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Gamow-Teller strength in the region of ^{100}Sn

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New calculations are presented for Gamow-Teller beta decay of nuclei near ^{100}Sn. Essentially all of the ^{100}Sn Gamow-Teller decay strength is predicted to go to a single state at an excitation energy of 1.8 MeV in ^{100}In. The first calculations are presented for the decays of neighboring odd-even and odd-odd nuclei which show, in contrast to ^{100}Sn, surprisingly complex and broad Gamow-Teller strength distributions. The results are compared to existing experimental data and the resulting hindrance factors are discussed.

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One of the primary new directions in nuclear spectroscopy is in the experimental study and theoretical understanding of nuclei near the limits of particle stability. The heaviest nucleus with an equal number of protons and neutrons predicted to be stable is ^{100}Sn, and experiments are being planned and carried out at several laboratories to produce and study the decay of this nucleus [1] and others [2,3] in this mass region. One of the most interesting aspects of these proton-rich nuclei is that most of the giant Gamow-Teller resonance lies within the beta-decay Q-value window. We report here on new calculations which show some of the unusual features which one may expect to see in these decays, the special problems associated with their experimental detection and the important nuclear structure information that will be obtained.

Our model space, which is similar to that of a number of other calculations [4–7], is designed for nuclei with $Z\leqslant 50$ and $N\geqslant 50$ and starts from a closed-shell configuration for ^{100}Sn. We will later discuss the effects of going beyond the closed shell configuration. In the model space we designate by SNA, proton holes are allowed to occupy the $0f_{5/2}$, $1p_{3/2}$, $1p_{1/2}$, and $0g_{9/2}$ orbitals, and the neutron particles occupy the $0g_{7/2}$, $1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$, and $0h_{11/2}$ orbitals. The single-particle energies (SPE) and two-body matrix elements (TBME) for the protons in model space SNA are those of Ji and Wildenthal [8] which were obtained from a least-squares fit to energy levels of the $N=50$ isotones. For the neutron residual interaction, we started with a set of TBME obtained from a similar least-squares fit to the $N=82$ isotones with a ^{132}Sn [9] core in which the protons fill the same set of orbitals as do the neutrons outside of the ^{100}Sn core. We then subtracted a calculated Coulomb interaction and scaled the resulting TBME by a factor of $(132/100)^{0.3}$. The scaling approximately takes into account the change in size of the valence wave functions between ^{132}Sn and ^{100}Sn. The proton-neutron interaction was calculated from the bare G matrix of Hosaka [10], which is based on the Paris potential. Finally, the neutron single-particle energies were determined from a consideration of the "single-particle" states observed for the odd-even $N=51$ nuclei and will be discussed below.

We are interested in calculating the level structure and decay properties for as many nuclei as possible away from ^{100}Sn. We are also constrained by computational limitations to the consideration of J-T Hamiltonian matrix dimensions below about 10 000. In model space SNA this constraint limits the β^+decay calculations to those initial nuclei with $N_p+N_n\leqslant 4$, where N_p are the number of valence proton holes and N_n are the number of valence neutron particles. To go to larger N_p values, we investigated model space SNB in which only the $1p_{1/2}$ and $0g_{9/2}$ proton orbitals are active. The interaction is, of course, model-space dependent, and we replace the Ji-Wildenthal SPE and TBME with the seniority conserving interaction of Gloeckner and Serduke [11]. With these changes (and keeping the neutron and proton-neutron parameters the same), we recalculated the Gamow-Teller decay spectrum of ^{98}Cd and found it to be essentially the same as that obtained in the larger SNA model space. (This result disagrees with similar comparisons made in Refs. [4,5]. This is related to the fact that the previous work did not take into account the renormalization of the proton-proton interaction going from SNA to SNB.)

Finally, we come back to a discussion of the neutron single-particle energies and the related proton-neutron interaction which are particularly important for the Gamow-Teller decay properties. The ground states of all known odd-even isotones with $N=51$ from ^{89}Sr to ^{97}Pd have $J^\pi=5/2^+$. One-neutron transfer reactions on ^{88}Sr and ^{90}Zr establish these as $1d_{5/2}$ single-particle states and also provide information on the location of the excited $0g_{7/2}$, $1d_{3/2}$, and $2s_{1/2}$ states [12]. In addition, it is known that the excitation energy of the $7/2^+$states comes down linearly from about 2.0 MeV in ^{91}Zr to about 0.6 MeV in ^{97}Pd [7]. A reduction of the gap between the $0g_{7/2}$ and $1d_{5/2}$ single-particle states is obtained in the SNB model space due to the relatively large proton-neutron TBME connecting the $0g_{9/2}$ and $0g_{7/2}$ orbitals. However, the reduction compared to experiment is too strong by about 30%. Better agreement can be obtained by renormalizing the proton-neutron G matrix elements by a factor of 0.7. This renormalization improves agreement with experiment for the absolute change in the neutron SPE between ^{89}Sr and ^{97}Pd, and also improves the agreement with the location of the strong GT states in the β^+decay of ^{98}Cd.

Thus, we have adopted this renormalization for all calculations within the SNB model spaces. The absolute single-particle energies in units of MeV relative to a ^{100}Sn closed shell in model space SNB are for protons -3.38 ($1p_{1/2}$) and -2.99 ($0g_{9/2}$), and for neutrons -10.15 ($0g_{7/2}$), -10.10 ($1d_{5/2}$), -8.09 ($1d_{3/2}$), -8.40 ($2s_{1/2}$), and -7.85 ($0h_{11/2}$). It is interesting to note the crossover of the $0g_{7/2}$ and $1d_{5/2}$ states in ^{101}Sn relative to the other $N=51$ nuclei, and it would be very important to have an experimental confirmation of the ground state spin and level structure of ^{101}Sn. The low-lying position of the $0g_{7/2}$ orbital is very important for the $M1$ and GT properties in this mass region. Standard Skyrme Hartree-Fock and Woods-Saxon potential models, whose parameters are determined from the properties of nuclei near the valley of stability, predict the $0g_{7/2}$ orbital to be *less* bound than the $1d_{5/2}$ orbital by 0.5 to 2.0 MeV [13].

Levels schemes and decay properties of many nuclei have been calculated and compared to experiment. High-spin yrast levels in ^{104}Sn, ^{105}Sn, ^{106}Sn, ^{102}In, ^{103}In, ^{98}Cd, ^{100}Cd, ^{101}Cd, ^{102}Cd, ^{97}Ag, ^{98}Ag, ^{99}Ag, ^{96}Rh, and ^{97}Rh calculated in the SNB model space were found to agree with experiment to within a few hundred keV. (Our results for ^{104}Sn and ^{106}Sn are in somewhat better agreement with experiment than those obtained with the G matrix approach of Engeland *et al.* [14].) In addition, the splittings of the low-lying $1/2^-$ and $9/2^+$states in the odd-proton nuclei and the $5/2^+$and $7/2^+$states in the odd-neutron nuclei are reproduced, and the closely spaced states in the low-lying odd-odd multiplets [5,15] are reproduced about as well as the results of previous calculations [6,16].

We concentrate, in this Rapid Communication, on the Gamow-Teller (GT) β^+decay properties of nuclei near ^{100}Sn. We will compare with recent experiments and comment on the significance of the predictions for future experiments. First we discuss the decay of the even-even $N=50$ isotones which have been the subject of several previous theoretical calculations [4–6,17]. In Fig. 1 experimental B(GT) values deduced from the β^+decay of ^{94}Ru [18], ^{96}Pd [15], and ^{98}Cd [5] are compared to the SNB calculation. For purposes of comparison, the theoretical B(GT) values have been divided by four which is approximately the hindrance factor observed in the ^{98}Cd decay (but not the calculated hindrance factors to be discussed below). We will concentrate first on the shape of the GT strength distribution and then discuss to the origin of the hindrance. The dashed line represents the experimental sensitivity limit—that is, a B(GT) of this value would result in a gamma transition which is too weak to be observed in the present experiments. The calculated GT strength distributions are in reasonable agreement with experiment. The small Q_{ec} window for the ^{94}Ru decay allows for only a small fraction of the GT strength to be observed experimentally. But, by the time one reaches ^{98}Cd, the Q_{ec} window is large enough to allow for most of the calculated strength to be observed experimentally. The Skouras and Manakos calculations [4] obtain the mean energy of the GT distribution of ^{98}Cd 0.5 to 1.0 MeV too high compared to experiment. In the present calculation, the mean energy of the GT distribution was lowered and brought into better agreement with experiment when the proton-neutron TBME were renormalized by the factor of 0.7 discussed above.

The total GT strength extracted from the ^{98}Cd decay experiment is $3.5^{+0.8}_{-0.7}$ compared to a total of 13.4 calculated to

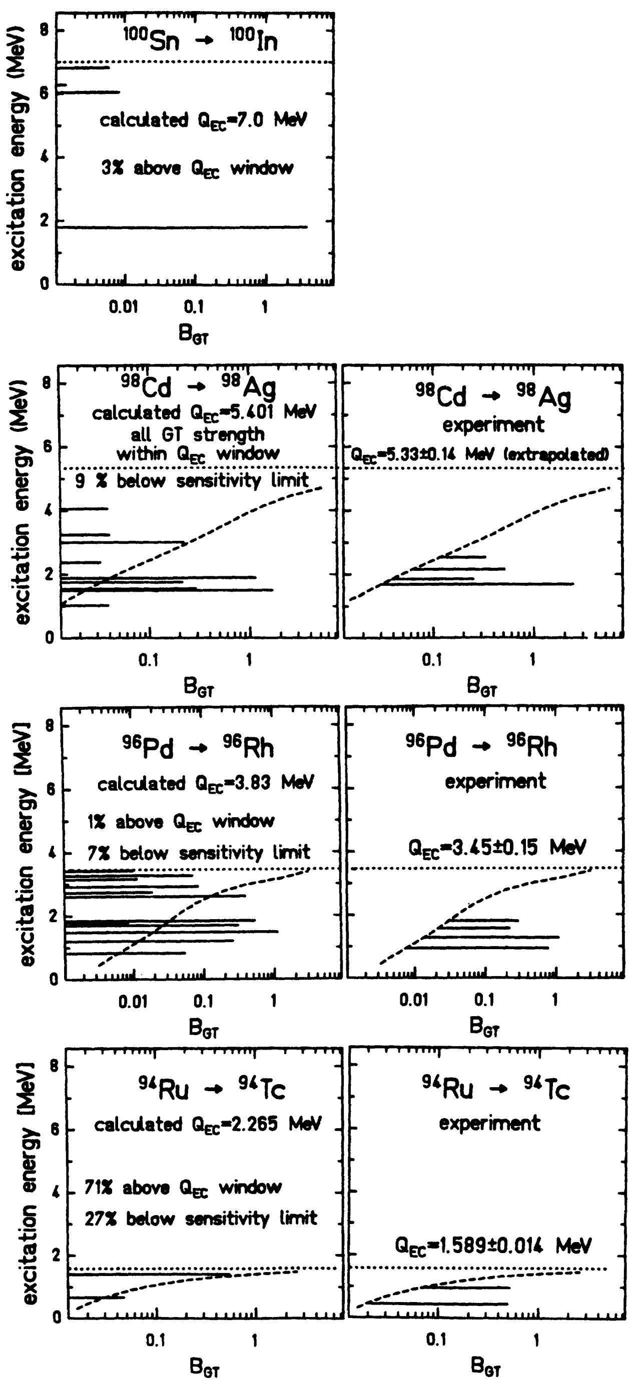

FIG. 1. Gamow-Teller strength distributions for the even-even $N=50$ isotones. The theoretical calculations on the left are compared to experiment on the right. For this comparison the theory has been divided by a factor of 4 (see the text for a detailed discussion of the experimental and theoretical hindrance factors). The amount of GT strength which lies outside the sensitivity limit and Q_{ec} window is indicated.

lie within the sensitivity limit. Experiment is thus hindered by about a factor of $h_{\rm exp}=3.8^{+0.7}_{-0.6}$ compared to theory. Understanding this hindrance is important in general and in particular for the calculations of the nuclear double-beta decays [19] which are used to set limits on the neutrino mass.

In the $0d1s$ shell nuclei ($A=16-40$) one observes a factor of $h_{\rm high}=1/0.6=1.67$ hindrance when experimental GT strengths are compared to those calculated within the *full* $0d1s$ model space [20]. From comparison of $M1$ and GT matrix elements one can deduce that about two-thirds (in the amplitude) of this comes from higher-order configuration mixing while one-third comes from the delta-particle nucleon-hole admixture [21]. Observation of approximately the same hindrance factor for the total β^- strength in heavy nuclei deduced from (p,n) reactions [22] indicates that the mass dependence of higher-order and delta admixture effects is not large, and one may expect about the same factor of $h_{\rm high}=1.67$ to contribute in the ^{100}Sn region. This leaves another factor of $h_{\rm exp}/h_{\rm high}=2.3\pm0.4$ to be understood.

The calculation for ^{100}Sn in the SNB model space is extremely simple—just a single $0g_{9/2}$ proton hole—$0g_{7/2}$ neutron particle final state with a B(GT)=17.8. Instead of this simple calculation, we show in Fig. 1 a calculation within a two-particle–two-hole (2p2h) model space. The 2p2h model space [23] allows for 2p2h admixture in the ^{100}Sn initial state and 2p2h and 3p3h admixture in the ^{100}In 1^+final states, and thus explicitly includes the core-polarization correction calculated in perturbation theory by Towner [24] and Johnstone [6], as well as some higher-order terms. The dimension of the final state is about 6000, and it is not possible to include more particle-hole states in the calculation or to carry out a similar calculation for ^{98}Cd. The results for ^{100}Sn are very interesting. The lowest 1^+state remains predominantly 1p1h in structure but the strength is reduced to 80% of that calculated in the SNB model space [25]. The final states which have a predominantly 2p2h and 3p3h structure do not start in the spectrum until about 6 MeV in excitation and carry only a few percent of the total GT strength. For the analogous calculations in the $0d1s$ and $0f1p$ shells [26,27] the simple state and complex states are nearly degenerate in energy resulting in a spreading of the GT strength over many states (a large spreading width). The very different result for ^{100}Sn is due to the relative reduction of the residual interaction compared to the $0g_{9/2}$-$0g_{7/2}$ spin-orbit splitting and to the fact that both the $0g_{9/2}$ and $0g_{7/2}$ orbitals lie next to the Fermi surface. As has been pointed out [28], it is the Coulomb interaction which pushes the proton $0g_{9/2}$ SPE above the neutron $0g_{7/2}$ SPE and opens up the Q-value window for this strong GT decay. The $0g$ hindrance factor we obtain for ^{100}Sn of $h_{0g}=1.25$ is smaller than the results obtained in perturbation theory by Johnstone [6] ($h_{0g}=1.60$) but consistent with the interaction-dependent range given by Towner [17] ($h_{0g}=1.29-1.71$).

Some Z dependence is expected for the $0g$ hindrance factor. The results of Towner and Johnstone for the ratio $h_{0g}(^{98}{\rm Cd})/h_{0g}(^{100}{\rm Sn})$ range from 1.23 to 1.30 and are much less interaction dependent than the actual range of values given above. Assuming a ratio of 1.30, our hindrance factor of $h_{0g}=1.25$ for ^{100}Sn would translate into a factor of $h_{0g}=1.62$ for ^{98}Cd compared to $h_{\rm exp}/h_{\rm high}=2.3\pm0.4$. We speculate in analogy with the $0d1s$ and $0f1p$ shell calculations [26,29] that higher-order mixing between the $0g_{9/2}$ and $0g_{7/2}$ orbitals is responsible for the remaining hindrance in the ^{100}Sn region—such calculations for the ^{100}Sn region may soon be possible within the Monte Carlo shell-model approach [29]. The experimental hindrance obtained for ^{100}Sn compared to that of ^{98}Cd will be important in deciding which hindrance mechanism is most important. [Starting with the SNB model space and $h=2.09$ ($=1.67\times1.25$) and $Q_{ec}=7$ MeV we obtain $T_{1/2}(^{100}{\rm Sn})=0.53$ s.]

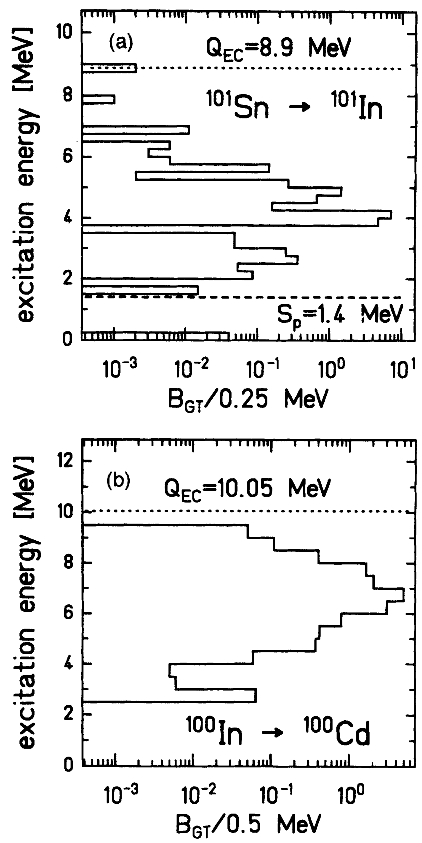

FIG. 2. The calculated Gamow-Teller strength distributions for (a) ^{101}Sn and (b) ^{100}In.

Similar calculations have been also performed for the GT decays of odd-A and odd-odd nuclei in the vicinity of ^{100}Sn. Before this work the GT strength distributions for the decays of non-even-even nuclei in the region of ^{100}Sn were presented only for ^{93}Tc and ^{95}Rh [6]. In this Rapid Communication we present as examples the GT strength distributions obtained for the decays of ^{101}Sn and ^{100}In—the closest neighbors of ^{100}Sn. Over 100 levels in ^{101}In are expected to be fed in the decay of ^{101}Sn (we assume a $0g_{7/2}$ single-particle ground state), see Fig. 2(a). Most of the strength is found at high excitation energies well above the proton separation energy ($S_p \approx 1.4$ MeV) in the ^{101}In isotope. This leads to a beta-delayed proton branching ratio above 40%, and explains why it was possible at all to detect a few tens of the protons assigned to the decay of ^{101}Sn which was produced in the heavy-ion fusion-evaporation reaction and identified at an on-line mass separator with an intensity of 40 atoms per hour [2]. Using $h=4$ we obtain $T_{1/2}(^{101}{\rm Sn})=1.5$ s which is not far from the experimental value of $T_{1/2}=3\pm1$ s [2].

The GT decay of ^{100}In, which has a theoretical ground state spin of 7^+, is shown in Fig. 2(b). Most of the strength is located in a broad symmetric peak centered at about 6 MeV. In addition, a small side peak at about 2.5 MeV can be seen. It is interesting to notice the similarity of calculated GT distribution for ^{100}In with the experimental one obtained for the decay of ^{104}In using the Total Absorption Gamma Spectrometer (TAGS) [30,31]. (The latter decay cannot be calculated due to the large number of neutron valence particles.) The TAGS method allows one, in principle, to obtain the "true" GT distributions even for such complex decays with high gamma multiplicity and statistical gamma cascades following beta decay. The ^{104}In decay is limited by a Q_{ec} value which is about 2 MeV lower than the one for ^{100}In, which results in a cutoff of the GT strength at higher excitation energies. However, the theoretical picture for ^{100}In resembles the main GT strength features measured already for ^{104}In. Using $h=4$ we obtain $T_{1/2}(^{100}\mathrm{In})=6.8$ s, which is close to the experimental result of $T_{1/2}$ of about 6 s [3].

In summary, we predict a very simple β^+ decay mode for ^{100}Sn. The experimental observation of beta-delayed gammas and/or protons will provide a test of the model, and the hindrance factor obtained for this decay compared to that of ^{98}Cd will provide a test of the hindrance mechanism. The calculated GT decays of ^{101}Sn and ^{100}In show the importance of being able to measure the total decay energy in a TAGS experiment.

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