Fluctuation dynamics of fragmenting spherical nuclei

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We use the quasiclassical nuclear model and the recently proposed early cluster recognition algorithm to investigate the dynamical evolution of the fragmentation pattern for suitably excited finite spherical spin-isospin symmetric system of 80 nucleons. We find that the asymptotic fragment structure (when clusters are far away from each other) is strongly related to the early most bound density fluctuations (MBDF's) in phase space (when the system is dense). We focus on the evolution of the microscopic structure of this MBDF, finding that the constituents of the MBDF are strongly correlated in time. Our calculations indicate that not only the final fragment spectra but also the microscopic structure of the final clusters carry information about the early stages of the evolution.

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I. INTRODUCTION

In the past years much experimental and theoretical work has been devoted to the analysis of fragmentation processes in nuclear systems. Many theoretical models have been proposed to analyze this phenomenon and they can be roughly classified in two major categories: first, those that assume that the fragmentation takes place at the end of a more or less complicated expansion process when the system has attained a certain degree of equilibration [1], and second, those that describe the full nonequilibrium dynamic evolution of the system that eventually leads to the clusterization process. Within this second class a further classification can be established between models that use different schemes to simulate the quantal Boltzmann kinetic equation [2] and those that rely on classical [5] and semiclassical [6] microscopic descriptions in which the quantal effects are sometimes taken into account. One of the relevant differences between these two approaches is the degree to which correlations are kept. In Boltzmann-like theories the main outcome is the one body distribution function and higher order correlations are systematically washed away. Much effort has been devoted to overcome this drawback by extending these theories to include fluctuations through the addition of Langevin-like terms [3], but recently it has been proved that these extra correlations are of the kinetic kind, i.e., those characteristic of the near-equilibrium regime [4]. In opposition the main goal of microscopic classical and semiclassical models is to keep correlations of all orders at all times.

In a previous paper [8] we have employed the so called quasiclassical nuclear model (QCNM) [7] and an alternative definition of cluster to analyze fragment production in simulations of intermediate energy nuclear collisions. It was shown that some properties of the final fragment spectra (mean total multiplicity, mean multiplicity for selected bins, mean maximun fragment, etc.) are closely related to the early formed most bound density fluctuations in (MBDF's) phase space.

In this calculation we use the same microscopic model and cluster recognition method to explore further the properties of the MBDF's, in particular the evolution of their microscopic structure for the case study of an expanding spherical system.

In this section we briefly review the properties of the QCNM and the cluster recognition method. In Sec. II we show the way in which the fragmenting system is built. In Sec. III we investigate the early formation of fragments and after showing some general properties of the expanding system, we show the early formation of clusters and focus on the time evolution of its microscopic structure. Finally, in Sec. IV, conclusions are drawn.

Model

The QCNM has already been described in a series of publications (see [7] and references therein) and for the sake of completeness we briefly describe its main properties. In this model the nuclear system is simulated via a classical interaction potential which reads

$$V = V_n + V_p + V_C, \tag{1.1}$$

where

$$V_p = V_{0p} \exp\left[-0.5\left(\frac{p_{ij}^2}{p_0^2} + \frac{q_{ij}^2}{q_0^2}\right)\right] \delta_{\tau_{ij}} \delta_{\sigma_{ij}} \tag{1.2}$$

is the term responsible for simulating the Pauli exclusion principle,

$$V_n = V_{0n}\left[\left(\frac{r_1}{r_{ij}}\right)^{a1} + \left(\frac{r_2}{r_{ij}}\right)^{a2}\right] \frac{1}{\exp\left(-\beta r_{ij} + \mu\right)} \tag{1.3}$$

is the so called nuclear term, and

$$V_C = \frac{\alpha_0}{r_{ij}} \tag{1.4}$$

is the Coulomb term. In Fig. 1 we show the way in which different nuclear properties are reproduced according to this scheme.

A special role in our analysis is played by the way in which we define and, consequently, recognize clusters. According to [8] a cluster is defined as that set of particles that satisfies the following condition:

$$\forall i \in C, \quad e_i = T_{i\text{c.m.}} + \sum_j V_{ij} < 0, \tag{1.5}$$

where $T_{i\text{c.m.}}$ is the kinetic energy of particle i calculated in the c.m. of the cluster and V_{ij} is the interaction energy between particles i and j belonging to the same cluster C, i.e., all particles must be bound.

For a system of particles we define the cluster decomposition as a partition of the total system into subsets such that for each subset condition (1.5) is satisfied, i.e., each subset is a cluster. Because this condition can be satisfied by more than one partition we introduce the following additional condition: Cluster structure is that cluster decomposition that maximizes the total binding energy of the system when each cluster is considered to be noninteracting with the others.

As can be easily seen, this is a very complicated task because the binding energy is calculated in the center of momentum of each subset of the partition, as a consequence the problem is highly self-consistent. To solve it, in Ref. [8] an algorithm in the spirit of simulated annealing was developed [early cluster recognition algorithm (ECRA)]. It is clear that this cluster definition is free from arbitrary "clusterization parameters." This method

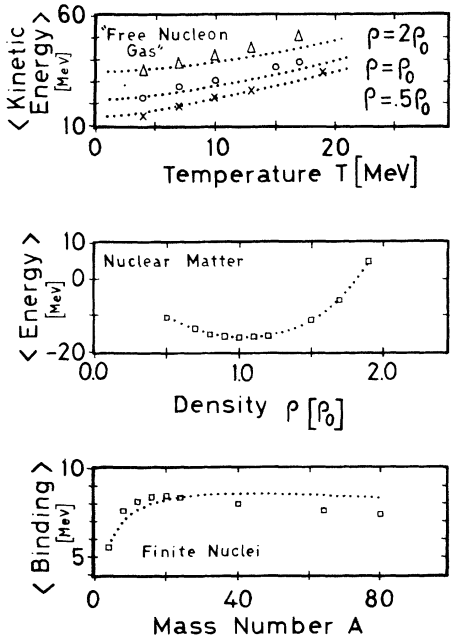

FIG. 1. The kinetic energy of a gas of nucleons that interact only via the Pauli potential (top), the binding energy of nuclear matter as a function of the density (middle), and the binding energy of finite nuclei as a function of their mass number (bottom).

is also statistical in nature, and although we can be confident that we will be quite close to the maximizing partition, we cannot be sure that we have reached it. It should be emphasized at this point that, to apply this method at any point in the evolution of the system, there will be a stage in which what we are recognizing as clusters is the set of most bound density fluctuations in phase space. During this stage particles are close together in q space and no cluster structure can be recognized by standard configuration (percolationlike) cluster recognition algorithms. At later, asymptotic, times clusters will be well separated in space and then we can call them fragments. In the following we apply this method to the analysis of an exploding system and explore the relation between the early recognized clusters (MBDF's) and the asymptotic ones (fragments).

II. CALCULATIONS

In order to work on a rather simple problem we focus on the analysis of suitably excited spherical systems. To perform this we take as an initial condition a spin-isospin symmetric system of 80 particles built via a Monte Carlo (MC) calculation in phase space at a temperature of 0.4 MeV. From the MC run, configurations well separated in "time" were kept in order to be sure that they are uncorrelated in both q and p space. An ECRA calculation was performed with each of them to confirm that they were stable bound systems. In order to build excited systems the following procedure was adopted: The cold nucleus was compressed in q space and consequently expanded in p space according to the following scale transformation:

$$Q_i' = Q_i \lambda^{-1/3}, \quad P_i' = P_i \lambda^{1/3} \tag{2.1}$$

with $\lambda > 1$. Also explored was the effect associated with an excitation procedure given by

$$P_i' = P_i(1 + \alpha). \tag{2.2}$$

Once this stage was accomplished the resulting time evolution was attained by solving the Hamilton equations of motion of the system using a standard self-adjusting Runge-Kutta scheme.

A set of values of λ ranging from 1.2 to 2.0 was taken and it was observed that as this parameter is increased, different behaviors of the system are encountered, from gentle oscillations plus some evaporation to total fragmentation in almost free nucleons.

We focus our analysis on the value $\lambda = 1.8$, which gives a total energy of -0.9 MeV per nucleon (the ground state of the system under consideration has a binding energy around -8 MeV/per nucleon). This excitation leads to a typical multifragmentation process in which light fragments up to mass 15 are found in the final mass spectra.

III. RESULTS

The following analysis was performed over 60 configurations built according to the above mentioned prescription. Some global properties of the evolution were cal-

culated as, for example, the time evolution of the radial density and radial flux. In Fig. 2(a) we show a time evolution of the radial projection of the mean density and it is seen that the system performs a monotonous expansion process. In Fig. 2(b) the evolution of the radial flux is displayed for different λ values.

We now focus on the relation between the early MBDF spectra and the final fragment espectra of the exploding system. In Fig. 3 we show the total multiplicity of clusters larger than 3 (dotted line) and the multiplicities for selected bins as a function of time. All these curves are remarkably constant. To have an idea of the relevant time scales, the same quantities calculated using a simple percolationlike cluster recognition algorithm, with a clusterization distance $R_{cl} = 6.0$ fm corresponding to the cutoff distance of the $V_n(r_{ij})$, Eq. (1.3), are displayed (thin lines). From this it can be realized that constancy of the number of clusters is attained long before spatial separation is reached. This result is similar to what was obtained in a previous work in which intermediate energy collisions were analyzed [8].

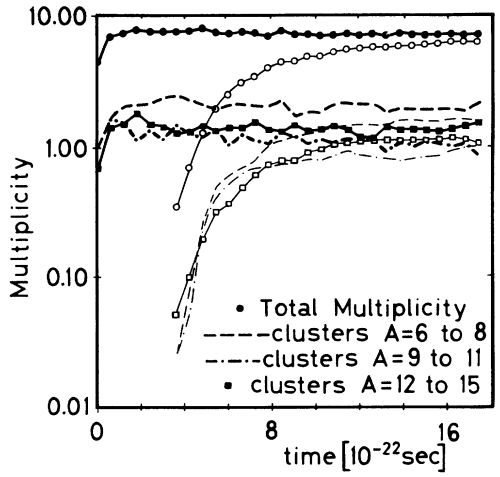

FIG. 3. Time evolution of the multiplicity. Thick lines result from the ECRA method; dots denote total multiplicity; crosses represent clusters of $A = 6 - 8$; squares represent cluster of $A = 12 - 15$; and no symbol denotes clusters of $A = 9 - 11$. Simple lines denote the time evolution of the multiplicity but resulting from a percolationlike method.

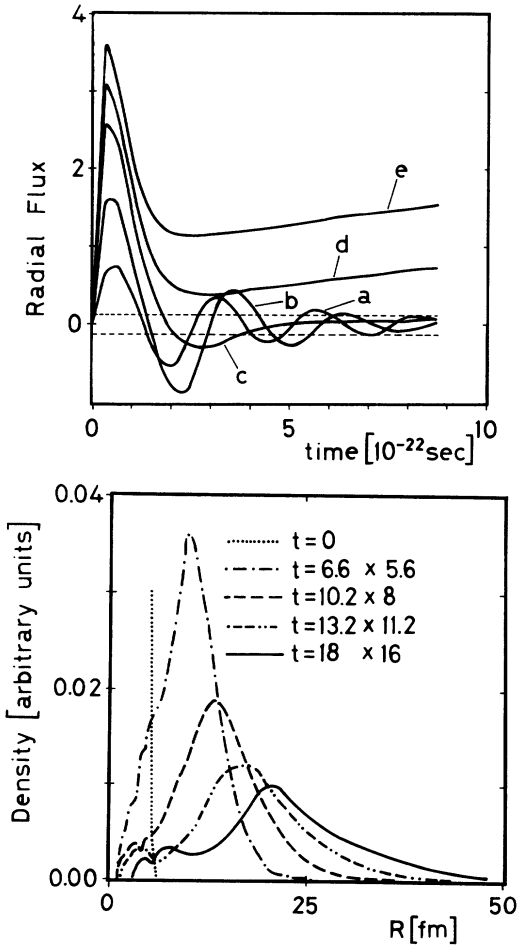

FIG. 2. (a) Time evolution of the radial flux for different initial energies E_0 of the excited system: *a*, $E_0 = -7.3$ MeV; *b*, $E_0 = -6.0$ MeV; *c*, $E_0 = -3.9$ MeV; *d*, $E_0 = -2.5$ MeV, *e*, $E_0 = -0.9$ MeV. (b) Radial density vs distance for the $E_0 = -0.9$ MeV system for different times t.

Our main objective is to get information about the time evolution of the microscopic structure of the MBDF and its relation to the final fragments. To get a glimpse of the complexity of the process under analysis we show in Fig. 4 the trajectories of these objects for a typical fragmenting process (see figure caption for details). Under

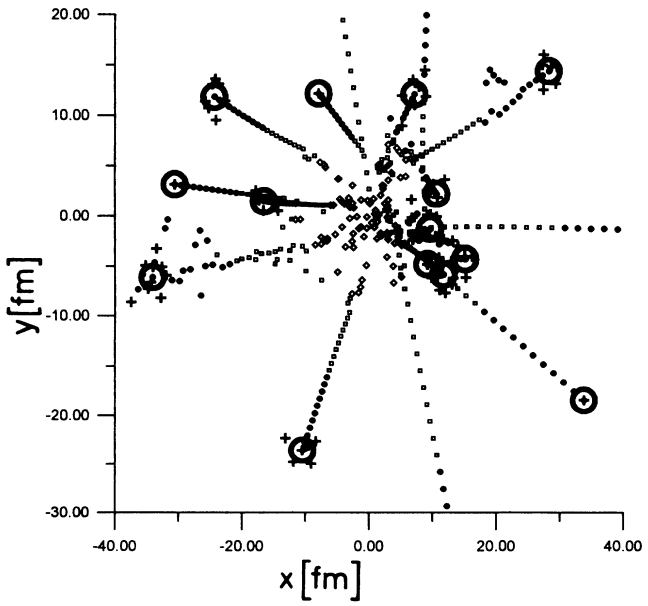

FIG. 4. Typical fragmentation process projected onto an arbitrary x-y plane as seen by the ECRA method. Crosses denote particle position at $t = 17.4$, open circles denote cluster position at $t = 17.4$ (circles radius is the mean rms radius of the clusters), triangles denote cluster position at $0 \leq t \leq 4.2$ (initial stage), rectangles idem at $4.8 \leq t \leq 10.8$ (intermediate stage), and dots idem at $11.4 \leq t \leq 17.4$ (final stage). Some free particles lay out of scope at the final stage.

detailed inspection it can be seen that early recognized clusters (MBDF's) persist nearly unaltered after a short consolidation-stabilization stage (i.e., smooth trajectories are seen because just a few particles are lost or gained along the path) up to the time at which, due to the expansion, they become spatially separated. This simple analysis suggests that the microscopic structure of the early formed clusters reaches constancy before spatial separation takes place. This result excludes another possible, opposite, scenario compatible with the constancy of the number of cluster, one in which, while the macroscopic pattern of fluctuations remains constant, constituent particles are exchanged until expansion freezes the microscopic structure.

In order to quantitatively explore this effect we find it convenient to define the following magnitudes.

(a) *The persistence coefficient (P).* Let $b(t) = 0.5N(N-1)$ be the number of pairs of particles in cluster C at time t. Let $a(t-\Delta t) = \sum 0.5[n(n-1)]$, the sum over all clusters present, at time $t-\Delta t$, of the number of pairs of particles present in each cluster and originally belonging to cluster C, in this way

$$P = \left\langle \sum [a(t-\Delta t)/b(t)] \right\rangle . \tag{3.1}$$

P will be equal to 1 if all particles remain together and 0 if the cluster breaks up completely. It then measures the tendency of the members of a given cluster to remain together. To have an idea of the behavior of this quantity, in Fig. 5 we show the value of this coefficient when 1,2,3,... particles are removed from clusters of 12, 10, and 8 particles. It is seen that this coefficient is highly sensitive to variations in mass number reaching a value around 0.8 when just one particle is removed for this range.

(b) *The pre-existence coefficient (E).* In this case E is defined as follows. Let there be a cluster C of size N at time t. We search for the cluster at time $(t-\Delta t)$ which contains the maximum number of particles belonging to C. Let its mass be L; then

$$E(N,t) = \langle L \rangle . \tag{3.2}$$

In Fig. 6(a) we show the results of calculating the coefficient P for different cluster sizes. It is apparent that it reaches a value close to one very early in the evolution of the system, and from a comparison with the results displayed in Fig. (3), we see that this happens long before the spatial separation of the clusters takes place.

In Fig. 6(b) the coefficient E is displayed for the same set of expansions as in Fig. 6(a). Once again the strong correlation between cluster composition at different times before the spatial separation is inferred.

From the results presented in Figs. 3 and 6 it is clear that the asymptotic fragment spectra is strongly related in both the number of fragments and its microscopic structure with the clusters (MBDF's) that are formed in the early dense stage of the explosive evolution of the excited system.

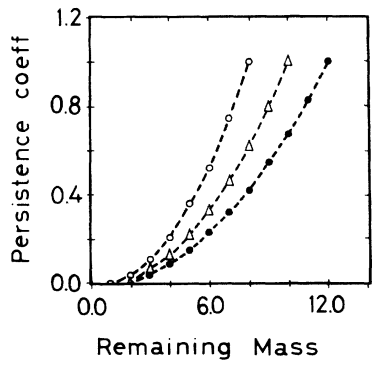

FIG. 5. The persistence coefficient as a function of the remaining fragment mass when the original cluster mass is 12 (squares), 10 (triangles), and 8 (circles).

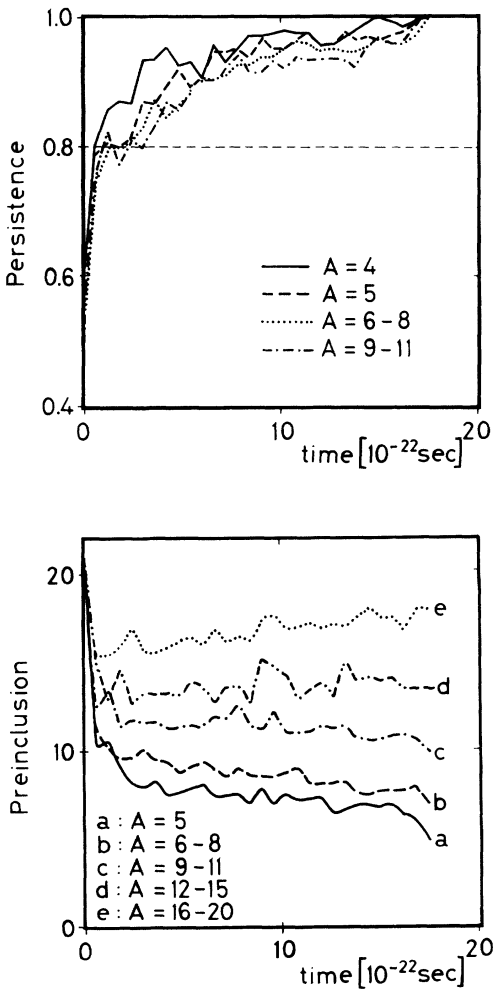

FIG. 6. (a) Time evolution of the persistence coefficient P. Solid line indicates clusters of $A = 4$ at the final stage, dashed line idem for $A = 5$, dotted line idem for $A = 6 - 8$, and dash-dotted line idem for $A = 9 - 11$. (b) Time evolution of the precoexistence coefficient E. a, solid line indicates clusters of $A = 5$ at the final stage; b, dashed line idem for $A = 6 - 8$, c, dash-dotted line idem for $A = 9 - 11$; d, full squares idem for $A = 12 - 15$; and e, dotted line idem for $A = 16 - 20$.

IV. CONCLUSIONS

From our calculations we conclude that the fragmentation process of excited spherical nuclei can be described as follows. As soon as the system is excited, a set of density fluctuations in phase space is built. From the infinite modes that can be considered, a very peculiar one is extremely relevant for the analysis of the subsequent evolution of the system, i.e., one that maximizes the total binding energy as described previously. This set of fluctuations evolves in time in such a way that after a short time most of its constituent particles remain together for the rest of the time evolution, as can be seen from the behavior of the P and E coefficients. As time evolves the system reaches a volume such that clusters become physically separated, but in this stage what we see as fragments are basically the time propagation of the already present MBDF's. In this way the fragments, in both its number and microscopic composition, carry information about the initial, highly excited stage of the evolution of the system. In this way we feel that a much clearer, although unexpected, picture of the fragmentation process has been detected. It must be noted at this point that a recent analysis of experimental data confirms this picture of the time evolution of fragmenting excited systems [9], at least at the level of the constancy of the number of fragments.

It is then clear that models based on the assumption of the existence of an intermediate regime in which the systems equilibrates and subsequently fragments are built up should be critically reanalyzed (according to our microscopical calculations there is no such equilibrated prefragmentation stage). Moreover, it is clear that the dynamics of fragmentation is driven by many-body correlations in phase space. As a consequence Boltzmann-Uehling-Uhlenbeck-like approaches should be refined further to include these effects.

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