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Multiphonon structure of γ-unstable or O(6) nuclei

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The multiphonon structure is shown for the O(6) limit of the interacting boson model. The phonon states are created by the O(6) quadrupole operator with proper symmetrization. All the $\sigma=N$ states can be described in this scheme in terms of phonon quanta and two-phonon anharmonicity, while the ground state is γ unstable. This structure is carried over into higher-lying $\sigma<N$ states.

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The O(6) dynamical symmetry [1] of the interacting boson model (IBM) [2] has been used for the description of quite a few nuclei, especially in the Xe-Ba [3] and Pt [4] regions. Besides such success in phenomenological description, the O(6) has attracted much interest regarding its interpretation in terms of a more intuitive picture. The commonly accepted picture [5–7] has been the γ-unstable rotor of Wilet and Jean [8]. Its relation to the rigid triaxial rotor of Davidov and Filippov [9] has also been discussed [10–15]. In this Rapid Communication, we shall present a completely different picture of O(6) limit. This is a multiphonon description with a strong ground state correlation, where the phonons are built upon a γ unstable ground state. The aim of this work is to show that the phonon description arises in a natural way from the basic properties of the O(6) Hamiltonian.

We focus upon the $\sigma=N$ eigenstates of O(6), where σ and N denote, respectively, the O(6) quantum number and the total boson number [i.e., SU(6) quantum number] [1,2]. The states with $\sigma<N$ are situated at higher energies for usual boson Hamiltonians, and can be described in a similar way, as stated at the end of this article.

The Hamiltonian we shall consider is

$$H=-\kappa(Q\cdot Q), \tag{1}$$

where κ denotes the strength parameter, the symbol $(\cdot)$ means a scalar product, and

$$Q=d^\dagger s+s^\dagger \tilde{d}, \tag{2}$$

with $\tilde{d}$ being the modified annihilation operator $[\tilde{d}_m=(-1)^m d_{-m}]$. This Hamiltonian is a linear combination of quadratic Casimir operators of O(6), O(5), and O(3) [1,2], and manifests the feature of the quadrupole collectivity of O(6). By this Hamiltonian, we do not lose the generality of the following discussions. We shall comment on this point later. The strength κ is supposed to be positive, and hence Eq. (1) means an attractive quadrupole-quadrupole interaction.

We now construct the ground state for the Hamiltonian in Eq. (1). This Hamiltonian can be rewritten as

$$\begin{aligned}H=-\kappa[\sqrt{5}\{[d^\dagger d^\dagger]^{(0)}ss+s^\dagger s^\dagger[\tilde{d}\tilde{d}]^{(0)}\}+2(d^\dagger\cdot\tilde{d})s^\dagger s\\ +(d^\dagger\cdot\tilde{d})+5s^\dagger s],\end{aligned} \tag{3}$$

where $[\ \ ]^{(L)}$ means the coupling to an angular momentum L. Here, on the right-hand side (RHS), the first two terms are those of the monopole pairing, the third is a monopole-monopole interaction, and the remaining terms are single-particle energies, because $(d^\dagger\cdot\tilde{d})$ is nothing but the d-boson number operator. Therefore, the ground state should be of the form

$$|0_1^+\rangle=\sum_n c_n\{[d^\dagger d^\dagger]^{(0)}\}^n(s^\dagger)^{N-2n}|0\rangle, \tag{4}$$

where $|0\rangle$ is the boson vacuum, and the c_n's stand for amplitudes. Here the RHS should be normalized.

We shall first show the commutation relation

$$\begin{aligned}[Q_M,\ Q_{M'}]&=d_M^\dagger\tilde{d}_{M'}-d_{M'}^\dagger\tilde{d}_M\\ &\equiv R_{M,M'}.\end{aligned} \tag{5}$$

The $R_{M,M'}$ operator in Eq. (5) can be expressed through $[d^\dagger\tilde{d}]^{(1)}$ and $[d^\dagger\tilde{d}]^{(3)}$ operators. We mention that

$$R_{M,M'}|0_1^+\rangle=0, \tag{6}$$

because $[R_{M,M'},\ [d^\dagger d^\dagger]^{(0)}]$ is identically zero. This relation plays a key role in the following procedure.

The commutation relation with the Hamiltonian then becomes

$$[H,\ Q_M]=4\kappa Q_M-2\kappa\sum_m(-1)^m Q_{-m}R_{m,M}\ . \tag{7}$$

This results in

$$[H,\ Q_M]|0_1^+\rangle=4\kappa Q_M|0_1^+\rangle, \tag{8}$$

which means that $Q_M|0_1^+\rangle$ is an eigenstate with the excitation energy 4κ. The state $Q_M|0_1^+\rangle$ is nothing but the first 2^+ state, as seen later.

We proceed to another illustrative example. The states with double Q's can be treated as

$$\begin{aligned}&HQ_MQ_N|0_1^+\rangle\\ &\quad=\{8\kappa Q_MQ_N+2\kappa Q_NQ_M-2\kappa(-)^N\delta_{M,-N}(Q\cdot Q)\\ &\qquad-\kappa Q_MQ_N(Q\cdot Q)\}|0_1^+\rangle,\end{aligned} \tag{9}$$

by using the relation

$$\left[\sum_m (-1)^m Q_{-m}R_{m,M},\ Q_N\right]=(-)^N\delta_{M,-N}(Q\cdot Q)-Q_NQ_M\ , \qquad (10)$$

which arises from the double commutator $[[H,\ Q_M],\ Q_N]$. For the states $[QQ]^{(L)}|0_1^+\rangle$with $L=4$ or 2, the excitation energy turns out to be 10κ, whereas it vanishes for $L=0$. The latter is natural, because $[QQ]^{(0)}|0_1^+\rangle \propto|\ 0_1^+\rangle$. In other words, the double action of the Q operator produces eigenstates of $L=4$ and 2, which are the first 4^+ and second 2^+ states, respectively, as seen later also.

We shall now consider the general cases. In Eq. (9), there are two important features; (i) both Q_MQ_N and Q_NQ_M appear on the right-hand side (RHS), (ii) the third term on the RHS produces nonvanishing effects only for two Q's coupled to $L=0$ because of $\Sigma_{M,N}(2M2N|LM+N)(-)^N\delta_{M,-N}=\sqrt{5}\,\delta_{L,0}$. Considering these points we construct a state as

$$|\Psi\rangle=\sum C(\{M_1,M_2,\ldots,M_n\})\mathscr{S}\{Q_{M_1}Q_{M_2}\cdots Q_{M_n}\}|0_1^+\rangle, \qquad (11)$$

where $\mathscr{S}$ implies a symmetrizer with respect to $M_1,M_2,\ldots,M_n$, and the C's mean amplitudes. By choosing proper C's, the state $|\Psi\rangle$ can have a good angular momentum, and one can introduce a set of $|\Psi\rangle$'s so that different $|\Psi\rangle$'s are orthogonal to each other. Here, we impose a condition on $|\Psi\rangle$ that any pair of two Q's is not coupled to angular momentum $L=0$. Therefore, in the case of $n=2$, only the total angular momenta $L=4$ and 2 (and their linear combinations) are allowed in Eq. (11).

We then consider $H|\Psi\rangle$. The first term of the RHS of Eq. (7) yields $4\kappa Q_{M_i}$ from the same Q_{M_i} at the same place. This keeps the state unchanged. On the other hand, R_{m,M_i} of the second term must form a commutation relation with one of the Q operators further right, because of Eq. (6). Using Eq. (10), one obtains from $[R_{m,M_i},Q_{M_j}]$,

$$+2\kappa Q_{M_1}\cdots Q_{M_j}\cdots Q_{M_i}\cdots Q_{M_n}|0_1^+\rangle, \qquad (12)$$

where $\mathscr{S}$ and the C's are omitted for brevity. Note that Q_{M_i} and Q_{M_j} are interchanged with a factor 2κ in Eq. (12) due to the double commutation discussed above. The first term on the RHS of Eq. (10) does not contribute because no pair of the Q's is coupled to $L=0$, as required in the construction of the state $|\Psi\rangle$. Thus, one ends up with

$$H|\Psi\rangle=\{4\kappa n+2\kappa\tfrac{1}{2}n(n-1)\}|\Psi\rangle+E(0_1^+)|\Psi\rangle, \qquad (13)$$

for all states constructed according to Eq. (11). Here $E(0_1^+)$ is the energy of the ground state. Table I shows the energy levels of some low-lying states, highlighting several characteristic features.

We would like to mention several points: (i) the energy level is determined only by n, i.e., the number of the Q's, (ii) the energy level can be expressed by n and $\frac{1}{2}n(n-1)$ which can be viewed as a one phonon energy and its anharmonicity, (iii) the symmetrizer in Eq. (11) produces only phononlike states, (iv) two Q's coupled to $L=0$ is forbidden. The first three points strongly suggest that the phonon structure dominate the present system. It is evident that the Q operator with the symmetrization plays the role of the phonon operator. Note that n stands for the number of the phonon quanta.

TABLE I. Classification scheme of lowest O(6) eigenstates (of σ=N) in terms of the number of phonon quanta (n) and the excitation energies (E_x) normalized by 4κ.

Number of phonon quanta (n)	$E_x/(4\kappa)$	Angular momenta of eigenstates
0	0	0 (ground state)
1	1	2
2	2.5	4, 2
3	4.5	6, 4, 3, 0
4	7	8, 6, 5, 4, 2
...	...	...

The fourth point is due to the strong ground state correlation, which can be seen in the structure of the Q operator; the $d^\dagger s$ term of Eq. (2) is the usual "phonon creation" operator as is in the U(5) limit of IBM [16]. The second term, $s^\dagger d$, corresponds to the so-called backward amplitude in the random phase approximation, and annihilates $L=0$ pairs of the d bosons (i.e., $[d^\dagger d^\dagger]^{(0)}$) when it is acting on $|0_1^+\rangle$. We need this second term with the equal strength as the first term, in order to make up the present scheme. On the other hand, it should be noticed that the backward-going contribution in the present case is probably much stronger than that obtained in the random phase approximation where the backward-going contribution should remain reasonably weaker than the forward one.

The symmetrization and the elimination of two Q's coupled to $L=0$ in Eq. (11) imply that the states constructed in Eq. (11) can be classified in terms of the τ quantum number of O(5) as a matter of mathematics. In fact, also from the comparison between energy levels of Eq. (13) and those of the O(6) limit, one finds that the states of n in Eq. (11) are nothing but the states of $\tau=n$ in the O(6) limit with the excitation energy rewritten as $\kappa\tau(\tau+3)$. Thus, it turns out that all the states of $\sigma=N$ are created by Eq. (11).

The possible use of the Q operator in the classification of the O(6) eigenstates has been mentioned in Ref. [17]. It was shown in Ref. [17] that low-lying O(6) states can be constructed by successive operations of the Q's, whereas the pattern of the energy levels (i.e., phonon quanta and anharmonicity), the dynamical origin of the phonon structure, and the precise manner of constructing the wave functions have remained untouched in Ref. [17]. Thus, the "Q construction" introduced in Ref. [17] means the interrelation among low-lying O(6) wave functions, and hence does not fully suggest the phonon structure.

It is of interest that one can obtain the present ground state exactly from the γ-unstable intrinsic states with the integration over the γ variable [6,7], while one can extract the ground state in a good approximation from the rigid-triaxial intrinsic state of $\gamma=30°$ for smaller boson numbers

[14,15]. Clearly the ground state is characterized also as a γ-unstable or triaxial state, and then it is most likely that the phonons introduced in this note preserve the γ softness to a good extent. This point should be better clarified in the future. We would like to point out that the present result does not contradict the γ-unstable or triaxial nature of the O(6) system as a whole. We should stress, on the other hand, that the excitation mechanism is indeed of the phonon nature. Combining with the conventional phonon picture for the U(5) limit, this new feature may be viewed as a support to a recent observation by Casten *et al.* [18] that low-lying collective levels of most even-even nuclei except for strongly deformed ones can be described in terms of phonons with anharmonic terms.

There are higher-lying states with $\sigma<N$ in the O(6) spectrum [1,2]. The lowest state of a given $\sigma(<N)$ is a 0^+ state, which contain $(N-\sigma)/2$ boson pairs with a specific structure. This pair is monopole, and is referred to usually as the $\mathscr{S}$ pair [1,2] (or the S pair [19]). The $\mathscr{S}$ pair is not included in the ground state in Eq. (4). This lowest state of $\sigma(<N)$ can be decomposed into a sector created solely by the $\mathscr{S}$ pairs and the rest [19]. In other words, this state is created by $(N-\sigma)/2$ times successive actions of the $\mathscr{S}$ pair-creation operator on the rest part. This rest part has a similar structure to the ground state in Eq. (4), but consists of $\sigma(<N)$ bosons. The phonon operator, Q, commutes with the $\mathscr{S}$ pair operators, and acts only to the rest part. Thus, the Q operator produces phonon excitations without disturbing the $\mathscr{S}$ pair sector. To be more precise, the phonon operator conserves the σ quantum numbers, and the phonon excitation occurs within a subspace belonging to the given σ. Thus, one can construct all the states of an O(6) nucleus in terms of the multiphonon excitation and the $\mathscr{S}$ boson pairing mode [19].

We have chosen the Hamiltonian in Eq. (1). There are three independent terms in the general O(6) Hamiltonian [1,2]. Besides the present term in Eq. (1), one of them is the total angular momentum, which does not change the wave function and yields the trivial variations of the energies. The third term can be the pairing interaction for the $\mathscr{S}$ boson pairs [1,2]. This interaction shifts all the levels of a given σ by the same amount. It does not change relative energies for the states belonging to the same σ. By including this interaction, the wave functions are not changed either. Thus, the above discussions based on the Hamiltonian in Eq. (1) are quite general for the O(6) limit.

In summary, we have presented that the low-lying $(\sigma=N)$ O(6) states are multiphonon states built upon the γ unstable ground state, where the ground state correlation is dominant and a rather large number of d bosons are contained reflecting a strong deformation. The energies are represented in terms of phonon quanta and two-phonon anharmonicity. This consequense appears to be different from the usual picture of O(6) as a γ unstable "rotor," although the triaxial nature is inherent in this multiphonon picture through the ground state. The $\sigma(<N)$ states are constructed as a product of the $\mathscr{S}$ pair sector and the present multiphonon states.

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