

Relativistic Hartree calculations of nuclear compressional properties

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Using the scaling model, we calculate within the framework of relativistic Hartree theory the nuclear breathing-mode energies corresponding to the NL1 and NL-SH parameter sets of the nonlinear σ - ω - ρ model. Both of these sets are found to be in disagreement with experiment and there is a clear need for an improved fit. However, as far as the nuclear-matter incompressibility K_v is concerned, neither the NL1 value, 212 MeV, nor the NL-SH value, 356 MeV, can be excluded.

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From the phenomenological point of view the most highly developed form of relativistic mean-field (RMF) theory is the nonlinear σ - ω - ρ model, with Lagrangian density:

$$\begin{aligned} \mathcal{L} = & \bar{\psi} \left(i\gamma^\mu \partial_\mu - M + g_\sigma \varphi - g_\omega \gamma^\mu \omega_\mu - g_\rho \gamma^\mu \vec{\tau} \cdot \vec{b}_\mu \right) \psi + \frac{1}{2} (\partial_\mu \varphi \partial^\mu \varphi - m_\sigma^2 \varphi^2) + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ & + \frac{1}{2} m_\rho^2 \vec{b}_\mu \cdot \vec{b}^\mu - \frac{1}{4} \vec{G}_{\mu\nu} \cdot \vec{G}^{\mu\nu} - \frac{1}{3} M b (g_\sigma \varphi)^3 - \frac{1}{4} c (g_\sigma \varphi)^4, \end{aligned} \quad (1)$$

where $F_{\mu\nu} \equiv \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$ and $\vec{G}_{\mu\nu} \equiv \partial_\mu \vec{b}_\nu - \partial_\nu \vec{b}_\mu$, in which \vec{b}_μ denotes an isovector. Two different realistic parameter sets have been presented, both having been obtained by fitting to the masses of several finite nuclei (Table I).

The first of these sets, labeled NL1 [1], suffers from a volume-symmetry coefficient J with the high value of 43.5 MeV, higher than anything that has emerged from any mass fit that we are aware of. It was later found [2] that this parameter set gave neutron-skin thicknesses that were much larger than experiment. This can be attributed [3] to the small value of the surface-stiffness coefficient Q , an inevitable consequence of a large value of J , once masses are fitted. In an attempt to remove this defect of NL1, Sharma *et al.* [4] introduced a new parameter set, NL-SH, in which J is reduced to the much more reasonable value of 36.1 MeV.

However, the value of the incompressibility coefficient K_v corresponding to parameter set NL-SH is 356 MeV, as opposed to 212 MeV for NL1, and no correctly analyzed experiment has ever given such a high value. Recent heavy-ion experiments [5], for example, give K_v as lying between 165 and 220 MeV, which strongly favors the NL1 set. The objective of the present paper is to see to what extent this preference for NL1 is supported by the measured energies E_{br} of the nuclear breathing mode, the giant isoscalar monopole resonance.

To extract a value of K_v from the breathing-mode data one takes a variety of interaction schemes, i.e., effective

forces in the case of nonrelativistic approaches and effective Lagrangians in the case of RMF approaches, each characterized by different values of K_v . Then for each of these interaction schemes one calculates E_{br} for the measured nuclei, using some appropriate model, e.g., the random phase approximation (RPA) or scaling, for the breathing mode. The first such analysis was performed in 1976 by Blaizot, Gogny, and Grammaticos [6] (BGG), who considered a number of nonrelativistic Hartree-Fock effective forces, and from the limited data available at the time extracted the value of 210 ± 30 MeV. However, since then much more precise data have been obtained by the Groningen group [7–9]. Nevertheless, adopting the same general approach as BGG [6], and taking a highly generalized and very flexible Skyrme force, it was found in Ref. [10] that fits to these new data can be obtained only if K_v lies in the interval determined by BGG [6].

This provides a further strong indication that param-

TABLE I. Parameter sets NL1 and NL-SH. Nucleon and meson masses are given in MeV. $C_i^2 = g_i^2 (M/m_i)^2$, $i = \sigma, \omega, \rho$.

	NL1	NL-SH
M	938	939
m_σ	492.250	526.059
m_ω	795.36	783.00
m_ρ	763.00	763.00
C_σ^2	373.176	347.533
C_ω^2	245.458	240.997
C_ρ^2	37.4175	29.0954
b	0.0024578	0.0012747
c	-0.0034334	-0.0013308

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eter set NL1 has much better compressibility properties than NL-SH. However, it must be realized that the analysis in Ref. [10] of the Groningen breathing-mode data is strictly nonrelativistic, and that with a relativistic treatment of the saturation mechanism a quite different range of values of K_v might emerge. Actually, a constrained relativistic Hartree calculation [11] with the *linear* σ - ω model [12] shows that the value $K_v = 545$ MeV given by this model leads to E_{br} 's that are much too high, and these authors suggest that a value of K_v in the range 200–250 MeV is indicated. However, the *nonlinear* model has quite different compressional properties, and merits a separate study. Thus in the present paper we attempt to estimate the E_{br} of the measured nuclei for the RMF parameter sets NL1 and NL-SH.

Unfortunately, we are not able to determine breathing-mode energies within the framework of RMF theories by direct calculations on the finite nuclei concerned. Instead, following the scaling model and Blaizot [13], we first define a finite-nucleus incompressibility for a nucleus of mass number A and asymmetry $I \equiv (N - Z)/A$ through

$$K(A, I) = \frac{M}{\hbar^2} \langle r^2 \rangle E_{br}^2, \quad (2)$$

where $\langle r^2 \rangle$ denotes the rms *matter* radius. Next we make the leptodermous expansion

$$K(A, I) = K_v + K_{sf} A^{-1/3} + K_{vs} I^2 + K_{Coul} Z^2 A^{-4/3} + \dots, \quad (3)$$

where, in the scaling-model approximation,

$$K_{sf} = \left(22 - 2 \frac{K'}{K_v} \right) a_{sf} + 36\pi r_0^2 \rho_{00}^2 \ddot{\sigma}, \quad (4)$$

$$K_{vs} = K_{sym} + L \left(\frac{K'}{K_v} - 6 \right), \quad (5)$$

and

$$K_{Coul} = \frac{3q_{el}^2}{5r_0} \left(\frac{K'}{K_v} - 8 \right). \quad (6)$$

The quantities K' , K_{sym} , L , r_0 , and ρ_{00} that we have introduced here, like J and K_v , are defined with respect to infinite nuclear matter (INM) as follows. If we express the energy per nucleon, e , of INM as a function of the total baryon density ρ and the asymmetry $\delta = (\rho_n - \rho_p)/\rho$, where ρ_n and ρ_p refer to neutron and proton densities, respectively, and $\rho = \rho_n + \rho_p$, then the above coefficients appear in the expansion [14]

$$e(\rho, \delta) = \left(a_v + \frac{1}{18} K_v \epsilon^2 - \frac{1}{162} K' \epsilon^3 + \dots \right) + \delta^2 \left(J + \frac{1}{3} L \epsilon + \frac{1}{18} K_{sym} \epsilon^2 + \dots \right) + \dots, \quad (7)$$

where $\epsilon = (\rho - \rho_{00})/\rho_{00}$, in which we denote by ρ_{00} the equilibrium (saturation) density in the symmetric case,

$\delta = 0$. We also define the charge-radius constant by $r_0 = (3/4\pi\rho_{00})^{1/3}$.

The remaining quantities, a_{sf} and $\ddot{\sigma}$, refer to symmetric semi-infinite nuclear matter (SINM). This is a one-dimensional system with $\rho_p(z) = \rho_n(z)$ for all z , and the limiting behavior

$$\lim_{z \rightarrow \infty} \rho(z) = 0, \quad (8)$$

$$\lim_{z \rightarrow -\infty} \rho(z) \equiv \rho_c = \rho_{00}. \quad (9)$$

That is, deep beneath the surface the local properties of symmetric SINM tend towards those of saturated symmetric INM. A specific surface energy for symmetric SINM is now defined according to

$$\sigma_{00} = \int_{-\infty}^{\infty} \{ \mathcal{E}(z) - a_v \rho(z) \} dz, \quad (10)$$

where $\mathcal{E}(z)$ is the local energy density in SINM and a_v is the energy per nucleon in symmetric INM at saturation, as defined in Eq. (7). Then $a_{sf} = 4\pi r_0^2 \sigma_{00}$. We note that the relations given above are valid regardless of the choice of theoretical methods; in particular they hold also for RMF approaches.

As for $\ddot{\sigma}$, the double derivative of σ is with respect to the limiting internal density ρ_c , as defined in Eq. (9), which means that one must in principle evaluate the specific surface energy σ of SINM over a range of densities ρ_c . This poses a problem, since to have ρ_c different from ρ_{00} in symmetric SINM requires the application of a constraint to the system, and Eq. (10) can be used only for $\rho_c = \rho_{00}$. One solution to this problem is to perform a full scaling calculation, but in this paper we make use of the simple relation given by Stocker [15]:

$$\ddot{\sigma} \equiv \left(\frac{d^2 \sigma}{d\rho_c^2} \right)_{\rho_c = \rho_{00}} = - \frac{5}{72} \frac{K_v t}{\rho_{00}}, \quad (11)$$

where t is the surface diffuseness, defined as the 90%–10% falloff distance of the surface density. This expression was obtained under the assumption of the existence of a ground-state energy-density functional that could be even relativistic. However, the ground-state density had to be approximated, as a result of which a possible K' dependence, believed to be weak, cannot be taken into account.

The tendency of this expression is to give *magnitudes* of $\ddot{\sigma}$, which is always negative, that are too large by up to 30%, as compared to the scaling value (see Refs. [16] and [17] for the nonrelativistic and relativistic cases, respectively).

This error in $\ddot{\sigma}$ is the largest source of error in our calculation of $K(A, I)$, compared to exact scaling calculations. Another source of error concerns the higher-order terms that are neglected in Eq. (3). An analysis based on Refs. [10] and [18] shows that the net contribution of these terms to $K(A, I)$ tends to be positive, of the order of 10 MeV or less. Finally, there is the question of the validity of the scaling model, which is implicit in Eq. (3) itself, and in the calculation of its coefficients according to Eqs. (4)–(6). By comparing with RPA calculations it

has been found [19] that the scaling model overestimates $K(A, I)$ slightly, but by not more than 10 MeV. Thus these last two sources of error, each of which is small, tend to cancel, and the error in $\check{\sigma}$ dominates, with the tendency to leave $K(A, I)$ underestimated.

In passing, one sees the possibility of fitting Eq. (3) directly to the data, and extracting empirical values for K_v , along with all the other coefficients. However, it is impossible to extract a unique value of K_v from the measured E_{br} in this way, all values over the range 100 – 400 MeV (and maybe over an even wider range) being compatible with the Groningen data [20]. Thus there is no alternative to the general strategy pioneered by BGG [6], i.e., to simply trying out different proposed interaction schemes.

We calculate both INM and SINM (symmetric) for the two parameter sets in the relativistic Hartree approach. This method is well known (see Ref. [12] for INM and Ref. [21] for symmetric SINM) and will not be discussed here. However, we give now the analytic expressions that we have used to calculate K_v , K' , L , and K_{sym} , since these do not seem to have been published before. We have

$$K_v = 3 \left\{ 3\rho \frac{g_\omega^2}{m_\omega^2} + \frac{k_F}{\epsilon_F^*} (k_F + M^* M^{*'}) \right\} , \quad (12)$$

$$K' = 9K_v + 3 \left[\frac{k_F}{\epsilon_F^*} \left(1 + \frac{k_F^2}{\epsilon_F^{*2}} \right) (k_F + 2M^* M^{*'}) - \frac{k_F^2}{\epsilon_F^*} \left\{ \left(1 - \frac{M^{*2}}{\epsilon_F^{*2}} \right) (M^{*'})^2 + M^* M^{*''} \right\} \right] , \quad (13)$$

$$L = \frac{3\rho}{2} \frac{g_\rho^2}{m_\rho^2} + \frac{1}{3} \frac{k_F^2}{\epsilon_F^*} \left(1 - \frac{1}{2} \frac{k_F^2}{\epsilon_F^{*2}} - \frac{1}{2} \frac{k_F}{\epsilon_F^{*2}} M^* M^{*'} \right) , \quad (14)$$

$$M^{*''} \equiv \frac{d^2 M^*}{dk_F^2} = \frac{3}{y^2 k_F} (x - M^{*'}) + \frac{\pi^2}{2y^2 k_F} \frac{x}{\sqrt{1+x^2}} \left[3F(x) \left\{ -(1+2x^2) + \frac{2+3x^2}{x} M^{*'} \right\} + \frac{G'(M^*)}{k_F^2} (xM^{*'} - 1 - 2x^2) + \frac{G''(M^*)}{k_F} (1+x^2)M^{*'} \right] . \quad (22)$$

For completeness we recall also the expression for J [12]:

$$J = \frac{\rho}{2} \frac{g_\rho^2}{m_\rho^2} + \frac{1}{6} \frac{k_F^2}{\epsilon_F^*} \quad (23)$$

All these quantities are to be evaluated for $\rho = \rho_{00}$, the saturation density of symmetric INM. Also, we have $k_F = (3\pi^2 \rho/2)^{1/3}$.

We show in Table II the values that we have calculated for all these coefficients for both parameter sets, along with the quantities a_{sf} and t , derived from SINM (all these numbers are as in Ref. [3]). This table also shows the value of $\check{\sigma}$ that we have calculated using Eq. (11), and of the values of K_{sf} , K_{vs} and K_{Coul} that we have derived using Eqs. (4) to (6), respectively.

Table III shows for both parameter sets the calculated

and

$$K_{sym} = -\frac{1}{3} \frac{k_F^2}{\epsilon_F^*} - \frac{1}{2} \frac{k_F^4}{\epsilon_F^{*3}} + \frac{1}{2} \frac{k_F^6}{\epsilon_F^{*5}} + \frac{k_F^3}{\epsilon_F^{*3}} \left(\frac{k_F^2}{\epsilon_F^{*2}} - \frac{1}{3} \right) M^* M^{*'} + \frac{1}{2} \frac{k_F^4}{\epsilon_F^{*5}} (M^* M^{*'})^2 - \frac{1}{6} \frac{k_F^4}{\epsilon_F^{*3}} (M^{*'})^2 - \frac{1}{6} \frac{k_F^4}{\epsilon_F^{*3}} M^* M^{*''} . \quad (15)$$

Here M^* is the Dirac effective mass, given by numerically solving the implicit equation

$$M^* k_F^2 F \left(\frac{M^*}{k_F} \right) + G(M^*) = 0 , \quad (16)$$

where

$$F(x) = \frac{1}{\pi^2} \left\{ \sqrt{1+x^2} - x^2 \ln \frac{\sqrt{1+x^2} + 1}{x} \right\} , \quad (17)$$

$$G(M^*) = -\frac{m_\sigma^2}{g_\sigma^2} (M - M^*) - bM(M - M^*)^2 - c(M - M^*)^3 , \quad (18)$$

and

$$\epsilon_F^* = \sqrt{k_F^2 + M^{*2}} . \quad (19)$$

Also

$$M^{*'} \equiv \frac{dM^*}{dk_F} = \frac{M^*}{y k_F} , \quad (20)$$

where

$$y = 1 - \frac{\pi^2}{2} \sqrt{1+x^2} \left\{ 3F(x) + \frac{1}{k_F^2} G'(M^*) \right\} , \quad (21)$$

with $x = M^*/k_F$. Finally,

values of $K(A, I)$ for the measured nuclei, and compares with the experimental values, extracted from Refs. [7–9], using Eq. (2). Clearly, for both parameter sets these predictions are in disagreement with the data. However, in view of the tendency for the ‘‘pocket formula’’ (11) to underestimate $K(A, I)$, it would seem that a better estimate of $\check{\sigma}$ might bring NL1 closer to the data, while for NL-SH the situation could only become worse.

Nevertheless, a more careful study shows that no matter what value is chosen for $\check{\sigma}$ the INM coefficients of NL1, whose values are calculated unambiguously, do not permit the A and I dependence of the data to be correctly reproduced. In fact, the main problem with NL1, as far as compressibility properties are concerned, lies with the low value of its INM coefficient

TABLE II. Coefficients of INM and SINM for RMF parameter sets NL1 and NL-SH. (See text for quantities in parentheses.)

	NL1	NL-SH
ρ_{00} (fm $^{-3}$)	0.1519	0.1460
r_0 (fm)	1.163	1.178
K_v (MeV)	211.7	355.8
K' (MeV)	31.98 (1207)	-600.9 (345.0)
J (MeV)	43.49	36.13
L (MeV)	140.2	113.7
K_{sym} (MeV)	143.0	79.82
a_{sf} (MeV)	18.56	18.96
t (fm)	2.30	1.83
σ_{00} (MeV fm $^{-2}$)	1.092	1.087
$\ddot{\sigma}$ (MeV fm 4)	-222.5 (-163.1)	-309.7 (-401.0)
K_{sf} (MeV)	-382.6 (-379.1)	-554.9 (-961.1)
K_{vs} (MeV)	-677.0 (101.5)	-794.4 (-492.1)
K_{Coul} (MeV)	-5.831 (-1.706)	-7.106 (-5.156)

$K' \equiv -27\rho_{00}^3(d^3e/d\rho^3)_{\rho_{00}}$: If we drastically increase this coefficient and at the same time make a small change in $\ddot{\sigma}$, as indicated in parentheses in the NL1 column of Table II, then without changing any of the other INM or SINM coefficients of NL1 the corresponding values of $K(A, I)$, shown in parentheses in the NL1 column of Table III, will improve dramatically.

However, within the context of the RMF theory described by the Lagrangian (1) no physical meaning can be attached to this modified set of INM and SINM coefficients if no modified set of meson parameters can be found to correspond to them. Actually, the required change in $\ddot{\sigma}$ may not be significant, especially in view of the fact that the ‘‘pocket formula’’ (11) tends to give too negative a value for this quantity anyway. As for the coefficients relating to symmetric INM, we have to fit not only the prescribed values of K_v and K' , but also the values of a_v , ρ_{00} , and M^* that are imposed by the fit to masses. But with the Lagrangian (1) symmetric INM is determined by just four meson parameters: g_σ^2/m_σ^2 , g_ω^2/m_ω^2 , b , and c . Thus it might be difficult to find a set of meson parameters that will allow us to fit simultaneously masses and the required incompressibility coefficients K_v and K' .

A similar improvement in the fit to the breathing-mode data is also possible in the case of parameter set NL-SH, as we again indicate in parentheses in Tables II and III. (The fact that good fits to the breathing-mode data are possible with such widely different values of K_v as 212 and 356 MeV is consistent with Ref. [20].) However, not only is a drastic modification in K' now required, but also $\ddot{\sigma}$ has to be made much more negative, making it still more unlikely that any corresponding set of meson parameters could be found.

To summarize, despite the uncertainty in our calculation of $\ddot{\sigma}$, we are able to conclude quite firmly that neither of the RMF parameter sets NL1 and NL-SH is compatible with the measured breathing-mode energies. Nevertheless, we have seen that as far as the incompressibility K_v is concerned, neither the NL1 value, 212 MeV, nor the

TABLE III. Finite-nucleus incompressibilities $K(A, I)$ (MeV). (See text for quantities in parentheses.)

	Expt.	NL1	NL-SH
^{90}Zr	112.8 ± 5.6	94.8 (121.5)	194.0 (114.9)
^{112}Sn	126.4 ± 2.2	97.4 (126.2)	198.6 (126.8)
^{114}Sn	126.8 ± 2.2	96.1 (127.2)	197.3 (126.9)
^{116}Sn	126.7 ± 2.6	94.5 (128.3)	195.5 (126.6)
^{120}Sn	126.7 ± 2.4	90.7 (130.4)	191.2 (125.5)
^{124}Sn	126.6 ± 2.6	86.0 (132.5)	186.1 (123.8)
^{144}Sm	134.2 ± 2.5	95.8 (132.5)	198.4 (136.7)
^{208}Pb	143.0 ± 6.2	84.9 (142.8)	187.8 (143.4)

NL-SH value, 356 MeV, can be excluded. However, it seems that it might be easier to fit the breathing-mode data with the lower value of K_v , in the sense that the required modification of NL1 would be much less extensive than that of NL-SH; this would be in agreement with all other indications on the correct value of K_v [5, 6, 10, 11]. In either case, one of the essential modifications would be a much higher value of K' . This means that even if NL1 should turn out to have close to the correct value for K_v , it predicts too stiff an equation of state beyond saturation. A further problem with NL1 concerns its incorrect symmetry properties, as pointed out in Refs. [2] and [4]. The discussion in Ref. [3] of this problem raises the question that a RMF theory with a Lagrangian of the simple form (1) might be incapable of fitting all the available data; the present paper does nothing to dispel this concern, but before any definitive statement could be made it would be necessary to do a lot more work on the fits. It should be noted that the breathing-mode energies obtained within the scaling approach [Eqs. (4)–(6)] are very sensitive to special quantities such as, e.g., K' .

An improvement might be obtained by going beyond the RMF theory and introducing exchange (Fock) terms and relativistic RPA correlations. However, so far no relativistic Hartree-Fock (RHF) parametrization comparable with the RMF parameter sets NL1 and NL-SH, i.e., fitted to the masses of several finite nuclei, is available. Moreover, it was only very recently that the first RHF calculations including σ self-interactions for finite nuclei, which are extremely involved, have been performed at all [22]. Concerning SINM, no RHF calculations are available yet. On the other hand, relativistic RPA correlations have been performed only within the linear RMF model so far [23]. For these reasons, it seems that it will be impossible for some time to come to investigate nuclear compressional properties within a self-consistent RHF framework including relativistic RPA correlations.

Finally, despite our somewhat negative conclusions, we have made it clear that in making parameter fits of RMF theories to nuclear data one should take account not only of masses and radii but also breathing-mode energies.

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