Normal and exotic collective states in the fermion dynamical symmetry model

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The comparison of bosonic and fermionic descriptions of collective nuclear structure are given within the framework of algebraic models. Despite the lack of the F spin in the fermion picture, unification of normal and exotic states can be achieved by the n-p quadrupole interactions. Results for the ^{134}Ba are used to illustrate the physics.

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Bosonic and fermionic descriptions for the nuclear many body system are complementary. The archetypical bosonic algebra is the original interacting boson model (IBM-1) [1]. Without distinguishing between proton and neutron bosons, it gave rise to a successful phenomenology for medium and heavy nuclei, and is built from the concept of dynamical symmetry [2] whose genesis is a group chain. The fermionic algebra, on the other hand, such as the fermion dynamical symmetry model (FDSM) [3], is necessarily more complex because it originates from the shell structure and uses protons and neutrons as building blocks. This demands that the fermion group chains with analogous IBM properties [4, 5] must contain fermion characteristics, such as the Pauli principle.

The comparisin becomes even more intriguing for the more realistic version of the boson model (IBM-2) where proton and neutron bosons are distinguishable [6]. This model has a new quantum number, the F spin [7] (an utterly bosonic concept) which will classify the low-lying states of nuclei as *normal* (symmetry or maximum F spin) or *exotic* (mix-symmetry or lower F spin) [8]. Technically, the F-spin algebra gives one the freedom to introduce a Majorana interaction (an F-spin scalar) [7] which is insensitive to the normal states and can phenomenologically "push" the exotic states to the proper energies. Preservation of the F-spin invariance requires that the Hamiltonian be symmetric under the exchange of protons and neutrons. *Predictions* of the excitation energies of these exotic states in nuclei can be made after the strengths of the Majorana interaction are ascertained by the systematics of such states from some known nuclei [9]. The most convincing experimental confirmation of such states came when Bohle *et al.* concluded via the measured transitions that the dipole scissor mode (a 1^+ state) [10] is exotic. Henceforth the F-spin description became a natural scheme in the boson language to investigate such states [10]. Recently it was also used to analyze other exotic states, for example, the vibrational isovector 2^+ states [11].

It is well known that the FDSM (and indeed any fermion models built from pairs [12]) has no closed F-spin algebra. Therefore, in this picture, it lacks the phenomenological freedom to separate the normal states from the exotic states and they should all be eigenstates of a Hamiltonian whose dominate feature must emphasize the intricate interplay between the long-range n-p quadrupole force and the short-range n-n and p-p pairing forces [13]. Furthermore, there is no reason to impose the constraint that the Hamiltonian be symmetric under proton-neutron exchange. Therefore classification and splitting of the normal states from the exotic states should emerge naturally, if at all, because from the boson phenomenology, there are manifestations of some isovector properties of the F-spin algebra for the exotic states. *This paper aims to show that the above mentioned scenario can be achieved.* As an example, we shall examine the spectrum and the electromagnetic properties of ^{134}Ba.

Actually the physical implications of the bosonic concepts of symmetry and mixed symmetry can be indirectly found in the FDSM in terms of proton-neutron exchange properties. The physics can roughly be understood as follows: The proton-neutron interactions will mix the separate proton and neutron states, and give rise to two classes of states. One class is invariant under the exchange of the protons and neutrons. We shall refer to these states as "normal" or "symmetry." Another class will not be invariant under the exchange and these states are referred to as "exotic" or "mixed-symmetry" states. These two classes of states will be viewed as quadrupole collective partner states in the n-p coupling scheme. In fact, we shall show in this paper that the splitting and the $M1$ transitions are primarily controlled by the n-p quadrupole interaction, while the n-n and p-p pairing interactions, which are insensitive to the splitting, give the proton and neutron pairing gaps, and ensure that due to Pauli blocking the identical pairs are lower in energy than coupled components such as $|D_\pi D_\nu\rangle$.

According to the FDSM, for the Ba isotopes in the 50-82 shell, the Hamiltonain can be written as

$$H_{\text{FDSM}} = G_{0\pi}S_\pi^\dagger S_\pi + G_{0\nu}S_\nu^\dagger S_\nu + \kappa P_\pi^2 \cdot P_\nu^2 + \lambda P_\pi^3 \cdot P_\nu^3, \tag{1}$$

where π (ν) denotes proton (neutron). The definitions of all the operators in Eq. (1) can be found in [3].

The four parameters in Eq. (1) are determined as follows: G_π (G_ν) is obtained by fitting the excitation energies of the 2_1^+ states of the semimagic isotones with neutron number 82 (the semi-magic Tin isotopes). In practice, pairing strengths are known to diminish in the presence of the n-p quadrupole interaction. Thus, for the Ba isotopes, one should renormalize somewhat the pairing strengths obtained from the semimagic nuclei. The octupole strength λ is only sensitive to adjust the quasi-γ band position and essentially has no effect on the exotic states [15]. Hence, to reveal the physics in question, there is only *one* basic adjustable parameter in these calculations: the strength of the n-p quadrupole. The degeneracies of the normal single-particle levels in the 50-82 major shell are $\Omega_{1\pi}=\Omega_{1\nu}=10$. The separation of the particles, be it neutrons or protons, between the normal and abnormal parity levels is given by a FDSM empirical formula [3]. For the ^{134}Ba case, there are six protons particles and four neutrons holes to form the S or D fermionic coherent pairs.

The $M1$ and $E2$ transition operators are

$$\begin{aligned} T(M1)^1_\mu &= \sqrt{\frac{3}{4\pi}}\,(g_\pi L_\pi + g_\nu L_\nu), \\ T(E2)^2_\mu &= e_\pi P^2_\mu(i)_\pi + e_\nu P^2_\mu(i)_\nu, \end{aligned} \tag{2}$$

respectively, where

$$\begin{aligned} P^r_\mu(i) &= \sqrt{\Omega_1/2}\left[b^\dagger_{ki}\tilde{b}_{ki}\right]^{0r}_{0\mu}, \quad r=1,2,3, \\ L_\sigma &= \sqrt{5}P^1_\sigma(i). \end{aligned} \tag{3}$$

In the above equations, e_π (e_ν) is the proton (neutron) effective charge determined by the $B(E2;2^+_1\to 0^+_1)$ for the ^{138}Ba (Sn isotopes) [14], and is 0.14 $e\,b$ (0.11 $e\,b$), and the g's are the g factors in the S-D subspace; the bare g factors are $g_\pi=1.0\mu_N$ and g_ν=0.

To show that the quadrupole n-p interaction is crucial for these exotic states, we have performed the following calculations. First we fix the parameters as $G_\pi=-0.08$ MeV, $G_\nu=-0.07$ MeV, and λ=0. Then we let κ to vary. The results, given in Fig. 1, vividly show how this interaction affects the spectra and the transitions. It is particularly interesting that, just as the IBM-2 calculations, the present fermion calculations also predict a number of 2^+ states. However, in the IBM-2 case, it is not an easy matter to clearly identify which of these are mixed-symmetry states and therefore a clear-cut determination of the Majorana strengths is a difficult task. The situation in the fermion case is quite different as we shall now discuss. When κ=0, the 2^+_1 (2^+_2) state is $|D_\nu\rangle$ ($|D_\pi\rangle$) [16]. Several higher energies 2^+ states are built from such D pairs. For example, the wave functions of the next three states are $2^+_3=|D^2_\nu\rangle$, $2^+_4=|D^2_\pi\rangle$, and $2^+_5=|D_\pi D_\nu\rangle$. Hence it is obvious that the $M1$ transitions from these states to the 2^+_1 state are forbidden. With the n-p interaction ($\kappa\neq 0$), the proton and neutron components will mix, thus giving rise to two new orthogonal partner states. For example, for the one D pair state, the partners are $\alpha|D_\pi\rangle+\beta|D_\nu\rangle$ and $\beta|D_\pi\rangle-\alpha|D_\nu\rangle$ [15,17]. (For more than one D pair, the situation is similar [15].) The former is identified as the normal (symmetric) state and the latter the exotic (mixed-symmetry) state. In realistic situations, there certainly will be additional mixings contained in these partner states. For example, $|D_\pi D_\nu\rangle$ is expected to be admixed in the 2^+_3 and 2^+_4 states. How-

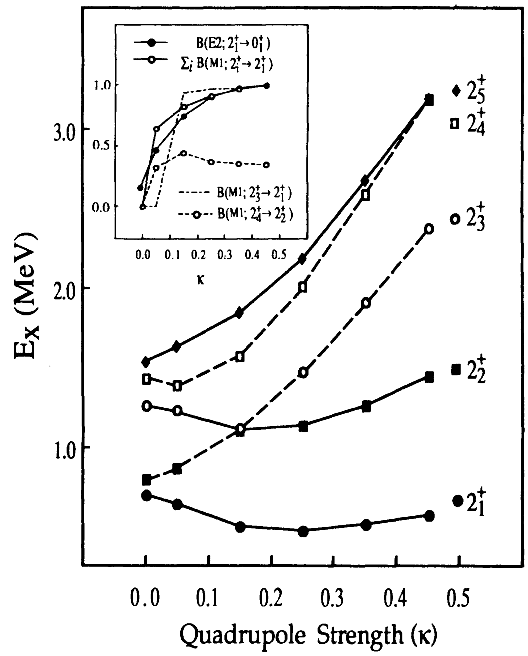

FIG. 1. The variations of the spectrum, $M1$ and $E2$ with the quadrupole n-p interaction. Note that the $M1$ and $E2$ results are normalized to the values at $\kappa=-0.45$ MeV. For the $2^+_4\to 2^+_2$ transition, they are normalized to the $2^+_3\to 2^+_1$ value at $\kappa=-0.45$ MeV.

ever, what is important *here* is that the $M1$ transitions can only proceed via the partner components. This can readily be appreciated in the κ=0 limit because there the $M1$ transitions from the excited 2^+ states to the first excited 2^+ state are forbidden.

From Fig. 1, one sees also that by increasing κ, the 2^+_2 state which contains the exotic components are rapidly shifted upwards before reaching $\kappa=-0.15$ (MeV). After that point, the levels are interchanged. The state which originally contains the two neutron D pairs (i.e., 2^+_3 state at κ=0) now becomes the 2^+_3 state. This is why the $M1$ transition of $2^+_3\to 2^+_1$ (see the inset of Fig. 1) abruptly jumps from zero after $\kappa=-0.15$. Indeed, prior to the exchange point, the 2^+_3 state is mainly the $|D^2_\nu\rangle$, which forbids the $M1$ transition to the 2^+_1 state. After this point, the 2^+_3 state takes on the exotic component, thus allowing an $M1$ transition to its partner component (mainly in the 2^+_1). The present calculations do show a significant $M1$ transition for $2^+_2\to 2^+_1$ before $\kappa=-0.15$. Therefore by summing the $B(M1)$'s for both 2^+_2 and 2^+_3 states, we can obtain a smooth dependence on the n-p quadrupole strength (see the inset of Fig. 1). It is worth mentioning that a similar smooth dependence on the n-p quadrupole strength is also observed in the $B(E2;2^+_1\to 0^+_1)$. In view of the recent speculation that the $E2$ and $M1$ transitions are conspicuously correlated [18], this similarity is particularly interesting. Within the present context, such a correlation can be understood as follows: The n-p quadrupole interaction, which is known to be the primary driving force for deformation (via the $0^+_1\to 2^+_1$ $E2$ transition), is now shown to be decisive in splitting the two partner energies and producing their $M1$ transitions. Hence this interaction appears to be the origin of these two types of transitions.

The same pattern is observed for the other partners states (2_3^+ and 2_4^+ at $\kappa = 0.0$), which suggests that the same physics prevails here. On the other hand, it should be noticed that the $B(M1)$ strength is reduced by half. This is due to the strong mixing with the nonpartner components in 2_4^+ and the 2_2^+ or 2_3^+ states. Finally, for all the transitions in Fig. 1, the $E2$ and the $M1$'s saturate in the presence of strong n-p coupling. Finally, we see that both 2_5^+ and 1_1^+ states—both mainly have $|D_\pi D_\nu\rangle$ as components—are also pushed up by the n-p term. From this analysis we see that the essential physics do emerge from the fermion point of view.

We now come to the full spectroscopy of ^{134}Ba. The results using the FDSM code FDU0 [19] are presented in Fig. 2. Suffice to say that the low-lying spectrum is in good agreement with the data, especially those between the predictions and the recently reported measurements of the exotic 2^+ states [11].

There are some details in Fig. 2 which deserve attention here. First, we predict that there is a 2_3^+ state at 1960 keV with a large $M1$ strength predicted. Experimentally, there are two nearly degenerate 2^+ states (separated by 50 keV) with appreciable $M1$'s at this energy [20]. This may be due to the fragmentation of the $M1$ strengths in the excited 2^+ states [22]. Therefore the sum of these two $B(M1)$'s should be comparable to the concentrated $M1$ strength in our calculation. The sum of these two experimental $B(M1)$'s, i.e., $\Sigma_{i=3,4} B(M1 2_i^+ \to 2_1^+)$, is equal to $0.199\mu_N^2$. With the bare g factors, the $B(M1)$ is overestimated ($0.38\mu_N^2$). However, by using the effective g factors ($g_\pi = 0.9$ and $g_\nu = 0.1$), a reasonable $B(M1)$ value is obtained ($0.24\mu_N^2$). As was pointed out, similar splittings in energies between the partner states with a strong $M1$ transition between them should also occur in the calculated 2_4^+ and 2_2^+ states. These states will correspond to the experimental 2_5^+ (2371 keV) and 2_2^+ (1168 keV) states respectively. The predicted $M1$ transition between them, using the same effective g factors, is $0.08(\mu_N^2)$. Finally, although not shown here, the 1_1^+ is predicted at 2486 keV with an appreciable $M1$ transition to the ground state. The possible candidates for this state in the data are those located at either 2335 keV or 2571 keV.

In Fig. 2(b), we show the predicted $E2$ and $M1$ transitions in a weak quadrupole n-p interaction case ($\kappa = -0.050$ keV). The purpose of this calculation is to demonstrate that in this limit, which is suitable for the vibrational nuclei, the $M1$ transitions differ significantly from the strong coupling case, which is suitable for the SO(6)-like nuclei. According to our analysis, for the weak limit, one would expect that there will be strong $M1$ transitions from 2_2^+ to 2_1^+ and from 2_4^+ to 2_3^+. However, for the strong n-p coupling, which ^{134}Ba is an example, because of the exchange of the level ordering as was discussed before, a strong $M1$ transition will occur

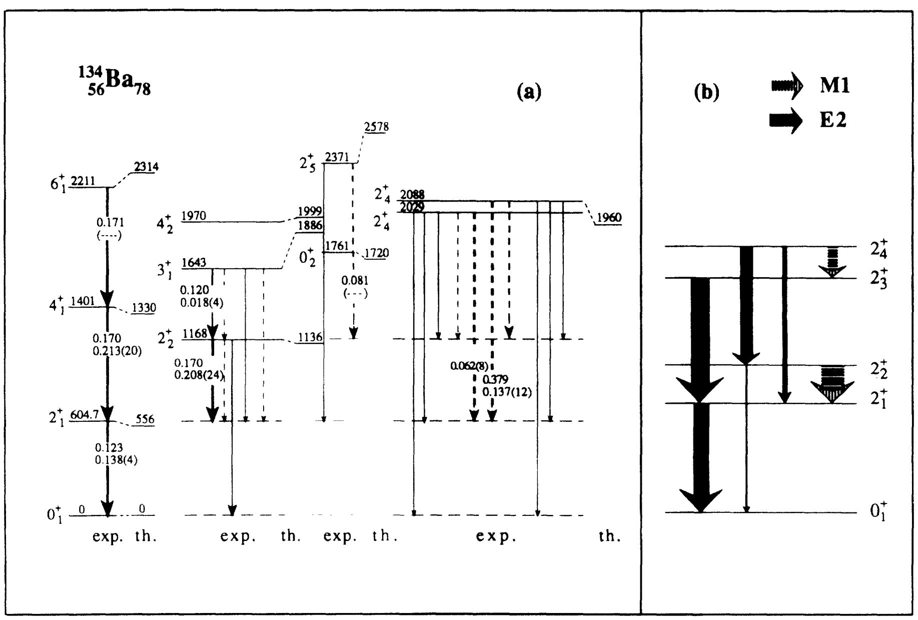

FIG. 2. (a) The theoretical spectrum and transitions of ^{134}Ba determined by the Hamiltonian given by Eq. (1) are compared with the data [11]. The parameters for the Hamiltonian are $G_\pi = -0.106$ MeV and $G_\nu = -0.094$ MeV. These values are reduced by 20% from the pairing strengths of ^{138}Ba and ^{128}Sn. Also, $\kappa = -0.34$ MeV and $\lambda = -0.25$ MeV. In (a), the calculated $E2$'s (solid arrows) and $M1$'s (dotted arrows) are compared with the data for those strong transitions. The two numbers in each transitions are theoretical predicted (upper) and experimental (lower) values. (—) means that there are no data. Transitions without inserted numbers mean that the predicted and measured values are very weak. (b) The $E2$ and $M1$ transitions for the artificially weakened n-p quadrupole coupling are presented. The width of the line is roughly proportional to the intensity.

between 2_3^+ and 2_1^+ states. These FDSM results are consistent with IBM-2 systematic analysis of the Xe-Ba-Ce region [23]. In that study, the strong $M1$ transitions between 2_3^+ and 2_1^+ are reproduced without the Majorana interaction, while the bosonic quadrupole n-p interaction is mapped from the fermion S-D subspace to account for the fermion many-body blocking effect, and the parameter of the one-body d boson term essentially represents the pairing effect. Finally, it is interesting that by using the same parameters, we can also predict equally well the systematics of spectroscopy in the neighboring Ba isotopes (mass 130–136). In fact, the appreciable $M1$ strengths between the excited 2^+ states are also predicted [15].

In summary, there are three points which deserve to be noted here.

First, we have presented two complementary pictures of bosons and fermions to describe the normal and the exotic states. We find that the bosonic concepts of symmetry and mixed-symmetry can subtly be interpreted within the fermion picture as well. However, there is one (important) dichotomy. In the fermion description, the n-p quadrupole interaction is responsible for splitting these two types of states and produces strong $M1$ transitions. This phenomenon is in close analogy to the L-S splitting of orbital and spin spaces. The examples given in the paper show that the many 2^+ normal and exotic states are in fact "partners" for the n-p quadrupole coupling and therefore must split in its presence. However, this emphasis of the n-p coupling appears to be opposite to the F-spin role in the boson picture because a corresponding boson n-p quadrupole term will break the F-spin symmetry. Indeed, one could regard the Majorana terms as a F-spin symmetry restoration interaction.

Second, we should stress that what is important here is that the FDSM represents a class of fermion models constructed from S and D pairs. In fact, similar and generic physics can be and has been obtained in other S and D pair shell models [12].

Finally, as was stressed often (e.g., [13]), the collective nuclear Hamiltonian must contain competing long and short range interactions. We like to emphasize that not only should the normal states be eigenstates of such a Hamiltonian, but the exotic states as well. In fact, we suspect that the proper placement of the positions of the exotic states and the prediction of their respective transitions must be another stringent constraint on the effective interactions of the Hamiltonian. This could be the contact point between the present fermion description and the approach for the lighter systems [21] for the exotic states.

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