangle\mid^2$. In the present simple model, the nonlocality manifests itself in the total transmission coefficient only through the effective mass and $H_{\rm NL}(x;b)$ terms which will modify $T(E,V(x)+\lambda_\beta;b)$ for each channel because, due to our assumption on the x independence of $M(x,\xi)$, ansatz (16), the overlap factor of the intrinsic states is not affected by the nonlocality. Although in realistic situations we expect much more involved expressions mixing the channel coupling and the nonlocal effects, we already expect the present approach to exhibit the essential features relevant to the understanding of the competition between the two effects for the fusion processes.

In order to fully calculate the coefficients $T(E,V(x)+\lambda_\beta;b)$ we would have to solve Eq. (14), being the general solution obtainable, in principle at least, by numerical procedure. But these coefficients have already been obtained through the use of the Weyl-Wigner quantum phase space formalism in the Feynman path integral approach [9], and are written for the case described by Eq. (17) as

$$T(E, V(x) + \lambda_\beta; b) = \left\{ 1 + \exp\left[2 \int_{x_1}^{x_2} \sqrt{\frac{2\mu(x;b)}{\hbar^2}\,[V(x) + \lambda_\beta - E]}\, dx \right] \right\}^{-1} . \quad (19)$$

Now, as is well known from momentum-dependent microscopic interaction calculations [10,11], the effective mass depends on the density function of the system. Here, for simplicity, we will assume a geometrical model for the effective reduced mass in which it changes with the density distribution profile of the colliding nuclei only at the radius of the barrier, R_B, defined by the total potential, i.e.,

$$\mu(x;b) = \begin{cases} \mu, & x > R_B, \\ \mu \Big/ \left(1 + \frac{\mu b^2}{2\hbar^2} \mid V^{(0)}(R_B) \mid \right), & 0 \le x \le R_B . \end{cases} \quad (20)$$

Furthermore, if one works with a parabolic approximation for $V(x)$ [18] and with the assumption that $(\mu b^2/2\hbar^2) \mid V^{(0)}(R_B) \mid$ produces only a small change in the reduced mass at the barrier, then we get

$$T(E, V(x) + \lambda_\beta; b) = \left\{ 1 + \exp\left[\frac{2\pi\,(V_B + \lambda_\beta - E)}{\hbar\omega_B} D(b, R_B) \right] \right\}^{-1} , \quad (21)$$

where V_B , R_B, and $\hbar\omega_B$ are the height, position, and curvature of the effective barrier, respectively, and

$$D(b, R_B) = 1 - b^2 \frac{f(R_B)}{4}, \quad (22)$$

with

$$f(R_B) = \frac{\mu}{2\hbar^2} \mid V_N(R_B) \mid . \quad (23)$$

It is important to notice that the factor $D(b, R_B)$ in Eq. (21), coming from the effective reduced mass, renormalizes the curvature of the barrier $\hbar\omega_B$, keeping unaltered the barrier height. It is interesting to observe that a similar result was already obtained by Frahn and Lemmer in their study of a particle bound in a harmonic oscillator potential [16]. In this way, it is clearly seen in this schematic model encompassing nonlocal effects, besides channel couplings, that the barrier is then modified in its curvature, in addition to the shift in its height, for each channel. Both these effects are of short range character and reveal the essential role of the fermionic structure of the colliding nuclei in the description of the barrier dynamics during the fusion process.

III. TWO-CHANNEL MODEL

A simplified version of the model discussed in the previous section, i.e., the two-channel model, can be now studied which is still of interest because not only can it simulate the coupling to a harmonic mode, representing a nuclear collective excitation in its weak limit, but also because it could be used for describing particle transfer channels [19]. In this connection, in the standard treatment [4] the coupling matrix $M_{\alpha\beta}$ is written as

$$M = \begin{pmatrix} 0 & F \\ F & -Q \end{pmatrix} , \quad (24)$$

where Q is the reaction Q value so that it can be directly diagonalized, thus giving the eigenvalues

$$\lambda_\pm = \frac{1}{2}\left[-Q \pm \left(4F^2 + Q^2\right)^{\frac{1}{2}} \right] . \quad (25)$$

The overlap probabilities corresponding to the eigenvalues are

$$P_\pm = \mid \langle 0 \mid \pm \rangle \mid^2 = \frac{F^2}{F^2 + \lambda_\pm^2}, \quad (26)$$

and making use of Eqs. (18), (21), and (26), we can write the total transmission function, namely,

$$\begin{aligned} T(E, V(x) + \lambda_\pm; b) &= P_+ \left\{ 1 + \exp\left[\frac{2\pi(V_B + \lambda_+ - E)}{\hbar\omega_B} D(b, R_B) \right] \right\}^{-1} \\ &+ P_- \left\{ 1 + \exp\left[\frac{2\pi(V_B + \lambda_- - E)}{\hbar\omega_B} D(b, R_B) \right] \right\}^{-1} . \end{aligned} \quad (27)$$

It is clear from this expression that the final contributions of the two basic distinct effects present in the model Hamiltonian to the transmission factor are of different character. While the channel coupling, in this version of the model, gives rise to two barrier heights ($V_B + \lambda_\pm$) with their corresponding weights, $P_\pm$, the nonlocality renormalizes the curvature of the unshifted barrier

$$\hbar\tilde{\omega}_B = \frac{\hbar\omega_B}{D(b, R_B)} . \quad (28)$$

It is also remarkable that this renormalization factor bears essential information related to the attractive nuclear potential at the barrier radius, Eq. (23), besides carrying the explicit nonlocality range dependence.

In order to show the interplay between nonlocal and channel coupling effects, we have chosen some values for the coupling strength F and the Q-value and calculated the transmission function, Eq. (27), being the parameters related to the barrier of a ^{58}Ni+^{58}Ni system. The parameters used were $Q = -1.454$ MeV, $F = -2.7$ MeV, so that we simulate at least the coupling to the ^{58}Ni first 2^+ state as discussed by Landowne and Pieper [20], $V_B = 100$ MeV, $\hbar\omega_B = 4$ MeV, and the results are shown in Fig. 1. To calculate $f(R_B)$ we assumed the Christensen-Winther potential [21] to describe the tail of the nucleus-nucleus potential and we obtain the value $f(R_B) = 1.75$ fm^{-2} with $R_B = 11.0$ fm. The range of the nonlocality is assumed to be $b = 0.94$ fm, that is, associated with pure nonlocal effects [6]. The results of this schematic model which embodies nonlocality besides coupling channels present a larger enhancement, in energies below the barrier V_B, than that obtained through the coupled-channels calculation alone. It is important to stress that in the local limit, $b \to 0$, the expression above gives back the usual expressions for the two-channel problem .

If we want to use the expressions for the transmission function, adapted to the case of a $l = 0$ partial wave, in the standard definition of the fusion cross section for all those cases for which the present model can be applied, we need to extend that expression for $l \neq 0$. This can be accomplished by the introduction of the simple assumption that the main effect arises from the standard centrifugal potential [22,6]. Thus, we see that, using the set of approximations

$$V_B \simeq V_0 + \frac{\hbar^2 l(l+1)}{2\mu R_0^2}, \tag{29}$$

$$\hbar\omega_B \simeq \hbar\omega_0, \tag{30}$$

$$R_B \simeq R_0 \tag{31}$$

[12] in Eq. (27), the final result is

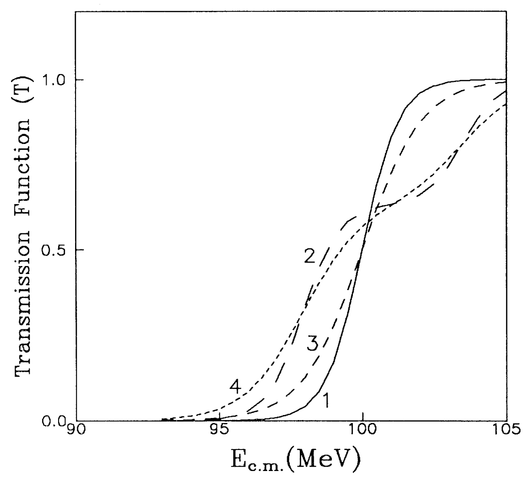

FIG. 1. The transmission function, Eq. (27), calculated for a ^{58}Ni+^{58}Ni system. Curve 1 describes the local no-coupling calculation, curve 2 corresponds to the coupled-channels calculation for the first 2^+ state, curve 3 depicts the pure nonlocal effects, and curve 4 represents the summed channel coupling and nonlocal effects.

$$\sigma_f^{\rm NLC}(E) = P_+ \frac{R_0^2\hbar\omega_0}{2ED(b,R_0)} \ln\left\{1 + \exp\left[\frac{2\pi\left[E-(V_0+\lambda_+)\right]}{\hbar\omega_0} D(b,R_0)\right]\right\} + P_- \frac{R_0^2\hbar\omega_0}{2ED(b,R_0)} \ln\left\{1 + \exp\left[\frac{2\pi\left[E-(V_0+\lambda_-)\right]}{\hbar\omega_0} D(b,R_0)\right]\right\}, \tag{32}$$

where V_0, R_0, and $\hbar\omega_0$ are the height, position, and curvature of the Coulombic barrier, respectively, for $l = 0$.

In order to show the summed result of the nonlocal and channel coupling effects in the fusion cross section, we have also calculated Eq. (32) for the case of a ^{58}Ni+^{58}Ni collision. Albeit the parameters F and Q previously chosen may not give a complete description of the fusion process in that reaction, the results already show some interesting features. It is worth noting that $\sigma_f^{\rm NLC}(E)$ is larger than the fusion cross section calculated with the channel coupling effects only at energies below and around the barrier and that they are identical at energies above the barrier (Fig. 2), this being an interesting feature of the fusion cross section $\sigma_f^{\rm NLC}$ due to the presence of the factor $D(b, R_0)$ in the exponential argument as well as in the denominator of the prelogarithm terms of Eq. (32). While this factor enhances the fusion cross section for energies below or around the barrier by redefining its curvature, it also has the correct behavior so as to give the expected trend for higher energies.

IV. CONCLUSIONS

We have discussed in this paper the interplay between nonlocal effects and channel couplings in a model Hamiltonian describing a nuclear collision and we have shown how they produce enhancements in the nuclear fusion cross section. To this end, and for the sake of simplicity, we have started from a model Hamiltonian which has the channel coupling terms as described by Dasso *et al.* [4] besides a nonlocal potential. This choice enabled us to discuss, in its dominant aspects, the two contribu-

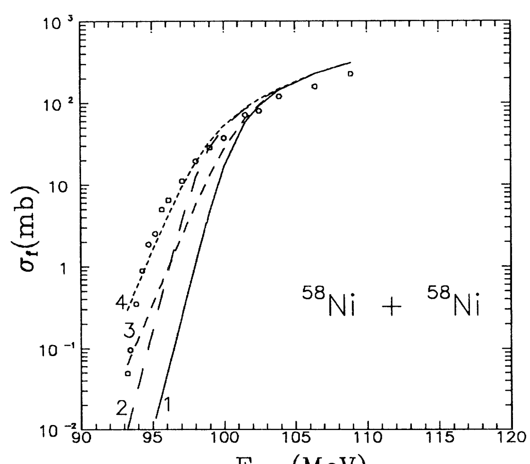

FIG. 2. The fusion cross section for a ^{58}Ni+^{58}Ni system. Curve 1 describes the local no coupling calculation, curve 2 is the result for the first 2^+ state coupled-channels calculation, curve 3 represents the pure nonlocal result, and curve 4 gives the summed channel coupling and nonlocal effects. Open circles are the experimental data.

tions for the nuclear fusion cross section, this approach being therefore at least a guide for the understanding of the main features present in realistic nuclear collision processes. The nonlocal contribution manifests itself through the effective reduced mass, thus allowing for the diagonalization of the channel coupling interaction in a standard form. This leads to a new equation in which, besides the family of shifted potentials generated by the channel coupling, the effective reduced mass will also be responsible for a modified transmission function, which has been already calculated by the authors in another paper [6]. The final expression for the transmission function exhibits two separated contributions, namely, the channel coupling effects which modify the barrier height through the addition of the eigenvalues of the diagonalization procedure in the channel space and the redefinition of the barrier curvature induced by the nonlocal effects embodied in the effective reduced mass. In fact we must expect a more entangled expression in realistic situations in which both effects are mixed, but here, due to the assumption of constant coupling strength, there occurs a complete separation between channel coupling and nonlocal effects. Nevertheless, the present approach is expected to describe the main contributions coming from those two effects.

In order to show quantitatively how nonlocal effects contribute to the enhancement of the channel coupling calculated transmission function, we have adopted the nuclear data from a ^{58}Ni+^{58}Ni collision and performed the calculations with a fixed value of the nonlocality range, as previously determined by the authors [6], and a Christensen-Winther nucleus-nucleus potential [21] for some specific cases of channel coupling parameters. As an illustration we have also used the same schematic model of channel coupling interaction to study the nuclear fusion cross section in those cases, in order to show their behavior when nonlocal effects are included. The results, although not completely realistic because of the simple modelistic treatment of the channel coupling interaction, already points to the importance of the inclusion both of nonlocal as well as channel coupling terms.

Finally, we want to emphasize the importance of a more realistic calculation to study the role of nonlocal effects in fusion processes as well as in elastic scattering within this framework. To this end we have to solve a general Schrödinger equation with a Perey-Buck-like nonlocal potential [7] and look for nonlocal effects in the relevant cross sections. This analysis is in progress and will be reported in another paper.

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