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# New interpretation of the lowest $K=0$ collective excitation of deformed nuclei as a phonon excitation of the $\gamma$ band

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Interacting-boson approximation calculations of the properties of the lowest $K=0$ intrinsic excitation of deformed nuclei suggest that this mode is not a $\beta$ vibration as is traditionally thought but rather it is a collective phonon *built on* the $\gamma$ vibration. This conclusion is in agreement with a wide body of experimental data concerning the $K=0$ excitation.

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The structure of the lowest $K=0$ excitation of deformed nuclei has always been an enigma. Traditionally described [1] as a "$\beta$" vibration, that is, as a quadrupole oscillation in the axially symmetric $\beta$ degree of freedom, its properties are empirically erratic [2–4] and, theoretically, have never been described as successfully as those of the $\gamma$ vibration [1,5–7]. Other components, such as pairing modes, have also been suggested [8] to play an important role. Although it seems to be a collective mode (it lies well below the pairing gap) its $B(E2)$ values to the ground state are 1–2 orders of magnitude less than those of the $\gamma$ band.

These perplexing aspects of the structure of the lowest $K=0$ excitation (which we will frequently denote here as the $0_2^+$ state, with $0_g^+$ referring to the ground state and $2_\gamma^+$ referring to the bandhead of the lowest $K=2$ intrinsic excitation) lay more or less simmering for a decade or so until the advent of interacting-boson approximation (IBA) predictions [9,10] for deformed nuclei, including the unexpected prediction that there should be *collective* $0_2^+\rightarrow\gamma$ $E2$ transitions with matrix elements comparable to those for $\gamma\rightarrow g$ transitions. Such transitions would be forbidden for a true $\beta$ vibration since they would involve a two-phonon change of structure (destruction of $\beta$ vibration, creation of $\gamma$ vibration). When these collective $0_2^+\rightarrow\gamma$ transitions were subsequently discovered [2,10,11] (first in $^{168}$Er and Gd, and then in Dy, Yb, and Hf nuclei) the question of the structure of the $0_2^+$ mode was again raised.

In a recent survey [2] of a number of properties of deformed nuclei, the empirical phenomenology of the lowest excited $K=0$ band was given. Despite considerable fluctuations in properties its most salient and pervasive aspects are as follows:

(i) $B(E2)$ values to the ground-state band that are weak compared to $\gamma\rightarrow g$ $B(E2)$ values:

$$B(E2{:}0_2^+\rightarrow g)\sim(10^{-2}-10^{-1})B(E2{:}\gamma\rightarrow g).$$

(ii) Collective $B(E2{:}2_\gamma^+\rightarrow 0_2^+)$ values, comparable (within a factor of 4 or so) to $B(E2{:}2_\gamma^+\rightarrow 0_g^+)$ values.

(iii) Excitation energies $E(0_2^+)$, that are almost always between 0.8 and 1.8 times that of the $\gamma$ band.

(iv) Properties which are linked to those of the $\gamma$ band. In particular, a plot of $K=0$ band properties against $\gamma$-band properties, such as $E(0_2^+)/E(\gamma)$ vs $B(E2{:}2_\gamma^+\rightarrow 2_g^+)/B(E2{:}2_\gamma^+\rightarrow 0_g^+)$ shows a strong correlation.

Two of these properties are recapitulated in Fig. 1, which is based on Ref. [2].

Were it not for the persistent tradition of perceiving the lowest $K=0$ mode as a $\beta$ vibration, it is obvious that the above collection of properties would naturally be considered indicative of a collective excitation built upon the $\gamma$ band.

It is remarkable that the IBA model predicts [2,9–11] exactly the empirical properties of the lowest $K=0$ excitation discussed above. Indeed these predictions are a robust feature of the model: they are inherent to the basic structure of the model and cannot be avoided. Therefore it would seem that a further study of $K=0$ bands in the framework of the IBA might shed useful light on their nature.

It is the purpose of this Rapid Communication to do this. We will investigate the possible model candidates for phonon excitations of the $\gamma$ band by studying the calculated $B(E2)$ values for the deexcitation of *all* $K=0$ excitations in the IBA-1 space and will also study the decay modes of the lowest $K=0$ excitation.

Our procedures and the results are simple. Taking off from the fact, empirically and in the IBA, of collective transitions from the lowest $K=0^+$ band to the $\gamma$ band, we investigate the decay of *all* calculated $K=0$ bands into the $\gamma$ band. We use the following IBA-1 Hamiltonian [11] which is a standard one for deformed nuclei:

$$H=-\kappa Q\cdot Q \tag{1}$$

where

$$Q=(s^+d+d^+s)+\chi(d^+d)^{(2)}, \tag{2}$$

and we use the consistent $Q$ formalism [12] in which

$$T(E2)=e_BQ, \tag{3}$$

where $e_B$ is a boson effective charge. In Eq. (1), $\kappa$ is an overall energy scale factor and is of no consequence for $B(E2)$ values and wave functions. Hence the only structural parameter is $\chi$, which can vary from 0, which gives the 0(6) symmetry, to $-\sqrt{7}/2=-1.32$, which gives the SU(3) symmetry. Virtually all deformed nuclei can be well described

 

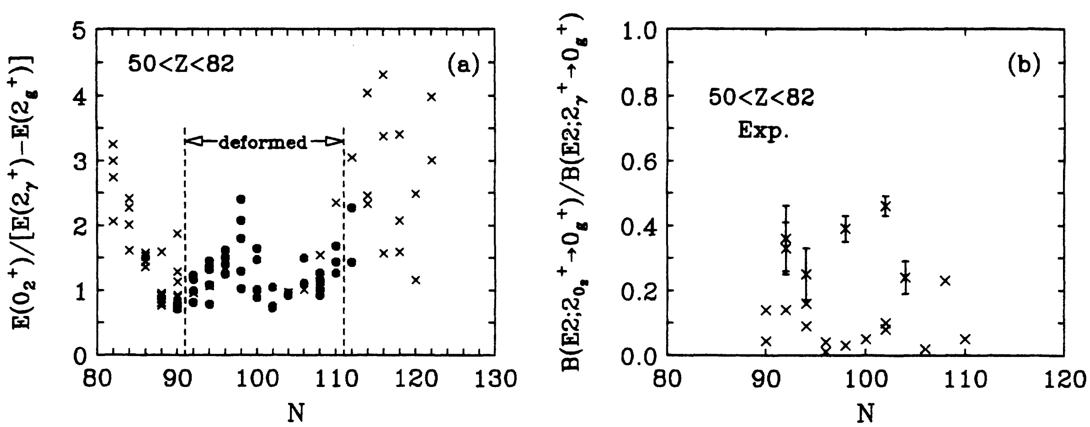


FIG. 1. Empirical properties of the lowest $K=0$ and $\gamma$ bands for the rare-earth region. Based on Ref. [2]. (a) Ratios of vibrational energies, $E(0_2^+)/[E(2_\gamma^+)-E(2_g^+)]$. (b) The ratio $B(E2{:}2^+[K=0_2^+]\to 0_1^+)/B(E2{:}2_\gamma^+\to 0_1^+)$.

[10] with values of $\chi$ near $\chi=-0.45$. Only a few [13] actually approach the limiting case of an SU(3) symmetry. Note that an $L\cdot L$ term in Eq. (1) has no effect on $B(E2)$ values.

We carried out a series of calculations, using Eqs. (1)–(3) with $\chi=-0.45$, of $B(E2{:}0_i^+\to 2_\gamma^+)$ values ($i>1$, i.e., of transitions from all excited $0^+$ states to the $2_\gamma^+$ level), as a function of boson number $N_B$ for $N_B=6-16$. The reason for emphasizing the boson number dependence is evident from a coherent state analysis of the ground, $K=0$, and $\gamma$ excitations of the IBA. Bijker and Dieperink [14] have shown that, for large $N_B$,

$$\langle g|T(E2)|g\rangle \sim e_B N_B , \tag{4}$$

$$\langle \gamma|T(E2)|g\rangle \sim e_B N_B^{1/2} , \tag{5}$$

$$\langle 0_2^+|T(E2)|g\rangle \sim e_B N_B^{1/2} , \tag{6}$$

$$\langle \gamma|T(E2)|0_2^+\rangle \sim e_B . \tag{7}$$

Thus, when intraband $B(E2{:}2_1^+\to 0_1^+)$ values are normalized to empirical values, fixing $e_B$, $0_2^+\to\gamma$ matrix elements will vanish in the $N_B\to\infty$ limit. In that limit, the properties of the lowest $K=0$ excitation go over into those of the traditional $\beta$ band with $B(E2)$ values to the ground band of $\sim\frac{1}{6}$ those of the $\gamma$ band, and with no $B(E2)$ values to the $\gamma$ band itself. However, for realistic $N_B$, the properties cited earlier apply and we can expect them to be accentuated for low boson number. For very low $N_B$ the distinction between rotational and vibrational motion begins to blur, especially for spins $I>2$ (intraband and interband matrix elements approach similar values, and "rotational" spacings are erratic). Nevertheless, for $0_i^+\to 2_\gamma^+$ transitions the trend is clear and informative.

The results are shown in Fig. 2, in terms of the ratio

$$R\left(\frac{0_2^+\to\gamma}{\Sigma 0_i^+\to\gamma}\right)\equiv\frac{B(E2{:}0_2^+\to 2_\gamma^+)}{\Sigma_{i>1}B(E2{:}0_i^+\to 2_\gamma^+)} . \tag{8}$$

This is simply the fraction of the total calculated $E2$ strength from all excited $0^+$ states to the $2_\gamma^+$ level that comes from the *lowest* excited $0^+$ state. The results are striking. The $0_2^+$ state nearly exhausts all the $B(E2)$ strength. Moreover, the *percentage* it represents steadily increases from $\sim 75\%$ for $N_B=16$ to 95% for $N_B=6-8$. *No other excited $0^+$* state decays significantly to the $2_\gamma^+$ level.

Figure 2 contains two other results that may help to illuminate the structure of the $0_2^+$ excitation. The lowest curve gives the calculated values of

$$R\left(\frac{2_1^+\to 0_2^+}{2_\gamma^+\to 0_2^+}\right)\equiv\frac{B(E2{:}0_2^+\to 2_1^+)}{B(E2{:}0_2^+\to 2_\gamma^+)} . \tag{9}$$

The ratios almost vanish: they are always less than 0.05: the IBA predicts that the lowest $K=0$ excitation decays much more strongly to the $\gamma$ band than to the ground-state band.

The middle curve of Fig. 2 shows that the ratio

$$R\left(\frac{2_\gamma^+\to 0_2^+}{2_\gamma^+\to 0_1^+}\right)\equiv\frac{B(E2{:}2_\gamma^+\to 0_2^+)}{B(E2{:}2_\gamma^+\to 0_1^+)} , \tag{10}$$

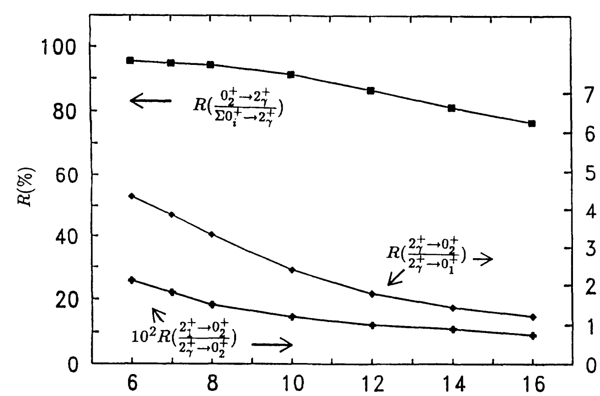


FIG. 2. $E2$ properties of the lowest $K=0$ excitation in the IBA. See text and Eqs. (8)–(10) for definitions of the ratios $R$. The topmost curve gives the percentage of the total calculated $E2$ strength from *all* excited $K=0$ bands to the $\gamma$ band [$\Sigma_i 0_i^+\to 2_\gamma^+$] that is contained in the $B(E2)$ value from the *lowest* $K=0$ excitation. Note that the two lower curves use the right-hand scale, and that the lowest has been multiplied by 100: the actual values are $\sim 0.01$.

the ratio of $\gamma \to 0_2^+$ to $\gamma \to g$ $B(E2)$ values, is of order unity. The lowest $K=0$ excitation decays to the $\gamma$ band with collective $B(E2)$ values typical of vibrational modes in deformed nuclei.

We can summarize these results. IBA calculations of the lowest $K=0$ excitation in deformed nuclei show that it decays to the $\gamma$ band with *collective* $B(E2)$ values, that *no other* excited $K=0$ excitation does so, and that the lowest $K=0$ excitation does *not* decay collectively to the ground band. These properties of the lowest $K=0$ intrinsic excitation agree very well with the experimental situation [2]. [$B(E2)$ values from higher lying $K=0$ excitations to the $\gamma$ band are seldom known experimentally.]

Thus, all the available results, both experimental and theoretical, suggest a new interpretation of the lowest $K=0$ excitation. Instead of being thought of in the traditional sense of a vibration in the $\beta$ degree of freedom (axially symmetric oscillation of a quadrupole ellipsoid), it has all the earmarks of a *collective phonon excitation built on the* $\gamma$ *band.* It is not clear whether this phonon excitation should be viewed as an *independent* $K=2$ excitation superposed on the $\gamma$ band or as a two-phonon double $\gamma$ vibration. What is clear at the moment is that, in the IBA, which in all other respects accounts for the energetic and decay properties of the lowest $K=0$ band, there is at least *no other* plausible candidate for a $\gamma\gamma K=0$ excitation since no other $K=0$ band decays to the $\gamma$ band. Further experimental study of this is necessary and encouraged with particular emphasis on searches for other empirical candidates for $K=0\,\gamma\gamma$ vibrations.

Given the apparent two-phonon nature of the lowest $K=0$ excitation in deformed nuclei, theoretical calculations (e.g., of random-phase approximation type) incorporating a multiphonon basis, are also called for. For the $K=4^+$ excitation (e.g., in $^{168}$Er), calculations incorporating multiphonon excitations of $K=2$ type have been successful (see, e.g., Ref. [15]). Analogous studies for the $K=0$ mode should be sufficiently thorough to allow for all plausible components in the wave functions, including the $\beta$ vibrational mode, pairing vibrations (whose role has been discussed in Ref. [9], multiphonon excitations based on the $\gamma$ vibration, and two quasiparticle components. They should investigate the relation of low-lying and higher $K=0$ intrinsic excitations as well and should take careful account to reproduce the collectivity of $K=0^+ \to \gamma\ E2$ transitions as well as the other empirical properties summarized earlier in this paper.

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