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Simplified $\alpha+4n$ model for the ^{8}He nucleus

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A simple five-body cluster orbital shell model approximation is proposed to describe the ground state wave function of the ^{8}He neutron-halo nucleus. The spatial angular correlations, the geometry of the system, the α-particle, and the valence neutron momentum distribution are calculated analytically. With a single free parameter the model is able to reproduce the experimental geometry of ^{8}He as well as the experimental transverse momentum distribution of the ^{6}He nucleus from ^{8}He fragmentation on a carbon target, at high energy.

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In studies of light nuclei far from stability, besides the famous example of the ^{11}Li neutron-halo nucleus [1], the neutron rich ^{6}He and ^{8}He isotopes are known to have extended valence neutron distributions, called neutron halos or "thick neutron skins [2]," For the cases of ^{11}Li and ^{6}He, microscopic calculations in different three-body (core $+2n$) approaches predict exotic correlations in the neutron halos (see [3] and references therein). The ^{8}He nucleus contains four valence neutrons and has the largest neutron to proton ratio among the known bound nuclei. The extended experimental investigations of ^{8}He began recently and the first results are now available [4,5]. We can also mention that spectra of fragments from the ^{8}He breakup processes on different targets at low [6] and intermediate [7] energies will soon be available. For these reasons it is necessary to have a theoretical model, which provides the opportunity to calculate the correlations and particle momentum distributions for the ^{8}He nucleus.

In the present work we generalize a three-body cluster orbital shell model approximation (COSMA), constructed in Ref. [8] on the basis of the cluster shell model [9], to a five-body $(\alpha+4n)$ case in order to describe the ^{8}He ground state wave function (WF). We recall here that the COSMA is translationally invariant, it treats the Pauli principle between valence neutrons strictly, and it combines the advantages of the shell and cluster models. The COSMA accounts rather well for both the experimental data and for the features of the more advanced theory for the cases of ^{6}He and ^{11}Li [3,10], indicating this should also be a viable approach for ^{8}He.

Unlike other five-body approaches [9,11], where the Schrödinger equation with pair-wise potentials was solved for both WF and binding energy, we construct a simple phenomenological WF with one free parameter, determined from experimental data. The resulting WF is used to calculate other observables. For the radial n-α motion, phenomonological oscillator wave functions are used.

In our five body $(\alpha+4n)$ approach for the ground state of ^{8}He $(J^{\pi}=0^{+})$ the total WF is written as $\Phi=F(\alpha)\cdot\Psi(\mathbf{r}_i,\sigma_i)$, $i=1,\ldots,4$. Here $F(\alpha)$ is the α-particle WF and Ψ is the "active" part of the total WF, which describes the motion of the α particle and the valence neutrons and depends on spin variables σ_i (of valence neutrons) and on a set of translation invariant coordinates $\mathbf{r}_i$, $i=1,\ldots,4$, relating the valence neutrons to the center of mass of the α core.[1] To construct Ψ we assume that each valence neutron occupies a $0p_{3/2}$ state relative to the α core. Then we can write Ψ as a Slater determinant constructed

[1]This assumption is justified by results of strict calculations [9], where it was shown that the effect of the $p_{1/2}$ orbit can be neglected in the $^{6-8}$He isotopes (see also [11]). The same conclusion concerning negligible contribution of the $p_{1/2}$ state to the ^{6}He WF, was obtained from the strict three-body calculation [12].

TABLE I. The r.m.s. distances in ^{8}He. $r_\alpha(r_v)$ is the distance between the α particle (valence neutron) and the ^{8}He c.m., $r_{\alpha v}$ is the distance between the α particle and valence neutron, r_{12} is the distance between two valence neutrons, and $r_p(r_n)$ is the distance of a point-proton (neutron) from ^{8}He c.m.

r_α (fm)	$r_{\alpha v}$ (fm)	r_v (fm)	r_{12} (fm)	r_p (fm)	r_n (fm)
0.87	3.47	3.13	4.9	1.69	2.74
				1.76±0.03 [2]	2.60±0.04 [2]

with the help of four single-particle wave functions $\psi_i(\mathbf{r}_k,\sigma_k)$ corresponding to different projections of the $j=\frac{3}{2}$ angular momentum:

$$\psi_i(\mathbf{r}_k,\sigma_k)=\varphi(r_k)\sum_{m_i\nu_i}\langle 1m_i\tfrac{1}{2}\nu_i|\tfrac{3}{2}(\tfrac{3}{2}+1-i)\rangle \times Y_{1m_i}(\mathbf{r}_k/r_k)\chi_{\nu_i}(\sigma_k), \quad (1)$$

$$\varphi(r)=\sqrt{\frac{8}{3\sqrt{\pi}r_0^3}}\left(\frac{r}{r_0}\right)\exp\left(\frac{-r^2}{2r_0^2}\right). \quad (2)$$

Here χ_{ν_i}, $\nu_i=\pm 1/2$, is the spin wave function and $\varphi(r)$ is just the standard $0p$ radial oscillator wave function. Since the valence neutrons completely fill the $0p_{3/2}$ subshell, the WF Ψ is fully antisymmetrized against valence neutron permutations and has quantum numbers $J^\pi=0^+$. The WF Ψ contains only one free parameter, the oscillator length r_0 in Eq. (2). This parameter will be chosen based on the experimental rms matter radius of ^{8}He. Using our model WF Ψ it is straightforward to obtain a relation connecting the matter rms radii of ^{8}He and ^{4}He via r_0. The result is

$$8R^2(^8\text{He})-4R^2(^4\text{He})=\tfrac{35}{4}r_0^2. \quad (3)$$

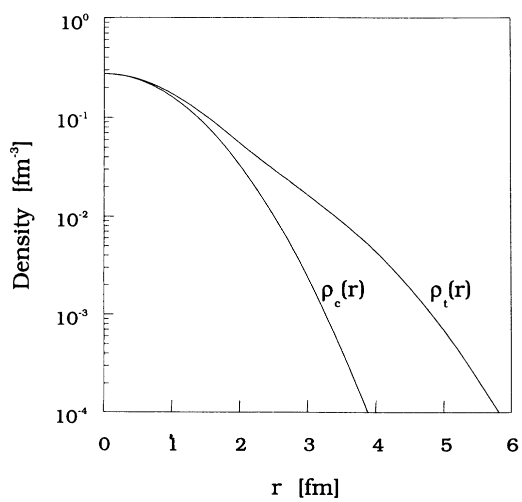

FIG. 1. One nucleon density distributions: the total density $\rho_t(r)=\rho_c(r)+\rho_v(r)$ [(4) and (5)]; the core density $\rho_c(r)$ (4).

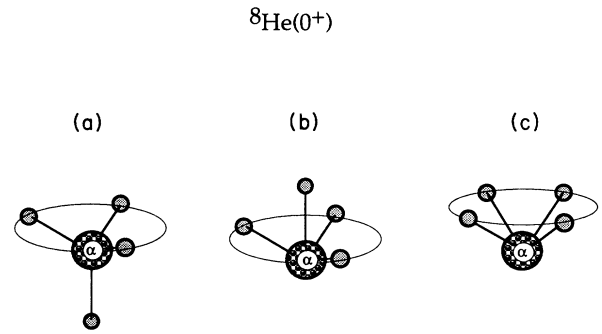

FIG. 2. Three configurations with maximum probability for the angular part of the spatial correlation function [(6)] are shown.

With the values 1.45 fm [3] and 2.52 fm [13] for the matter rms radii of α particle and ^{8}He respectively, the corresponding value for the oscillator length is $r_0=2.20$ fm.

Having fixed the r_0 parameter we are able to calculate all the rms distances in the system. To calculate the rms distance of a core nucleon from the ^{8}He c.m. we assume a Gaussian distribution for core nucleons with the same $R(^4\text{He})=1.45$ fm [3]. The results are shown in Table I and they are in a good agreement with experimental data [2].

In many practical applications it is necessary to know one-nucleon densities for ^{8}He. It is possible to derive such densities from the strict COSMA WF, as in the ^{11}Li case [3,14], but here we suggest more simple formulas for the core nucleon and valence neutron density distributions $\rho_c(r)$ and $\rho_v(r)$. To obtain these formulas, we used a Gaussian distribution for nucleons in the α particle and for the valence neutron distribution the same functional form as in COSMA, but with r calculated from the ^{8}He c.m. The resulting formulas are

$$\rho_c(r)=\frac{1}{\pi}\sqrt{\frac{2}{\pi}}\frac{1}{a^3}\exp\left(\frac{-r^2}{2a^2}\right) \quad \text{where } a=\frac{1.69}{\sqrt{3}}\text{ fm}, \quad (4)$$

$$\rho_v(r)=\frac{8}{3\pi\sqrt{\pi}}\frac{r^2}{b^5}\exp\left(\frac{-r^2}{b^2}\right) \quad \text{where } b=1.99 \text{ fm}. \quad (5)$$

With the parameters a and b from (4 and 5), these densities give the correct ^{8}He matter radius, the same $r_p=1.69$ fm, and the value $r_v=3.14$ fm, which is only slightly larger than the value in the Table I. It should be noted that these densities have already been used in calculations of angular distribution for ^{8}He$+p$ elastic scattering [5,15] and have demonstrated their applicability. In Fig. 1 $\rho_c(r)$ and the total one-nucleon density $\rho_t(r)=\rho_c(r)+\rho_v(r)$ are shown. The pronounced neutron halo is clearly seen in Fig. 1.

To determine the neutron-halo correlation function for ^{8}He $[f_{\text{corr}}(\mathbf{r}_1,\ldots,\mathbf{r}_4)]$, which contains information about the neutron halo structure and the valence neutron correlations, we need to square WF Ψ and sum it over all spin variables. With the COSMA WF Ψ this calculation can be performed analytically [taking into account the explicit form (1)] with the result

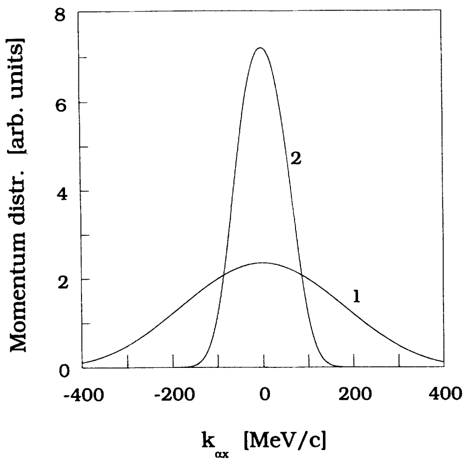

FIG. 3. The α-particle momentum distributions calculated with COSMA WF. See explanations in the text.

$$f_{\rm corr}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)=\varphi(r_1)^2\varphi(r_2)^2\varphi(r_3)^2\varphi(r_4)^2 \times\frac{1}{(4\pi)^4}\frac{3}{4}(S_{12}^2S_{34}^2+S_{13}^2S_{24}^2+S_{14}^2S_{23}^2), \tag{6}$$

where $S_{ij}^2=1-(\mathbf{r}_i\cdot\mathbf{r}_j)^2/(r_ir_j)^2$. As expected, the expression for $f_{\rm corr}$ is invariant under permutation of any two valence neutrons. The angular part in (6), which determines correlations, is a function with pronounced maxima and minima. Only three configurations with maximum probability are shown in Fig. 2.[2] The configuration in Fig. 2(a) corresponds to the most symmetrical arrangement of four valence neutrons in space. The configuration in Fig. 2(b) looks like ^4n and the configuration in Fig. 2(c) resembles ^4n or a pair of "dineutrons." Note that all configurations in Fig. 2 should have a large probability for the ^{8}He β decay to the triton channel [16], according to the almost "prepared" triton correlation.

Another important information about the ^{8}He structure is connected with the momentum distributions of particles in ^{8}He. To calculate these momentum distributions, we need to know the correlation function (6) in the momentum representation. In the framework of COSMA it is easy to show that the correlation function in the momentum representation has the same form as in coordinate space but with coordinates $\mathbf{r}_i$ replaced with quantities $\mathbf{q}_i (i=1, \ldots, 4)$. These quantities can be expressed in terms of the particle momenta $\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4,\mathbf{k}_\alpha$ in the c.m. frame in the form

$$\mathbf{q}_i=\mathbf{k}_i, \quad \mathbf{k}_\alpha=-(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4). \tag{7}$$

Note that the momenta $\mathbf{k}_i$ here are the particle momenta relative to the total c.m., and not to the α core.

[2]The correlation function depends on six angular variables. One extra connection between the angles, which has not been mentioned here, should be added for proper investigation of all possible correlations.

In the frame of the model all necessary calculations are done analytically and we obtain for the α particle transverse momentum distribution, $\rho^\alpha(k_{ax})$, and the neutron transverse momentum distribution, $\rho^n(k_{nx})$,

$$\rho^\alpha(k_{ax})\sim e^{-k_{ax}^2/4k_0^2}\left[1+\frac{1}{9}\left(\frac{k_{ax}}{k_0}\right)^2+\frac{1}{108}\left(\frac{k_{ax}}{k_0}\right)^4\right], \tag{8}$$

$$\rho^n(k_{nx})\sim e^{-k_{nx}^2/k_0^2}\left[1+\left(\frac{k_{nx}}{k_0}\right)^2\right] \quad \text{where } k_0=(\hbar/r_0). \tag{9}$$

Curve 1 in Fig. 3 shows the distribution (8) calculated with the value $r_0=2.20$ fm. This curve can be fitted with a pure Gaussian, $\exp(-k_{ax}^2/2\sigma^2)$, giving $\sigma=172$ MeV/c. This distribution is broad in comparison with the α-particle transverse momentum distribution obtained in COSMA for ^{6}He [8]. First, the fact that we are dealing with four valence neutrons instead of two, as in the ^{6}He case, makes the width σ a factor of $\sqrt{2}$ larger when integrating over all neutron momenta. Second, the oscillator length, $r_0=2.20$ fm, is smaller for ^{8}He [$r_0(^6{\rm He})=2.73$ fm]. This means that ^{8}He is a more compact system and due to Heisenbergs uncertainty principle it should have broader momentum distributions. Finally, the effect of the polynomial in the transverse momentum distribution is different for ^{8}He and ^{6}He. The polynomial in (8) makes the distribution broader. For ^{6}He the effect is opposite, since the coefficient for k_{ax}^2 is negative. This effect reflects the fact that halo correlations (which are different for ^{8}He and ^{6}He) give an essential contribution to the shape and the width of the α-particle momentum distribution.

It is necessary also to comment on the α-particle momentum distribution which could be measured in ^{8}He fragmentation experiments. It was shown in [12] that the dominant reaction mechanism for ^{6}He fragmentation at high energy is connected with one neutron removal process. In this case the α-particle momentum distribution from ^{6}He fragmentation

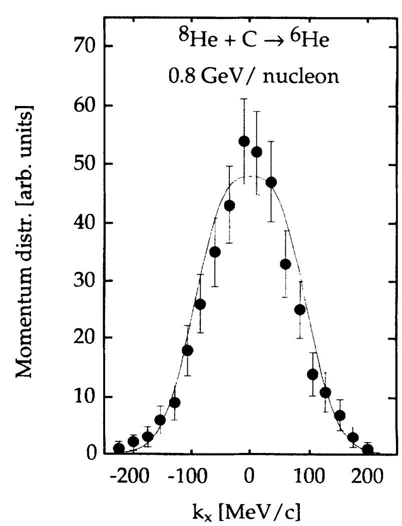

FIG. 4. The ^{6}He transverse momentum distribution. The result of the calculation is shown by the solid line. The experimental data are from [4].

is close to the α-particle distribution in ^{6}He (sudden approximation or Serber mechanism). However, for ^{8}He fragmentation the one neutron removal process would not generate α particles in a straightforward way, but mainly through a creation of $^6He^*(2^+)$ and a sequential decay of this state. We have estimated the α-particle momentum distribution from such a process and it is shown in Fig. 3 (curve 2). This distribution is essentially narrower than curve 1 in Fig. 3. This result shows that the α-particle momentum distribution is very sensitive to the reaction mechanism and therefore may provide important information about the fragmentation mechanism for ^{8}He.

To calculate the ^{6}He transverse momentum distribution from fragmentation of ^{8}He on a carbon target at 0.8 GeV/nucleon we use the valence neutron transverse momentum distribution (9). We assume in accordance with [12] that the dominant reaction mechanism for ^{8}He fragmentation (on light target at high energy) is connected with the one neutron removal process. After one neutron removal we have the ^{7}He subsystem ($n+{}^6$He) with quantum numbers $J^\pi=(3/2)^-$. The ^{7}He motion is determined by the motion of the extra neutron in ^{8}He due to momentum conservation. The residual $n+{}^6$He subsystem being in the $p_{3/2}$ state of relative motion should undergo a final state interaction at the ^{7}He resonance. To describe the ^{7}He decay we use the Breit-Wigner formula with parameters known from experiment ($E_r=0.44$ MeV, $\Gamma=0.16$ MeV). The ^{6}He transverse momentum distribution $\rho^{^6\mathrm{He}}(k_x)$ being the result of the ^{7}He motion and sequential decay of ^{7}He has a form

$$\rho^{^6\mathrm{He}}(k_x)\sim\int\rho^n[\tfrac{7}{6}(k_x-k_x^0)]I_{\mathrm{BW}}(k_x^0)dk_x^0, \quad (10)$$

where $I_{\mathrm{BW}}(k_x^0)$ is the Breit-Wigner formula expressed through the ^{6}He momentum and integrated over two components of the momentum. The ^{6}He transverse momentum distribution calculated from (10) is shown in Fig. 4 by the solid line together with the experimental data [4]. The calculation describes the experimental distribution well. Since ^{6}He is heavy, the sudden approximation gives a result for the ^{6}He momentum distribution, which is close to the curve in Fig. 4 obtained with taking the $n+{}^6$He final state interaction into account.

A simple five-body cluster orbital shell model approximation is proposed to describe the ground state wave function of the ^{8}He neutron-halo nucleus. In spite of its simplicity this approximation has all the necessary features of a microscopical description: (i) a translationally invariant set of coordinates for relative motion; (ii) an implicit form of antisymmetrization of the total WF, consisting of an exact antisymmetrization of the valence neurons and a neutron-core Pauli blocking in an approximate manner. The model WF has simple properties, which allow us to calculate analytically many characteristics—geometric parameters, momentum distributions, and correlation function—and to make estimations of different values. The single parameter of the model is fitted to experimental data to determine the WF.

The model is able to reproduce the experimental neutron and proton spatial extensions in ^{8}He as well as the experimental transverse momentum distribution of the ^{6}He nucleus form ^{8}He fragmentation on a carbon target, at high energy.

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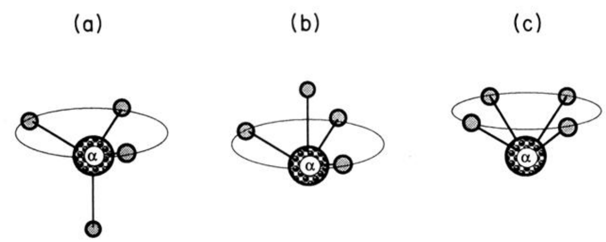
$^8He(0^+)$



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