PHYSICAL REVIEW C VOLUME 50, NUMBER 1 JULY 1994

Alleged contra-rotation of neutrons and protons

E. R. Marshalek
Department of Physics, University of Notre Dame, Notre Dame, Indiana 46556
(Received 17 December 1993)

The claim made in a recent Letter that neutrons and protons in the ground states of deformed nuclei rotate in opposite directions is based on an artifact arising from using the angular momentum defined relative to the laboratory frame rather than relative to the center-of-mass frame. This can be demonstrated already for a simple two-body system such as a diatomic molecule.

PACS number(s): 21.60.Ev, 21.10.Re, 21.60.Fw

In his recent Letter [1] and a subsequent conference proceeding [2], Otsuka arrived at the startling conclusion that the neutrons and protons in the ground states of deformed nuclei collectively rotate in opposite senses. If this were so, a radical revision of our understanding of nuclear structure, and, indeed, nuclear forces would be required. The collective mode closest to this description is the "scissors mode" [3], but this involves a small-amplitude torsional oscillation of neutrons against protons, not a free large-amplitude contra-rotation. In spite of the revolutionary implications, the conclusions in Refs. [1,2] were based on an examination of several venerable but conventional nuclear models, namely, the Nilsson model [4] with pairing and angular-momentum projection, the interacting boson model (IBM-2) [5], and also the shell model (results to be published). While the emergence of the same qualitative results from such a diversity of nuclear models might at first sight appear to bolster Otsuka's claims, on second thought one wonders how such a salient feature as contra-rotation could have escaped notice all these years and what common property might be shared by all these models to give rise to this feature. First of all, it will be shown here that the common model-independent feature is an elementary result of angular-momentum algebra. Second, it will be argued that Otsuka's claim, based on using angular momenta defined relative to the laboratory frame as declared in the title of his Letter, is spurious since he should have used angular momenta defined relative to the *center-of-mass* frame. The argument given here is based on a much simpler two-component system, namely a two-body system, where results qualitatively similar to those of Otsuka can be obtained in the laboratory frame. However, in this model there is obviously no contra-rotation of the two particles.

Consider a two-component quantum system where the constituents carry respective angular momenta $\mathbf{j}_1$ and $\mathbf{j}_2$, with the total angular momentum $\mathbf{J}=\mathbf{j}_1+\mathbf{j}_2$. In Otsuka's paper, $\mathbf{j}_1,\mathbf{j}_2$ correspond to the neutron and proton angular momenta. Since $\mathbf{j}_1\cdot\mathbf{j}_2=\frac{1}{2}(|\mathbf{J}|^2-|\mathbf{j}_1|^2-|\mathbf{j}_2|^2)$, the angle between the two angular-momentum vectors as defined by Otsuka is

$$\cos\theta\equiv\frac{\langle\mathbf{j}_1\cdot\mathbf{j}_2\rangle}{[\langle|\mathbf{j}_1|^2\rangle\langle|\mathbf{j}_2|^2\rangle]^{1/2}}=\frac{\frac{1}{2}[\langle\mathbf{J}^2\rangle-\langle|\mathbf{j}_1|^2\rangle-\langle|\mathbf{j}_2|^2\rangle]}{[\langle|\mathbf{j}_1|^2\rangle\langle|\mathbf{j}_2|^2\rangle]^{1/2}}. \tag{1}$$

Consider a general normalized state vector of the form

$$|\Psi_{JM}\rangle=\sum_{\alpha,j_1,j_2}C_\alpha(j_1j_2)|\alpha;(j_1j_2)JM\rangle, \tag{2}$$

obtained by coupling the subsystems to a resultant angular momentum J (α refers to quantum numbers other than angular momentum). It is immediately clear that if $J=0$ ($\langle\mathbf{J}^2\rangle=0$) then $j_1=j_2=j$ and $\langle|\mathbf{j}_1|^2\rangle=\langle|\mathbf{j}_2|^2\rangle$, which implies $\cos\theta=-1$ from Eq. (1). This explains the ubiquity of Otsuka's results for different models involving the coupling of neutrons and protons. In addition to this result for the ground state, Otsuka also describes an approximately parabolic increase of $\langle\mathbf{j}_1\cdot\mathbf{j}_2\rangle$ with increasing J for the neutron-proton systems. While a quantitative calculation is model dependent in this case, the qualitative behavior can be obtained for a simple two-body Hamiltonian as shown next.

Consider the general spinless[1] two-body system described by the Hamiltonian

$$H=\frac{|\mathbf{p}_1|^2}{2m_1}+\frac{|\mathbf{p}_2|^2}{2m_2}+V(\mathbf{r}_1-\mathbf{r}_2), \tag{3}$$

which can be treated in the textbook manner by introducing the center of mass (c.m.) and relative coordinates. In order to obtain eigenvectors with a finite norm as in Otsuka's work, one may employ the device of working with the modified Hamiltonian $\tilde{H}\equiv H+\frac{1}{2}M\Omega^2|\mathbf{R}|^2$, where the vector $\mathbf{R}$ is the position of the c.m. and $M\equiv m_1+m_2$. In this way, the Hamiltonian for the c.m. motion becomes that of a three-dimensional isotropic harmonic oscillator centered at the origin. Ultimately, the spurious c.m. excitations can be pushed up to high energies by making the frequency Ω arbitrarily large.

Clearly, to meaningfully check the opening angle between the angular momenta of the two particles, one should calculate $\langle\tilde{\mathbf{l}}_1\cdot\tilde{\mathbf{l}}_2\rangle$, where $\tilde{\mathbf{l}}_i\equiv(\mathbf{r}_i-\mathbf{R})\times[\mathbf{p}_i-(m_i/M)\mathbf{P}]$, $i=1,2$, is the angular momentum of each particle *relative to the c.m. frame* ($\mathbf{P}$ is the total momentum relative to the laboratory frame). Upon transformation to the c.m. and relative coordinates, one easily obtains $\tilde{\mathbf{l}}_i=(m_i/M)\mathbf{l}$, where $\mathbf{l}\equiv\mathbf{r}\times\mathbf{p}$ is the total angular momentum relative to the c.m. frame, with $\mathbf{r}\equiv\mathbf{r}_1-\mathbf{r}_2$ being the relative vector and

[1]The inclusion of spin is straightforward, but irrelevant in the present context.

$\mathbf{p} \equiv (m_2 \mathbf{p}_1 - m_1 \mathbf{p}_2)/M$ its conjugate momentum. In this case, there is no question of contra-rotation since $\langle \tilde{l}_1 \cdot \tilde{l}_2 \rangle = (\mu/M)\hbar^2 \ell(\ell+1) \geqslant 0$, where ℓ is the angular-momentum quantum number of the relative motion and $\mu \equiv m_1 m_2/M$ is the reduced mass.

Suppose that instead one follows Otsuka and calculates $\langle l_1 \cdot l_2 \rangle$, i.e., using the angular momenta relative to the laboratory frame and a wave function that is the direct product of a relative wave function and that of the c.m. oscillator ground state. After a straightforward calculation, one finds

$$\langle |l_1|^2 \rangle - \frac{m_2^2}{M^2} \hbar^2 \ell(\ell+1) = \langle |l_2|^2 \rangle - \frac{m_1^2}{M^2} \hbar^2 \ell(\ell+1)$$

$$= -\langle l_1 \cdot l_2 \rangle + \frac{\mu}{M} \hbar^2 \ell(\ell+1) = \gamma, \tag{4}$$

where

$$\gamma \equiv \frac{2}{3}(\langle |\mathbf{P}|^2 \rangle \langle |\mathbf{r}|^2 \rangle + \langle |\mathbf{R}|^2 \rangle \langle |\mathbf{p}|^2 \rangle) - \frac{3\mu\hbar^2}{M}$$

$$= \frac{\mu^2}{M} \hbar\Omega \langle |\mathbf{r}|^2 \rangle + \frac{\hbar}{M\Omega} \langle |\mathbf{p}|^2 \rangle - \frac{3\mu\hbar^2}{M}, \tag{5}$$

which is exact. Therefore, this result depends on the c.m. zero-point fluctuations. For any state with $\ell = 0$, such as the ground state, Eq. (4) implies $\cos\theta = -1$, *which holds independently of any intrinsic deformation*, as expected from the earlier discussion. If the potential $V(\mathbf{r}_1 - \mathbf{r}_2)$ has a deep minimum at $|\mathbf{r}| = r_0$ to give a diatomic molecule, and if Ω is chosen very large $[\hbar\Omega \gg \hbar^2/(\mu r_0^2)]$, then only the first term on the far right of (5) contributes significantly, and one finds from (4) that

$$\langle l_1 \cdot l_2 \rangle \simeq -\frac{\mu^2}{M} \hbar\Omega r_0^2 + \frac{\mu}{M} \hbar^2 \ell(\ell+1), \tag{6}$$

and

$$\cos\theta \simeq -1 + \frac{M\hbar}{2\Omega\mu^2 r_0^2} \ell(\ell+1), \tag{7}$$

which gives the parabolic dependence on ℓ similar to that found by Otsuka.

The important lesson taught by this simple model is that the separation of the c.m., while essential, is not enough; the simultaneous transformation of the angular momenta to the c.m. frame is also required to obtain correct results. Since this was not done in Refs. [1,2], the conclusions concerning the existence of contra-rotation are unfounded. There are other physical reasons for disbelieving the existence of such a mode. For example, it is easy to see that time-dependent mean-field theories, while permitting the scissors mode, disallow uniform contra-rotation, unless, of course, the neutron-proton interaction were to vanish (which indeed would constitute a radical revision of nuclear forces). The cranking model [6], whose citation by Otsuka suggests harmony with his picture, actually describes neutrons and protons rotating in the same sense.

The above discussion also indicates that Otsuka's result $\cos\theta = -1$ is not limited to the ground states of even-even deformed nuclei, but should also hold for doubly closed-shell nuclei, where the notion of "neutron and proton ellipsoids" does not apply. A corrected calculation properly taking account of the c.m. degrees of freedom can certainly be carried out for the shell model starting with a translationally invariant effective interaction, but this would be more difficult within the framework of the Nilsson model with pairing or the IBM-2. The Nilsson model, being a phenomenological mean-field approximation, is not translationally invariant to begin with, so that the spurious c.m. motion may be significantly mixed with physical modes. However, methods have been suggested for adding residual interactions in such cases to restore translational invariance [7]. The IBM-2 is even more problematical since it is a phenomenological boson model whose relation to microscopic physics is still tenuous. Therefore, Otsuka may have exposed a real deficiency in this model, the correction of which would require the identification of c.m. degrees of freedom. Perhaps this can be done by either adding dipole bosons or deriving the model from a microscopic shell model, taking into account the c.m. degrees of freedom.

This work was supported in part by the Department of Energy under Grant DE-FG02-91ER40640.

[1] T. Otsuka, Phys. Rev. Lett. **71**, 1804 (1993).
[2] T. Otsuka, T. Mizusaki, and M. Honma, Nucl. Phys. **A570**, 265c (1994).
[3] R. R. Hilton, S. Iwasaki, H. J. Mang, P. Ring, and M. Faber, in *Microscopic Approaches to Nuclear Structure,* edited by A. Covello (Italian Physical Society, Bologna, 1986), p. 357.
[4] S. G. Nilsson, C. F. Tsang, A. Sobiczewski, Z. Szymański, S. Wycech, C. Gustafson, I. Lamm, P. Möller, and B. Nilsson, Nucl. Phys. **A131**, 1 (1969).
[5] F. Iachello and A. Arima, *The Interacting Boson Model* (Cambridge University Press, New York, 1987), p. 131.
[6] Z. Szymański, *Fast Nuclear Rotation* (Clarendon Press, Oxford, 1983).
[7] J. Meyer-ter-Vehn, Z. Phys. A **289**, 319 (1979).