

## Study of $^{110}\text{Cd}$ from the $^{110}\text{In}^m$ $\beta$ decay

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The  $\gamma$ -ray transitions following the  $\beta$  decay of  $^{110}\text{In}^m$  (half-life 69 min) were observed using Compton-suppressed Ge detectors. The activity was produced by the  $^{111}\text{Cd}(p, 2n)$  reaction at a bombarding energy of 18.9 MeV. Singles in multispectra mode and  $\gamma\gamma$ -coincidence experiments were performed. Approximately 150 transitions were assigned to  $^{110}\text{Cd}$ , based on their measured half-life and/or observation in coincidence with well-known lines. The  $\log_{10}(ft)$  values of nearly all observed levels were determined and an analysis of the beta-decay strength using microscopic theory was performed. The experimental data were interpreted in the framework of the interacting boson model.

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### I. INTRODUCTION

The even cadmium nuclei have been studied by many different groups since they appeared as typical examples for the application of nuclear models. The  $\beta$  decay of  $^{110}\text{In}$  presents an excellent way to get information about low-energy levels (0–3.5 MeV) of  $^{110}\text{Cd}$  because the  $Q$  value of  $^{110}\text{In} \rightarrow ^{110}\text{Cd}$  is 3.94 MeV. The  $^{110}\text{In}^g$  decay ( $T_{1/2} = 4.9$  h,  $I^\pi = 7^+$ ) and the  $^{110}\text{In}^m$  decay ( $T_{1/2} = 69$  min,  $I^\pi = 2^+$ ) have both been observed in the present study but only the latter showed interesting results and is reported. The  $^{110}\text{In}^m$  decay had already been observed by Sarantites *et al.* [1] and Meyer and Van Hise [2]. Because better experimental techniques are now available, this decay has been remeasured in order to improve the  $^{110}\text{Cd}$  level scheme at low spins and to resolve some open problems like the questionable existence of the 1809 keV level. This existence (or inexistence) is of importance to avoid a bias in the comparison of experimental results and model calculations.

In Sec. II the experimental procedure and the results are presented. Section III reports on the model-independent level scheme construction, Sec. IV on the quasiparticle random-phase approximation (QRPA) framework for the beta-decay observables and its results, and in Sec. V the interpretation of the results in the framework of the interacting boson model (IBM) is presented.

### II. EXPERIMENTAL PROCEDURES AND RESULTS

The  $^{110}\text{In}$  activity was produced by the  $^{111}\text{Cd}(p, 2n\gamma)$  reaction using beams of 18.9 MeV protons, obtained from the Philips variable energy cyclotron at the Paul Scherrer Institute (PSI) at Villigen, Switzerland. Three self-supporting targets of about 15 mg/cm<sup>2</sup>, enriched to 95%  $^{111}\text{Cd}$ , were used.

#### A. Singles measurement

The singles spectra were measured with two different detectors placed at a distance of 12 cm from the target: a Compton-suppressed Ge detector with a volume of 90 cm<sup>3</sup> and a resolution of 1.0 keV at 121 keV measured energies between 50 and 1800 keV; a second detector, a high purity Ge *n*-type detector, had a volume of 98 cm<sup>3</sup> and a resolution of 2.2 keV at 1400 keV and was used to observe the  $\gamma$ -ray spectrum between 400 and 3800 keV. The three  $^{111}\text{Cd}$  targets were alternately and repeatedly irradiated for 5–10 min, then each singles measurement was divided into four bins of 15 min each. The irradiation/measurement cycle was repeated seven times. Since  $^{110}\text{In}^g$  (half-life 4.9 h) is also produced during the irradiation, an identification by determination of the half-life of each line was performed. In addition, the irradiated targets were observed several hours after the end of the above described experiments in order to subtract properly the long-half-life components for lines appearing in the two decays and to complement the identifications based on half-lives for weak transitions. The energy and relative efficiency calibrations for both detectors were performed with  $^{152}\text{Eu}$  and  $^{88}\text{Y}$  sources placed at the tar-

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get position, using intensities and energies proposed by Lederer *et al.* [3]. All the  $\gamma$  rays were normalized to an intensity of 100 for the 657.8 keV transition (see Table I).

### B. Coincidence measurement

Twenty measurement cycles, which consisted of target irradiation for 10 min followed by coincidences measured for 1 h, were performed. Four Ge detectors, three of which were Compton suppressed, with volumes between 65 and 127 cm<sup>3</sup> and placed 12 cm from the target, were used. The fast-slow coincidence circuits had an overall time resolution of about 12 ns. The data were stored event by event on magnetic tape. The size of the coincidence matrix was 2048  $\times$  2048 and about  $3.5 \times 10^6$  events

were stored. The reconstruction of the different coincidence spectra was performed off line. Selected results for a few coincidence gates are given in Table II and some examples of coincidence spectra are shown in Fig. 1.

### III. DISCUSSION OF THE LEVEL SCHEME

The <sup>110</sup>Cd level scheme was extended primarily through the use of coincidence relations. A few transitions assigned on the basis of other evidence are noted in Table I with appropriate comments. Only the levels populated in the present experiment are presented in Table III and in Fig. 2. To assign spins and parities, the selection rules for the  $\beta$  decay [ $\log_{10}(ft)$  values], and for  $\gamma$  transitions, are used. The  $\log_{10}(ft)$  values of the transitions are discussed in Sec. IV.

TABLE I. Summary of the  $\gamma$ -ray data obtained from the <sup>110</sup>In<sup>m</sup> 2<sup>+</sup> decay.

$E_\gamma$ [keV]	$\Delta E_\gamma$ [keV]	I	$\Delta I$	$E_i - E_f$ Assignments	Comments	$E_\gamma$ [keV]	$\Delta E_\gamma$ [keV]	I	$\Delta I$	$E_i - E_f$ Assignments	Comments
295.38	0.05	0.052	0.003	2078.9 - 1783.5	a,b	1775.25	0.28	0.014	0.003	2433.1 - 657.8	c
338.24	0.11	0.025	0.004		a,d	1783.47	0.04	0.300	0.020	1783.5 - 0.0	c,e
416.50	0.11	0.027	0.004		a,d	1851.15	0.13	0.046	0.004	3634.7 - 1783.5	a,b
603.06	0.05	0.090	0.004	2078.9 - 1475.8	a,b	1975.20	0.05	0.144	0.004	2633.0 - 657.8	a,b
657.75	0.05	100.000	4.200	657.8 - 0.0	a,b	2002.37	0.05	0.132	0.003	3475.4 - 1473.1	a,b
686.92	0.09	0.111	0.009	2162.8 - 1475.8	b	2129.40	0.03	2.200	0.031	2787.3 - 657.8	a,b
790.81	0.18	0.018	0.003	3078.4 - 2287.4	c	2211.33	0.03	1.776	0.027	2869.2 - 657.8	a,b
815.31	0.04	0.304	0.013	1473.1 - 657.8	a,b	2243.30	0.10	0.042	0.003		a,d
818.05	0.03	0.890	0.110	1475.8 - 657.8	a,b	2259.38	0.10	0.045	0.003		a,d
884.70	0.04	0.160	0.080	1542.5 - 657.8	b	2317.41	0.04	1.315	0.022	2975.3 - 657.8	a,b
957.30	0.18	0.030	0.005	2433.1 - 1475.8	a,c,f	2420.51	0.04	0.535	0.010	3078.4 - 657.8	a,b
958.56	0.05	0.069	0.005	3314.4 - 2355.5	a,b	2444.05	0.04	0.299	0.006	3101.9 - 657.8	a,b
1001.71	0.06	0.050	0.005	2477.4 - 1475.8	a,b	2477.16	0.08	0.051	0.002	2477.2 - 0.0	a,c
1023.05	0.05	0.064	0.012	3101.9 - 2078.9	a,b	2535.55	0.04	0.219	0.005	3193.4 - 657.8	a,b
1073.55	0.04	0.106	0.005	1731.3 - 657.8	a,b	2598.55	0.13	0.026	0.002		a,d
1085.57	0.04	0.039	0.012	2869.2 - 1783.5	a,b	2656.55	0.04	0.406	0.009	3314.4 - 657.8	a,b
1125.77	0.03	1.060	0.060	1783.5 - 657.8	b	2745.45	0.06	0.090	0.003	3403.3 - 657.8	a,b
1151.70	0.06	0.050	0.004	3314.4 - 2162.8	a,b	2787.30	0.07	0.072	0.003	2787.3 - 0.0	a,c
1235.67	0.04	0.297	0.008	3314.4 - 2078.9	a,b	2808.59	0.04	0.519	0.013	3466.8 - 657.8	a,b
1314.25	0.10	0.042	0.003	2787.2 - 1473.1	c	2817.61	0.07	0.058	0.003	3475.4 - 657.8	a,b
1344.88	0.15	0.019	0.004	3128.4 - 1783.5	a,c	2869.28	0.10	0.033	0.002	2869.2 - 0.0	a,c
1387.22	0.07	0.033	0.004		a,d	2891.88	0.06	0.084	0.003		a,d
1393.63	0.07	0.054	0.004	2869.2 - 1475.8	a,b	2939.44	0.25	0.012	0.002		a,d
1410.08	0.08	0.033	0.004	3193.4 - 1783.5	a,b	2975.29	0.06	0.112	0.004	2975.3 - 0.0	a,c
1421.10	0.04	0.460	0.040	2078.9 - 657.8	a,b	3043.97	0.05	0.126	0.004		a,d
1475.76	0.03	0.420	0.060	1475.8 - 0.0	a,b	3059.20	0.15	0.017	0.002		a,d
1505.03	0.04	0.110	0.030	2162.8 - 657.8	b	3078.42	0.04	0.265	0.008	3078.4 - 0.0	a,c
1555.76	0.21	0.013	0.004	3634.7 - 2078.9	c	3102.00	0.18	0.005	0.001	3101.9 - 0.0	a,c
1583.18	0.20	0.013	0.005	3314.4 - 1731.3	a,b	3128.25	0.10	0.030	0.002	3128.4 - 0.0	a,c
1602.57	0.04	0.123	0.004	3078.4 - 1475.8	a,b	3280.85	0.10	0.033	0.002		a,d
1626.17	0.06	0.055	0.004	3101.9 - 1475.8	b	3403.48	0.15	0.022	0.002	3403.3 - 0.0	a,c
1629.62	0.05	0.085	0.005	2287.4 - 657.8	a,b	3467.14	0.49	0.003	0.001		a,d
1652.70	0.09	0.028	0.003	3128.4 - 1475.8	a,c	3475.34	0.03	0.590	0.019	3475.4 - 0.0	a,c
1666.23	0.07	0.042	0.003	3208.8 - 1542.5	a,b	3596.99	0.04	0.114	0.004		a,d
1697.97	0.04	0.260	0.030	2355.8 - 657.8	b	3726.51	0.18	0.004	0.001	3726.6 - 0.0	a,c
1717.70	0.10	0.030	0.003	3193.4 - 1475.8	a,c	3771.70	0.04	0.062	0.002	3771.8 - 0.0	a,c
1744.10	0.07	0.050	0.003	3475.4 - 1731.3	a,b						

<sup>a</sup>Identified with the multispectra method (otherwise identification by coincidence with a well known transition).

<sup>b</sup>Placed in the level scheme by coincidences.

<sup>c</sup>Placed in the level scheme by the Ritz principle.

<sup>d</sup>Not placed into the level scheme.

<sup>e</sup>Observed in other studies of <sup>110</sup>Cd and placed at the same place in the level scheme.

<sup>f</sup>Transition was unresolved from a much stronger  $\gamma$ -ray at approximately the same energy. Energy has been taken as the difference between the level energies. Intensity has been determined by the fit program.

### Discussion level by level

The discussion below is limited to levels which have not been observed before, to those which were expected to be observed but were not, and to those for which the population or depopulation mode has been revised with respect to previous works. Table III summarizes the results on all observed levels.

**1809.5 keV level.** This level was observed in a  $(p, p')$  experiment [4]. Sarantites *et al.* [1] in their study of the  $^{110}\text{In}^m$   $\beta$  decay suggested that this level would depopulate by a 1151.5(8) keV transition. A weak transition with the energy 1151.9 keV was observed by Kern *et al.* [5] in the study of the  $^{108}\text{Pd}(\alpha, 2n)^{110}\text{Cd}$  reaction. Due to the very good energy correspondence, it was identified as the former. A clear spin assignment was not possible. Later, Kumpulainen *et al.* [6] studied the same reaction at a lower  $\alpha$ -beam energy and concluded that the transition belonged in  $^{111}\text{Cd}$ . Wesseling *et al.* [7] confirmed the existence of the 1809 keV level in their study of  $^{110}\text{Cd}$  by  $(e, e')$  inelastic scattering and proposed an  $I^\pi$  value of  $4^+$ , although  $2^+$  or  $3^+$  could not be excluded. However, it is very unlikely that a level with spin 4 could be significantly populated in the  $^{110}\text{In}^m$   $\beta$  decay. Finally, the level

was not observed by Pignanelli *et al.* [8] in their study of  $^{110}\text{Cd}$  by the  $(p, p')$  and  $(d, d')$  reactions. The existence (or inexistence) of this level is of importance to compare and identify calculated and experimental results. In the present work, a 1151.70(6) keV transition, with a well defined half-life of 69 min, was observed in coincidence with the 1505.0 keV line and probably with the 686.6 keV line (see Table II and Fig. 1), indicating that the 1151.7 keV transition populates the level at 2162.8 keV and not directly the 657.8 keV level. Thus there is no evidence of a level at 1809 keV in the present work or in the more recent  $(p, p')$  and  $(d, d')$  study by Pignanelli *et al.* [8]. In agreement with the result of a discussion on the completeness properties of the  $(\alpha, 2n\gamma)$  reaction [9] it is therefore concluded that this level does not exist.

**2078.8 keV level.** A  $3^-$  level at 2078.8 keV was found by Sarantites *et al.* [1]. It decays by two transitions of 1421.4 and 603.6 keV. Later on, measuring the  $^{110}\text{Ag}$  decay, Kawase *et al.* [10] assigned a level with  $I^\pi = 0^+$  having nearly the same energy, decaying via a 295.3 keV transition. The coincidence relations of the 295.38(5), 603.06(5), and 1421.10(4) keV  $\gamma$  rays (see Table II) confirm the proposal of Kern *et al.* [5] that the  $3^-$  level is depopulated by the 295.4 keV transition as well.

TABLE II. Selected coincidence gates.

gate	display (a)
295.4	657.8, 1125.8, 1235.7
396.8	244.7, 648.5, <b>657.8</b> , <b>884.7</b> , <b>937.5</b> , 1421.1
581.9	461.8, <b>657.8</b> , <b>884.7</b> , (937.5), <b>997.3</b> , 1421.1
584.2	<b>657.8</b> , <b>884.7</b> , <b>937.5</b>
603.0	818.1, 1235.7, 1475.8, (2129.4)
626.3	187.3, <b>244.7</b> , 467.2, 648.5, <b>657.8</b> , <b>708.2</b> , (759.9), <b>884.7</b>
641.7	<b>657.8</b> , <b>884.7</b> , <b>937.5</b>
686.9	(581.9), 657.8, 818.1, (884.7), (937.5), (1151.7), 1475.8
707.4	229.2, 409.6, 461.8, 626.3, <b>657.8</b> , (708.2), <b>884.7</b> , <b>937.5</b> , 1475.8, (1542.0)
759.9	<b>657.8</b> , <b>884.7</b> , <b>937.5</b>
901.6	<b>657.8</b> , <b>677.6</b> , 744.3, 818.1, 884.7, 1475.8, 1562.4
937.5	(461.8), 584.2, <b>641.8</b> , <b>657.8</b> , <b>707.4</b> , 759.9, 884.7, 1045.4
958.6	657.8, (884.7), 1698.0
1001.7	818.1, 1475.8
1045.4	<b>657.8</b> , <b>708.2</b> , <b>884.7</b> , <b>937.5</b>
1125.8	295.4, 467.2, 626.3, <b>657.8</b> , (935.6), 1085.6, (1410.1), 1851.2
1151.7	<b>657.8</b> , (686.9), 1505.0
1163.3	<b>657.8</b> , <b>884.7</b>
1235.7	603.1, 657.8, 1421.1
1299.4	657.8, 884.7
1421.1	120.3, 461.8, 581.9, <b>657.8</b> , 1023.1, <b>1235.7</b> , 1387.2
1505.0	<b>657.8</b> , 763.9, 844.1, 937.5, 1151.7
1583.2	(884.7), 1073.6
1626.2	657.8, 818.1, 1475.8
1629.6	657.8
1666.2	657.8, 884.7
1698.0	<b>657.8</b> , 884.7, 958.6
1802.5	657.8, 884.7
1851.2	657.8, (884.7), 1125.8
1975.2	<b>657.8</b>

<sup>a</sup>A semiquantitative judgment about the strength of coincidences is indicated: parentheses indicate questionable evidence and bold print means the signal is strong or very strong. In many cases the gates have been set in various, slightly different ways. The table summarizes the obtained results.

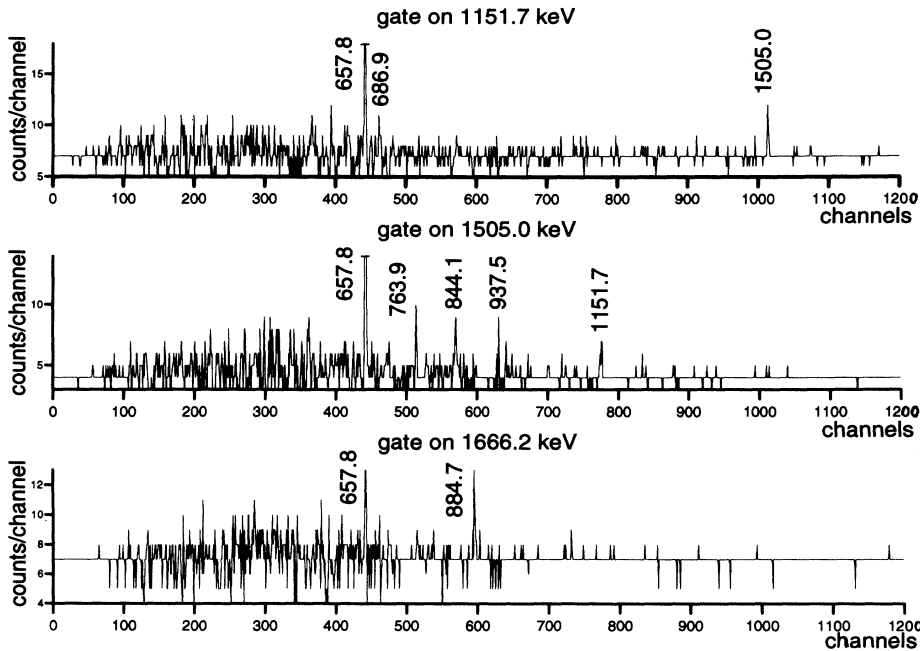


FIG. 1. Selected coincidence gates supporting the new placements of the 1151 keV and the 1666 keV transitions.

*2355.8 keV level.* The level energy has been changed from 2356.38 to 2355.81(5) keV. The spin value can only be limited to the range 1–3.

*2433.1 keV level.* Kern *et al.* [5], Araddad *et al.* [11], and Kiang *et al.* [12] observed this level to which they assigned the spin 3. This assignment is supported by the population from the  $5^+$  level at 2926.8 keV [10,12] and by the slope and side feeding intensity measured in the  $(\alpha, 2n\gamma)$  reaction [5]. The assignment of spin 3 is at variance with that of Blasi *et al.* [13] who performed a  $(\vec{d}, t)$  reaction. Their analysis of the analyzing power indicates  $I^\pi = 2^+$ . The origin of this discrepancy is not understood. It might be due to an impurity problem or to the presence of an unresolved doublet. Hereafter, the spin for the 2433.1 keV level will be written as  $(2,3)^+$ .

*2477.3 keV level.* A 1001.7 keV transition in coincidence with the lines at 818.1 and 1475.8 keV is observed (compare with Table II), along with a  $\gamma$  ray of energy 2477.3 keV, which is compatible with the sum of the 1001.7 and 1475.8 keV transitions. Therefore a level at 2477.3 keV is proposed. The  $\log_{10}(ft)$  value indicates  $I^\pi = (1-3)^-$ , but since there is a transition to the ground state, and given the fact that transitions of any multipolarity may exhibit a large  $\log_{10}(ft)$  value corresponding to small nuclear matrix elements, the adopted value is  $2^+$ , in correspondence with the assignment proposed by Blasi *et al.* [13].

*2633.0 keV level.* Sarantites *et al.* [1] interpreted the peak at 1975.2 keV as a doublet, but there is no evidence of this in the present work (see *3449.6 keV level*). The transition of 1975.2 keV is in coincidence with the 657.8 keV line (see Table II), and therefore depopulates a level at 2633.0 keV, which was first observed with the  $(n, n'\gamma)$  reaction [14]. Araddad *et al.* [11] proposed  $I^\pi = (3^+), 4^+$

for this level, while Pignanelli *et al.* [8] and Blasi *et al.* [13] assigned  $I^\pi = 2^+$ . The direct  $\beta$  population excludes spin 4. The assignment  $I^\pi = 2^+, (3^+)$  is therefore suggested.

*3128.4 keV level.* The existence of this level is assured by the observation of a ground-state transition with the energy 3128.3 keV and  $T_{1/2} = 69$  min. Two transitions with energies 1344.9 and 1652.7 keV are too weak to show coincidences but can be placed on the basis of the Ritz principle. The  $\log_{10}(ft)$  value favors  $I^\pi = (1,2)^-$  but positive parity is not excluded.

*3208.8 keV level.* The 1666.2 keV transition shows coincidences with the transitions of 884.7 and 657.8 keV (see Table II and Fig. 1). Therefore a new level at 3208.8 keV is proposed. Considering the  $\log_{10}(ft)$  value and the  $\gamma$  selection rules,  $I^\pi = (2,3)^-$  is tentatively assigned. There is no evidence to support the placement of the 1666.2 keV transition proposed by Sarantites *et al.* [1] and by Meyer and Van Hise [2], who assigned it as depopulating a level at 3449.6 keV.

*3314.4 keV level.* This well-known level is additionally depopulated by the 1151.7 keV transition (see *1809.5 keV level*). The 1151.7 keV transition cannot be identified with the 1151.9 keV transition seen by Kern *et al.* [5]. This is because other transitions depopulating the 3314.4 keV level have not been observed in the  $(\alpha, 2n\gamma)$  experiment as would be expected. The  $I^\pi$  value of the 3314.4 keV level can be restricted to  $(1,2)^+$ .

*3449.6 keV level.* This level has been proposed by Meyer and Van Hise [2]. It should be depopulated by the 1666.2, 1973.7, and 1976.4 keV transitions. As already discussed, two transitions of 1973 and 1977 keV are not observed but instead only one with an energy of 1975.2 keV which depopulates the level at 2633.0 keV

(see 2633.0 keV level). The previous placement of the 1666.2 keV transition is also incorrect (see 3208.8 keV level) and thus there is no evidence for a level at 3449.6 keV.

**3634.7 keV level.** Sarantites *et al.* [1] observed a transition at 1852(2) keV which could not be placed in the level scheme. In the present study a transition at 1851.2 keV is observed to be in coincidence with the lines at 1125.8 and 657.8 keV (compare with Table II). This implies that there is a new level at 3634.7 keV which also depopulates by a new transition of 1555.8 keV. The spin and parity of the 3634.7 keV level can be restricted to

$(1-3)^+$ . This new level can perhaps be identified with the  $2^+$  level at 3632 keV seen by Pignanelli *et al.* [8].

**3701 keV level.** Sarantites *et al.* [1] proposed a level at 3701 keV. This level should be depopulated by transitions with 3044, 1618, and 1347 keV, respectively. A transition with energy 1618.5 keV was observed, but this line was identified to be the single escape peak of the 2129.4 keV transition. Also a 1344.9 keV transition was observed but placed elsewhere (see 3128.4 keV level). Therefore only the 3044.0 keV transition supports the existence of the doubtful 3701 keV level.

**3726.6 keV level.** A well-identified transition of 3726.6

TABLE III. Levels populated in the decay of  $^{110}\text{In}^m$   $2^+$ .

$E_{\text{exc}}$ with error this work	$E_{\text{exc}}$ with error previous <sup>a</sup>	$\log(ft)$ this work	$\log(ft)$ prev. <sup>b</sup>	Trans. type (d)	$I\pi$ following $\log(ft)$	$I\pi$ adopted	$I\pi$ previous (c)
0.000	0.000				$0^+$	$0^+$	$0^+$
657.787 0.022	657.763 0.002	5.6 (2)	5.6	0	$(1,2,3)^+$	$2^+$	$2^+$
1473.064 0.045	1473.120 0.040	7.7 (2)	8.1	0,1,1a	$(0,1,2,3,4)^e$	$0^+$	$0^+$
1475.771 0.025	1475.797 0.003	7.0 (2)	7.0	0,1,1a	$(1,2,3)^e$	$2^+$	$2^+$
1542.511 0.038	1542.449 0.003	7.8 (5)	7.9	0,1,1a	$(0,1,2,3,4)^e$	$4^+$	$4^+$
1731.331 0.050	1731.360 0.170	8.1 (3)	8.1	0,1,1a	$(0,1,2,3,4)^e$	$0^+$	$0^+$
1783.523 0.030	1783.490 0.020	6.7 (2)	6.7	0,1	$(1,2,3)^+$	$2^+$	$2^+$
2078.864 0.035	2078.830 0.030	7.2 (2)	7.3	0,1,1a	$(1,2,3)^-$	$3^-$	$3^-$
2162.777 0.044	2162.809 0.003	7.3 (2)	7.6	0,1,1a	$(1,2,3)^e$	$3^+$	$3^+$
2287.431 0.068	2287.440 0.080	7.7 (2)		0,1,1a	$(1,2,3)^e$	1,2,3	$2^+$
2355.812 0.049		7.1 (2)		0,1,1a	$(1,2,3)^-$	1 - 3 <sup>f</sup>	$2^+$
2433.070 0.200	2433.230 0.030	7.7 (2)		0,1,1a	$(1,2,3)^-$	$(2,3)^+ f$	$(2,3)^+$
2477.378 0.066		7.2 (2)		0,1,1a	$(1,2,3)^-$	$2^+ f$	$2^+$
2633.006 0.070		7.0 (2)		0,1,1a	$(1,2,3)^-$	$2^+, (3)^+ f$	$(2,4)^+$
2787.236 0.040	2787.340 0.130	5.8 (2)	5.8	0	$(1,2,3)^+$	$1^+, 2^+$	$1^+, 2^+$
2869.164 0.036	2869.190 0.080	5.8 (2)	5.8	0	$(1,2,3)^+$	$2^+$	$2^+$
2975.257 0.047	2975.290 0.070	5.7 (2)	5.8	0	$(1,2,3)^+$	$2^+$	$2^+$
3078.379 0.034	3078.350 0.080	5.9 (2)	5.9	0	$(1,2,3)^+$	$1^+, 2^+$	$1^+, 2^+$
3101.899 0.042	3101.890 0.120	6.2 (2)	6.3	0,1	$(1,2,3)^+$	$1^+, 2^+$	1,2
3128.402 0.081		7.3 (2)		0,1,1a	$(1,2,3)^-$	$(1,2)^- f$	
3193.425 0.049	3193.570 0.090	6.3 (2)	6.3	0,1	$(1,2,3)^+$	1,2,3	1,2,3
3208.750 0.100		7.1 (2)		0,1,1a	$(1,2,3)^-$	$(2,3)^- f$	
3314.439 0.040	3314.360 0.150	5.6 (2)	6.0	0	$(1,2,3)^+$	$1^+, 2^+ f$	$1^+, 2^+$
3403.310 0.076	3403.450 0.110	6.4 (2)	6.4	0,1	$(1,2,3)^+$	$1^+, 2^+$	(1 <sup>-</sup> )
3466.416 0.058	3466.060 0.100	5.6 (2)	5.6	0	$(1,2,3)^+$	$(1-3)^+$	$1^+, 2^+$
3475.419 0.033	3475.410 0.080	5.4 (2)	5.3	0	$(1,2,3)^+$	$1^+, 2^+$	$1^+, 2^+$
3634.670 0.150		6.2 (2)		0,1	$(1,2,3)^+$	$(1-3)^+ f$	
3726.580 0.240		6.9 (2)		0,1	$(1,2,3)^+$	$1^+, 2^+ f$	
3771.769 0.053	3771.900 0.300	5.5 (2)	5.5	0	$(1,2,3)^+$	$1^+, 2^+$	$1^+, 2^+$

<sup>a</sup>Values taken from Ref. [5].

<sup>b</sup>Values taken from Ref. [27].

<sup>c</sup>Values taken from Refs. [5,11,12,13,27].

<sup>d</sup>A measured  $\log(ft)$  value can only be used to discard a degree of forbiddness, since any  $\beta$ -ray transition can be hindered by small matrix elements. We used the following limits:  $\log(ft) > 4.0$ : allowed transition (0),  $\log(ft) > 6.0$ : forbidden transition (1),  $\log(ft) > 7.0$ : forbidden unique transition (1a).

<sup>e</sup>The  $\log(ft)$  values do not exclude negative parity, but the levels are known to have positive parity.

<sup>f</sup>See level discussion.

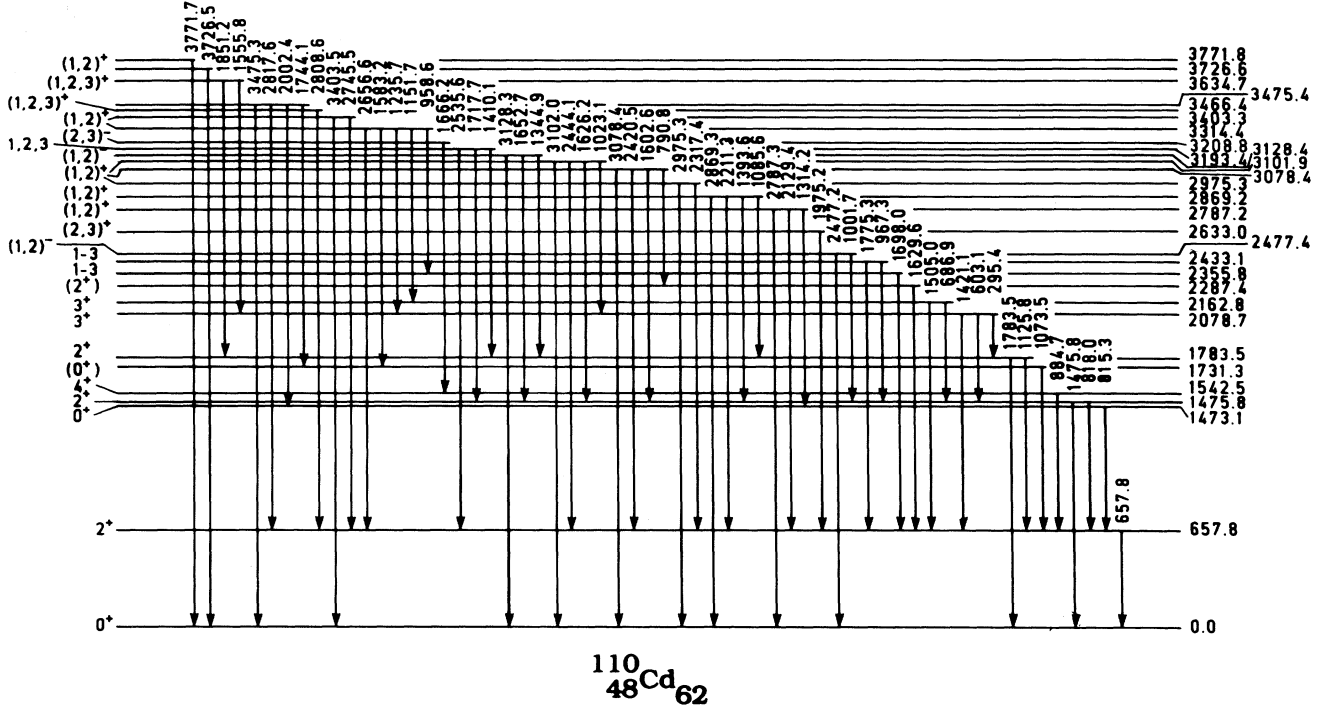


FIG. 2. The level scheme of  $^{110}\text{Cd}$  obtained from the observation of the  $^{110}\text{In}^m$   $\beta$  decay.

keV depopulates this level. The  $I^\pi$  value can be limited by the  $\log_{10}(ft)$  value to  $(1,2)^+$ . The level can be identified with the one at 3736(19) keV, seen by Auble *et al.* with the  $(^3\text{He},d)$  reaction [15].

#### IV. THE BETA-DECAY STRENGTHS

##### A. Observed $\log_{10}(ft)$ values

Using tabulated  $\log_{10}(ft)$  values [16], some very remarkable  $\log_{10}(ft)$  values are obtained for  $\beta$ -ray transitions populating the known positive-parity states up to 2.3 MeV (see Table III). First, the allowed transitions to the well-established  $2^+$  and  $3^+$  states have high  $\log_{10}(ft)$  values ranging from 5.6 to 7.7, well above the values generally observed for allowed ground-state transitions. This behavior is not uncommon in vibrational nuclei, since large  $\log_{10}(ft)$  values for allowed transitions to the  $2^+_1$  levels of the final nuclei have also been observed in the  $\beta^+$  decays of  $^{102}\text{Ag}^m$ ,  $^{104}\text{Ag}^m$ ,  $^{58}\text{Cu}$ ,  $^{58}\text{Co}$ , and  $^{60}\text{Cu}$  as well [3]. Secondly, it is observed that three twice forbidden transitions leading to two  $0^+$  states and one  $4^+$  state, in the same excitation energy range, have  $\log_{10}(ft)$  values between 7.7 and 8.1, only a little larger than for allowed transitions and this is quite astonishing. Finally, several transitions with  $\log_{10}(ft)$  values between 5.4 and 6.9 are observed to feed higher lying levels. They are believed to be of allowed character. A theoretical study of these features was thus undertaken.

##### B. QRPA framework for the beta-decay observables

Theoretical beta-decay  $\log_{10}(ft)$  values were calculated in the framework of the QRPA (quasiparticle

random-phase approximation) theory. The  $\log_{10}(ft)$  values for the  $\beta^-$  and the  $\beta^+$  decay transitions can be obtained from the expression [16,17]

$$\log_{10}(ft) = \log_{10} \left[ \frac{6050 \text{ s}}{S/S_0} \right], \quad (1)$$

where the integrated shape factors  $S$  and  $S_0$ , for the allowed and first forbidden transitions, are given in [18]. The various nuclear matrix elements embedded in Eq. (1) are calculated by using the QRPA commutator techniques [18] along with the quasiparticle form [19] of the beta-decay transition operators.

The decaying initial state of the odd-odd parent nucleus is generated as a QRPA phonon of the form

$$|I_i^{\pi_i} M_i\rangle = \sum_{pn} [X_{pn}(I_i^{\pi_i}, 1) A^\dagger(pn, I_i M_i) - Y_{pn}(I_i^{\pi_i}, 1) \tilde{A}(pn, I_i M_i)] |\text{QRPA}\rangle, \quad (2)$$

where  $I_i$  is the initial angular momentum,  $M_i$  its projection, and  $\pi_i$  the parity. The factors  $X_{pn}(I_i^{\pi_i}, 1)$  and  $Y_{pn}(I_i^{\pi_i}, 1)$  are the forward- and backward-going amplitudes of the first  $pn$ -QRPA phonon of multipolarity  $I_i^{\pi_i}$ . In the  $pn$ -QRPA approach [19] the ground state and the excited states of the odd-odd nucleus are obtained as linear combinations of quasiproton-quasineutron excitations of the (correlated) ground state ( $|\text{QRPA}\rangle$ ) of the adjacent even-even nucleus, in the present case  $^{110}\text{Cd}$ . Here the operators  $A^\dagger(pn, I_i M_i)$  create and operators  $\tilde{A}(pn, I_i M_i)$  annihilate a pair of coupled unlike ( $pn$ ) quasiparticles [18, 19] obtained in the BCS calculation.

The final states of angular momentum  $I_f$ , its projection  $M_f$ , and parity  $\pi_f$ , are obtained by diagonal-

izing the QRPA matrix built of two-quasiproton and two-quasineutron excitations [20]. The resulting QRPA phonons can be written as

$$|I_f^{\pi_f} M_f; k\rangle = Q^\dagger(I_f^{\pi_f} M_f; k)|\text{QRPA}\rangle \quad (3)$$

with

$$Q^\dagger(I_f^{\pi_f} M_f; k) = \sum_{a,a'} [Z_{aa'}(I_f^{\pi_f} k) A^\dagger(aa', I_f M_f) - W_{aa'}(I_f^{\pi_f} k) \tilde{A}(aa', I_f M_f)] , \quad (4)$$

where  $k$  enumerates states with the same angular momentum  $I_f$  and parity  $\pi_f$ . The quasiparticle pair creation and annihilation operators  $A^\dagger$  and  $\tilde{A}$  for proton-proton and neutron-neutron quasiparticle pairs, as well as the amplitudes  $Z_{aa'}$  and  $W_{aa'}$ , are defined in [18]. Furthermore,  $a$  denotes all quantum numbers needed to specify a single-quasiparticle harmonic-oscillator state for protons ( $a = p$ ) or neutrons ( $a = n$ ).

The two-phonon states are taken to be of the form

$$|I_f^+ M_f\rangle_{\text{two-ph}} = \frac{1}{\sqrt{2}} [Q^\dagger(2^+; 1) Q^\dagger(2^+; 1)]_{I_f M_f} |\text{QRPA}\rangle , \quad (5)$$

where the QRPA phonons  $Q^\dagger(2^+; 1)$  are defined in Eq. (4) and the square brackets denote angular-momentum coupling to the final angular momentum  $I_f$  with projection  $M_f$ .

The  $\beta^-$  and  $\beta^+$  transition matrix elements between the initial states (2) and the final states (3) or (5) can be evaluated using the techniques of [18] and the corresponding analytical expressions are given in [21, 22].

### C. Computational procedure and results

To test the QRPA wave functions of the ground state and excited states of  $^{110}\text{Cd}$  both the  $\beta^-$  feeding [ $^{110}\text{Ag}(1_{\text{g.s.}}^+) \rightarrow ^{110}\text{Cd}(I_f^{\pi_f})$ ] as well as the  $\beta^+$  feeding [ $^{110}\text{In}(2_i^{\pi_i}) \rightarrow ^{110}\text{Cd}(I_f^{\pi_f})$ ] to the final states of multipolarity  $I_f^{\pi_f}$  have been calculated. The two-body matrix elements were obtained using the  $G$  matrix based on the Bonn one-boson-exchange potential [23].

The single-particle basis, both for protons and for neutrons, consisted of the  $N = 3$  and  $N = 4$  oscillator major shells, supplemented by the  $h_{11/2}$  intruder orbital from the next higher major shell, leaving the nucleus  $^{40}\text{Ca}_{20}$  as the core. To a first approximation, the single-particle energies were obtained from the Coulomb-corrected Woods-Saxon potential [24]. These energies were shifted slightly in the vicinity of the proton and neutron Fermi surface in order to yield in a BCS calculation the observed low-energy spectrum [3] of the adjacent odd-mass isotone or isotope.

The proton-proton and neutron-neutron pairing strength was determined by comparing the calculated pairing gap [19] with the experimental pairing gap deduced from the empirical proton and neutron separation energies [25]. The magnitudes of the proton-neutron two-body  $G$ -matrix elements were scaled by reproducing [19] the semiempirical excitation energies [26] of the Gamow-Teller giant resonance (GTGR) in the odd-odd isotopes in the neighborhood of  $^{110}\text{Cd}$  and by fitting the  $\log_{10}(ft)$  value [27] of the  $\beta^-$  transition  $^{110}\text{Ag}(1_{\text{g.s.}}^+) \rightarrow ^{110}\text{Cd}(0_{\text{g.s.}}^+)$ . Finally, the overall scale of the proton-proton and neutron-neutron quadrupole (octupole) two-body matrix elements was set by adjusting the calculated energy of the lowest  $2^+$  ( $3^-$ ) state in  $^{110}\text{Cd}$  to its mea-

TABLE IV. Experimental  $\log_{10}(ft)$  values for the  $\beta$  decay of  $^{110}\text{In}^m$  compared with calculated ones. Two different calculations are presented, one with a positive and one with a negative initial parity. In the latter case a renormalization of the axial-charge matrix element was performed and is presented in the column  $1.6M_5$ .

$E_{\text{exp.}}$ [MeV]	$E_{\text{theory}}$ [MeV]	$I^\pi$ exp.	$I^\pi$ theory	$\log(ft)$ of $\beta^-$ decay from $^{110}\text{Ag}$		$\log(ft)$ of $\beta^+$ decay from $^{110m}\text{In}$			
				exp.	theory	exp.	th. ( $I^\pi = 2^+$ )	th. ( $I^\pi = 2^-$ )	$1.0M_5$
0.66	0.66	2 <sup>+</sup>	2 <sup>+</sup>	5.5	5.7	5.6	6.1	6.6	6.6
1.73	1.32	0 <sup>+</sup>	0 <sup>+</sup>	8.1	5.5	8.1	-(a)	8.2	8.2
1.47	1.32	2 <sup>+</sup>	2 <sup>+</sup>	7.4	8.3	7.0	7.7	6.2	6.0
1.54	1.32	4 <sup>+</sup>	4 <sup>+</sup>	-	-	7.8	-(a)	8.8	8.8
2.08	2.08	3 <sup>-</sup>	3 <sup>-</sup>	7.5	10.1	7.2	7.3	5.5	5.5
2.36	2.40	1, 2, 3	2 <sup>+</sup>	-	7.8	7.1	5.8	6.3	6.1
2.63	2.70	2 <sup>+</sup>	2 <sup>+</sup>	-	8.7	7.0	6.9	7.1	6.9
2.79 - 2.98	2.94	2 <sup>+</sup>	2 <sup>+</sup>	-	-	5.7 - 5.8	5.7	7.0	6.8
3.08	3.13	1 <sup>+</sup> , 2 <sup>+</sup>	2 <sup>+</sup>	-	-	5.9	6.8	6.9	6.8
3.10	3.19	1 <sup>+</sup> , 2 <sup>+</sup>	2 <sup>+</sup>	-	-	6.2	6.7	6.8	6.6
3.19	3.28	1, 2, 3	2 <sup>+</sup>	-	-	6.3	6.9	7.1	6.9
3.21	3.43	2 <sup>-</sup> , 3 <sup>-</sup>	3 <sup>-</sup>	-	-	7.1	6.4	5.0	5.0
3.31	3.39	1 <sup>+</sup> , 2 <sup>+</sup>	2 <sup>+</sup>	-	-	5.6	7.0	7.1	6.9
3.40	3.45	1 <sup>+</sup> , 2 <sup>+</sup>	2 <sup>+</sup>	-	-	6.4	6.0	7.0	6.9
3.47	3.55	(1, 2, 3) <sup>+</sup>	2 <sup>+</sup>	-	-	5.4 - 5.6	6.3	7.1	6.9
3.63	3.67	(1, 2, 3) <sup>+</sup>	2 <sup>+</sup>	-	-	6.2	6.2	6.5	6.5

<sup>a</sup>The theoretical method does not include twice-forbidden transitions.

sured value.

After fixing the values of the effective two-body matrix elements with the available spectroscopic data, the beta-decay observables, other than the  $\log_{10}(ft)$  of the  $\beta^-$  transition  $^{110}\text{Ag}(1_{g.s.}^+) \rightarrow ^{110}\text{Cd}(0_{g.s.}^+)$ , are left as parameter-free predictions. These predictions are compared with the corresponding experimental data in Table IV. In this comparison those experimental states of  $^{110}\text{Cd}$  were omitted where IBM computations [5, 28, 29] indicate that the intruder component is dominating. In Table IV, two calculations for the  $\beta^+$  branch, namely, one with initial multipolarity  $2^+$  and the other with multipolarity  $2^-$  for the isomeric state of  $^{110}\text{In}$ , for reasons given hereafter, are displayed.

#### D. Discussion of the theoretical $\log_{10}(ft)$ values

The parity of the isomeric state of  $^{110}\text{In}$  has not been measured directly but only adopted as being positive because Katoh *et al.* [30] in 1962 did not observe  $\beta$  transitions to the ground state and to the  $4^+$  state in  $^{110}\text{Cd}$ . For completeness, the theoretical calculations have been performed for both positive and negative parities of the isomeric state to see if there are qualitative differences in the decay-strength distribution between the resulting two types of  $\beta^+$  decay.

Table IV shows the comparison between the theoretical and experimental strength distributions both for the  $\beta^-$  decay from  $^{110}\text{Ag}(1_{g.s.}^+)$  as well as for the  $\beta^+$  decay from  $^{110}\text{In}^m(2_i^\pm)$ . As one can see the correspondence between the theoretical and experimental excitation energies is good even up to high excitation energies, though the identification beyond 2.3 MeV of excitation energy may be ambiguous. This rather satisfactory correspondence between such many energy levels opens up an opportunity to differentiate between the two theoretical calculations, one assuming  $2^+$  and the other  $2^-$  for the initial state in  $^{110}\text{In}$ , by comparison with the present measured  $\beta^+$  strength distribution.

In the theoretical  $^{110}\text{In } 2^+$  wave function the dominating component is the  $\pi 0g_{9/2} \otimes \nu 1d_{5/2}$  proton-neutron pair, the  $\pi 0g_{9/2} \otimes \nu 0g_{7/2}$  pair having a small but non-negligible amplitude, and the other proton-neutron pairs having only a very small contribution. Contrary to this, the  $2^-$  wave function carries several proton-neutron components with rather large amplitudes, the most prominent being  $\pi 1p_{1/2} \otimes \nu 1d_{5/2}$ ,  $\pi 1p_{3/2} \otimes \nu 2s_{1/2}$ ,  $\pi 1p_{1/2} \otimes \nu 1d_{3/2}$ ,  $\pi 1p_{3/2} \otimes \nu 0g_{7/2}$ ,  $\pi 0g_{7/2} \otimes \nu 0h_{11/2}$ , and  $\pi 1p_{3/2} \otimes \nu 1d_{5/2}$ . This spreading of the amplitudes is due to the position of the proton and neutron Fermi surfaces, enabling a larger amount of low-energy proton-neutron quasiparticle excitations for the negative parity.

The forbidden calculation for the  $2^- \rightarrow 2^+$  transitions has been performed by using the pure axial-charge matrix element  $M_5 = \int \gamma_5$  (see Ref. [16]), indicated as column "1.0 $M_5$ " in Table IV, and a renormalized one (column "1.6 $M_5$ " in Table IV). This change in the magnitude of the axial-charge matrix element stems from meson-exchange currents enhancing the impulse-approximation value of  $M_5$  [31]. To a good approximation the meson-

exchange effect can be taken into account by multiplying the impulse-approximation value of  $M_5$  by a constant factor of 1.6 [31, 32], and this simple multiplicative factor has been used in the present calculation.

The numerical results indicate that for all the decay transitions  $2^- \rightarrow 2^+$ , the ratio  $\Lambda_0 = M_5/\xi M_1$  ( $M_1 = i \int \sigma \cdot r$ , see Refs. [16, 33]) changes from  $\Lambda_0 = 0.7$  to  $\Lambda_0 = 1.1$  due to the renormalization. This change does not affect the  $\log_{10}(ft)$  values in a too drastic way (see Table IV), and in some cases (e.g., the transition to the first  $2^+$  state) is washed out by the contribution of the other nuclear matrix elements (mainly  $M_6$ ).

Both the allowed ( $I^\pi = 2^+$  in Table IV) and the forbidden ( $I^\pi = 2^-$  in Table IV) calculations describe rather well the  $\log_{10}(ft)$  values of transitions to the  $2^+$  states in  $^{110}\text{Cd}$ . On the basis of these transitions it is difficult to give any preference to either initial spin assignment. However, the decay transitions to the two  $3^-$  states would prefer the  $2^+$  initial spin assignment. In addition, one particular problem with the negative initial parity,  $2^-$ , is that the theoretically predicted ground-state transition [calculated  $\log_{10}(ft)=8.3$ ] has not been measured.

On the other hand, a transition to a  $4^+$  state has been identified, and if one assumed a negative initial parity most of the problems concerning the  $\log_{10}(ft)$  values mentioned in Sec. IV A for  $^{110}\text{Cd}$  would disappear, and the parity problem pointed out in the footnote in Table III is eliminated. It is clear that a change of parity of the  $I = 2$  ( $T_{1/2} = 69$  min) level in  $^{110}\text{In}$  will have important consequences on the interpretation of the structure of that isotope since a whole set of parity assignments depends on the parity of this level. Such a reanalysis is necessary if one adopts negative initial parity. It is, however, outside the scope of this paper.

#### V. INTERPRETATION OF THE RESULTS IN THE FRAMEWORK OF THE IBM

In the  $(\vec{d}, t)$  study of Blasi *et al.* [13] a number of states were found to be populated quite strongly, indicating the presence of significant two-quasiparticle amplitudes in the wave functions. Some of these levels are suggested below to belong to high  $n_d$  or intruder configurations for which one would not expect large spectroscopic strengths in single-nucleon transfer. In order to take into account the two-quasiparticle (2qp) amplitudes in the wave functions, IBM + 2qp calculations need to be performed, as was done in Ref. [13]. However, such calculations are outside the scope of the present work, and, as will be demonstrated below, the use of the IBM and intruder pictures reproduces quite well the energy level systematics. Therefore, the levels are labeled by the IBM and intruder quantum numbers even though there may be significant two-quasiparticle admixtures.

##### A. Normal configuration in the exact U(5) dynamical symmetry

Since the IBM-1 U(5) description of the normal configuration of  $^{112}\text{Cd}$  by D el eze *et al.* [34] reproduced quite well the level energies of this nucleus in a very simple way,



it seems natural to study  $^{110}\text{Cd}$  by the same method. Therefore no distinction between proton and neutron degrees of freedom is made, and the Hamiltonian is written in terms of the Casimir operators associated with the subgroups appearing in the group chain

$$U(6) \supset U(5) \supset O(5) \supset O(3). \quad (6)$$

Neglecting terms that contribute only to the binding energy, the dynamic symmetry associated with (6) gives rise to the following Hamilton operator:

$$\hat{H} = \varepsilon \hat{C}_1[U(5)] + \varepsilon' \hat{C}_2[U(5)] + \delta \hat{C}_2[O(5)] + \gamma \hat{C}_2[O(3)] \quad (7)$$

with  $\varepsilon$ ,  $\varepsilon'$ ,  $\delta$ , and  $\gamma$  free parameters and with  $\hat{C}_i[G]$  the Casimir operators of  $i$ th order of the group  $G$ . The eigenvalues of (7) can be expressed analytically in the  $U(5)$  basis  $||N\rangle\{n_d\}(\nu)_\alpha L\rangle$ , i.e.,

$$E = \varepsilon n_d + \varepsilon' n_d(n_d + 4) + \delta \nu(\nu + 3) + \gamma L(L + 1), \quad (8)$$

where  $n_d$  is the number of  $d$  bosons,  $\nu$  the number of  $d$ -boson pairs not coupled to zero angular momentum, and  $L$  the angular momentum of the state. The missing label  $\alpha$  classifies states to a given  $\nu$  having the same  $L$ .

Using this eigenvalue equation, a least-squares fit to the experimental excited states (members of the ground state and quasi- $\gamma$ -bands) was performed. Because it was possible to add two levels (the  $4^+$  state at 2220.1 keV and the  $6^+$  state at 3121.7 keV) to the quasi- $\gamma$ -band proposed by Sakai [35], three levels can be associated reliably with each of the  $n_d = 3$  and 4 multiplets, constraining the fit parameters. Because  $\varepsilon'$  was very small, the second term of (8) was neglected. A similar situation occurs for  $^{112}\text{Cd}$ . The theoretical spectrum was obtained with the parameters  $\varepsilon = 824.2$  keV,  $\delta = -22.9$  keV, and  $\gamma = 8.2$

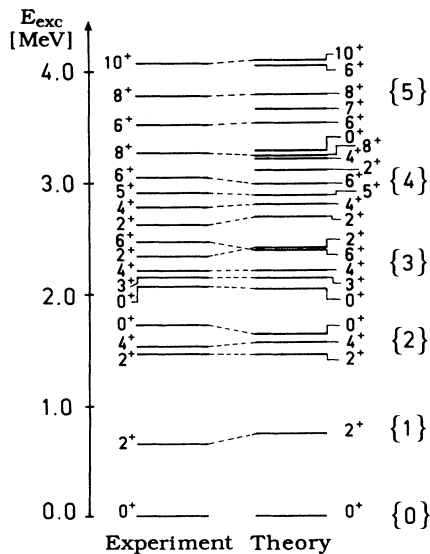


FIG. 3. Comparison between the experimental levels belonging to the ground-state configuration and a theoretical level scheme calculated in the IBM-1 framework using three free parameters and neglecting mixing (see text).

keV and fits very well the experimental data (see Fig. 3).

Several observations can be made from the comparison between experimental and theoretical states.

(1) The fit for the excited states is good up to high energies. It is possible to identify the complete set of levels belonging to the three-phonon states and a nearly complete set of four-phonon states. For lack of absolute transition rates this identification is done only on the basis of spins and energies. In essence the good agreement thus only reflects the “global” properties of the excited states (see Ref. [36]).

(2) This agreement points out, in particular, that the mixing interaction with the intruder configuration is only of the order of 50 keV [5, 28], so that the energies are only relatively little shifted (at most a few percent).

(3) The mixing interaction, however, can disturb quite strongly the wave functions of levels which would be close in the unperturbed situation. The detailed IBM-2 calculations of D el eze *et al.* [28] support the proposed identifications, i.e., the computed wave functions have mainly the normal or intruder character corresponding to the results presented here.

(4) The present assignment of the  $n_d = 2$  states is at variance with the one of Arima and Iachello [37]. They proposed the  $0^+$  state at 1473.1 keV to belong to the  $n_d = 2$  triplet. This level is identified here to belong to the intruder band. This identification is in agreement with the more detailed studies in Refs. [5, 28, 29]. There is, however, a need to measure the lifetime of the 1473.1 keV state. Based on similar arguments, the level at 1731.3 keV is proposed to be a member of the  $n_d = 2$  triplet.

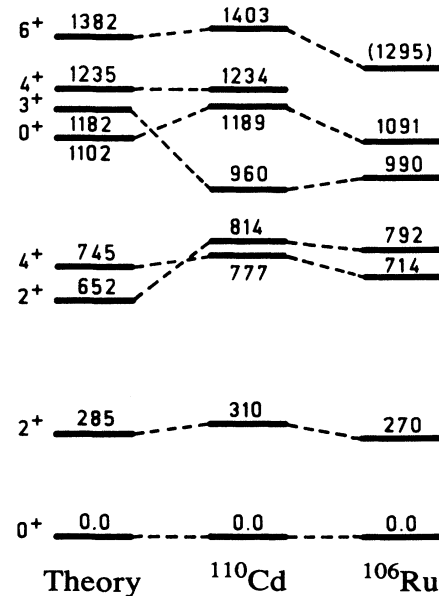


FIG. 4. The low-lying states of  $^{106}\text{Ru}$  compared to the experimental and theoretical intruder states in  $^{110}\text{Cd}$ . The excitation energies in  $^{110}\text{Cd}$  refer to the  $0^+$  “configuration” head at 1473 keV. Note that the spin of the experimental level at 960 keV is not determined with certitude. The experimental data concerning  $^{106}\text{Ru}$  are taken from Refs. [39] and [40].

Because the U(5) limit cannot reproduce the whole set of experimental states with positive parity, another configuration space, called the intruder configuration, has to be taken into account to describe some of the other levels.

### B. Intruder configuration in the exact O(6) dynamic symmetry

In the analysis of  $^{112}\text{Cd}$  performed by Déléze *et al.* [34] the well-known intruder states observed in [5] are described in the O(6) dynamical limit of the IBM-1. In this limit the group chain is

TABLE V. Comparison of experimental and IBM-2 positive-parity excitation energies in  $^{110}\text{Cd}$ . The list of levels is a compilation of published results. Spin and parity values are primarily taken from Ref. [5], or as otherwise noted with appropriate comments.

$E_{\text{exc}}$ Experiment	Spin Experiment	$E_{\text{exc}}$ Theory	Spin Theory	Comments
0	0 <sup>+</sup>	0	0 <sub>1</sub>	$n_d=0$
658	2 <sup>+</sup>	657	2 <sub>1</sub>	$n_d=1$
1473	0 <sup>+</sup>	1463	0 <sub>2</sub>	Intruder
1476	2 <sup>+</sup>	1385	2 <sub>2</sub>	$n_d=2$
1542	4 <sup>+</sup>	1457	4 <sub>1</sub>	$n_d=2$
1731	0 <sup>+</sup>	1739	0 <sub>3</sub>	$n_d=2$
1783	2 <sup>+</sup>	1740	2 <sub>3</sub>	Intruder
2079	0 <sup>+</sup>	2144	0 <sub>4</sub>	$n_d=3$
2163	3 <sup>+</sup>	2091	3 <sub>1</sub>	$n_d=3$ , quasi- $\gamma$
2220	4 <sup>+</sup>	2179	4 <sub>2</sub>	$n_d=3$ , quasi- $\gamma$
2251	4 <sup>+</sup>	2416	4 <sub>3</sub>	Intruder
2287	2 <sup>+</sup>	1934	2 <sub>4</sub>	Intruder
2332	2 <sup>+</sup> (a,d)	2362	2 <sub>5</sub>	
2356	2 <sup>+</sup> (a)	2461	2 <sub>6</sub>	$n_d=3$
2433	(2,3) <sup>+</sup> (b)	2761	3 <sub>2</sub>	Intruder
2480	6 <sup>+</sup>	2325	6 <sub>1</sub>	$n_d=3$
2561	4 <sup>+</sup>	2742	4 <sub>4</sub>	
2633	2 <sup>+</sup> (c)	2792	2 <sub>7</sub>	$n_d=4$
2662	0 <sup>+</sup> (e)	2750	0 <sub>5</sub>	Intruder
2707	4 <sup>+</sup>	2843	4 <sub>5</sub>	Intruder
2787	2 <sup>+</sup>	3284	2 <sub>8</sub>	
2793	4 <sup>+</sup>	2969	4 <sub>6</sub>	
2869	2 <sup>+</sup> (c)	3618	2 <sub>9</sub>	
2877	6 <sup>+</sup>	2936	6 <sub>2</sub>	Intruder
2915	4 <sup>+</sup>	3455	4 <sub>7</sub>	$n_d=4$

<sup>a</sup>Spin and parity assignments from Ref. [8].

<sup>b</sup>Spin and parity assignments from Refs. [11,12,13].

<sup>c</sup>Previous spin and parity assignments were controversial.  $I^\pi$  value listed is according to the present work.

<sup>d</sup>This level is unobserved in the present work although it has a low spin.

<sup>e</sup>This level [10,12] should not be confused with the close lying 3<sup>+</sup> level [13,27].

$$U(6) \supset O(6) \supset O(5) \supset O(3) \quad (9)$$

with its corresponding Hamilton operator

$$\hat{H} = \eta \hat{C}_2[O(6)] + \delta' \hat{C}_2[O(5)] + \gamma' \hat{C}_2[O(3)] \quad (10)$$

and the eigenvalues

$$E = \eta\sigma(\sigma + 4) + \delta'\tau(\tau + 3) + \gamma'L(L + 1). \quad (11)$$

As in  $^{112}\text{Cd}$ , the levels corresponding to the first term [representing the  $\sigma = (N - 2)$  irrep] could not be identified because the level density in this region is quite high, and therefore the  $\eta$  parameter has been chosen arbitrarily to be large and negative, so that the lowest state belongs to  $\sigma = N$ . The other two parameters have then been fitted with a least-squares procedure that gave the following values:  $\delta' = 61.2$  keV and  $\gamma' = 6.7$  keV. The excitation energies refer to the 0<sup>+</sup> "configuration" head at 1473 keV.

The improved knowledge of high-lying, low-spin states can now be used, for instance, to test recent claims [38] on the existence of intruder analog states related to  $I$  spin.  $I$  spin classifies the bosons depending on their  $p$ - $h$  character. If the Hamiltonian is  $I$ -spin invariant, intruder analog multiplets occur and allow, for instance, the relation of the intruder excitations to those in the normal configuration in the nucleus with four protons less.

After subtraction of the normal states having  $I = \frac{1}{2}$  and  $I_z = -\frac{1}{2}$ , where  $I$  refers to the intruder spin of the protons, the experimental level scheme should contain only states with  $I_z = -\frac{1}{2}$  and  $I$  larger than or equal to  $\frac{3}{2}$ , the lowest intruder states being expected to have  $I = \frac{3}{2}$ . If intruder analog states exist, these states should have their analog in the normal states of  $^{106}\text{Ru}$  which have  $I = \frac{3}{2}$ ,  $I_z = -\frac{3}{2}$ . Figure 4 shows the comparison of these levels. A good agreement is obtained although the second 4<sup>+</sup> level was not observed [39, 40] in  $^{106}\text{Ru}$ . It is noteworthy that this agreement is better than with the theoretical fit in the O(6) limit. Comparison with other nuclei having  $I = \frac{3}{2}$  states is hampered by lack of experimental data.

### C. Comparison between experimental levels and IBM-2 calculations

In Table V, the results of the IBM-2 calculation performed by Déléze *et al.* for  $^{110}\text{Cd}$  [28] are compared with the energies of the levels observed in this and in previous experiments [4, 5, 8, 10–13, 15]. Levels whose existence is doubtful are neglected. A good correspondence is obtained, although for three levels (at 2787, 2869, and 2915 keV) the IBM-1 description is better, but there are a few levels, those at 2332, 2561, 2787, 2793, and 2869 keV, that do not fit into the model. These are candidates for mixed symmetry states (in the IBM-2 calculations [28] a 2<sup>+</sup> at 2362 keV appears, maybe corresponding to the level at 2332 keV. Two additional 4<sup>+</sup> states are calculated as well at 2742 and 2969 keV. They could correspond to the two levels at 2561 and 2793 keV) or for two-quasiparticle states.

## VI. CONCLUSION

The present study shows that there is no evidence for a level at 1809 keV. As discussed in Ref. [9] such a level most probably does not exist. The level scheme was furthermore improved in several respects resulting in a better knowledge of the low-spin levels which are weakly or not observed in (particle, $xn$ ) reaction experiments. An incorporation of the present into previous results in the framework of IBM-1 shows that high phonon multiplets can be identified in the "normal" configuration. Several of the additionally observed levels can be assigned to the intruder O(6) configuration. Some interesting peculiarities regarding the  $\log_{10}(ft)$  values of the transitions feeding the low-energy (up to 2.3 MeV) levels are observed.

The QRPA approach to describe the beta decay does not exclude a negative parity for the initial isomeric spin-2 level in  $^{110}\text{In}$ . Such a negative initial parity could explain some surprising  $\log_{10}(ft)$  values observed for transitions to well-known levels in  $^{110}\text{Cd}$  in this analysis.

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