Dependence of nuclear shape transformations on the nuclear volume

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The dependence of nuclear shape transition on changes in the nuclear volume in ^{24}Mg has been studied within the framework of the constrained finite temperature Hartree-Fock approximation. The deformation parameters β and γ are very sensitive to changes in the nuclear volume. A first-order shape transition in ^{24}Mg takes place at the temperature of 1 MeV and at the volume of $V_c = 1.025\,V_0$, where V_0 is the zero temperature unconstrained volume of the system. Previously we have shown that a 2.5% compression of the system yields a downward shift in the critical temperature of 0.7 MeV. Similarly a 2.5% expansion of the system also yields a decrease in the critical temperature of 0.3 MeV. Finally a compression of the system leads to a corresponding reduction in the magnitude of the level density, while an expansion increases its value only slightly.

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I. INTRODUCTION

Nuclear shape transitions have been discussed extensively in the literature within the framework of finite temperature mean field (FTMF) approximation [1–22]. It has been shown, in unconstrained finite temperature Hartree-Fock (FTHF) calculations, that the finite nuclear systems undergo deformed-to-spherical shape transitions [1–13]. Furthermore in cranked finite temperature Hartree-Fock-Bogoliubov (CFTHFB) and finite temperature Hartree-Fock (CFTHF) calculations, shape transitions from prolate collective to oblate noncollective rotation take place for fixed nonzero values of the average angular momentum [14–24]. Clearly deformed-to-spherical transitions cannot occur in cranked mean field calculations as the equation of constraint must be satisfied for each value of angular momentum [23]. Despite large fluctuations in the order parameters, which tend to obscure these transitions [16–18], they can be identified by peaked structures in the specific heat. In canonical ensemble calculations, using either the exact shell model or the experimental nuclear energy eigenspectrum, similar structures are seen in the specific heat which confirms that such shape transitions do occur [21,22,24,25]. Furthermore the critical temperature is predicted remarkably well in FTMF calculations even in small model spaces [26]. Such transitions have been interpreted as thermal excitations from collective to noncollective portions of the nuclear spectrum [24,25]. The peaks in the specific heat in all cases are the result of a sudden increase in the many-body level density around the critical temperature [24,25]. Since this level density is also an experimentally measurable quantity, it is believed to be the relevant order parameter for shape transitions in deformed nuclei [25].

In FTMF calculations, as the system heats up, it is usually allowed to expand freely. Therefore the specific heat calculated, strictly speaking, is not $C_{\langle V\rangle}$, but $C_{\langle N\rangle}$ since only the average number of particles is held constant. In order to calculate $C_{\langle V\rangle}$ in the FTMF approximation one has also to constrain the average volume of the system to a fixed value. Such calculations enable one to investigate how structures in the specific heat, which are indicative of a shape transition in a deformed nuclear system, respond to changes in the nuclear volume. This is of particular interest in high energy heavy ion collisions as it may provide a means of determining the density of a compressed system and ultimately provide information about the nuclear equation of state. Previously [27] we performed a FTHF calculation at fixed volume in ^{24}Mg at different temperatures. We found that a 2.5% compression on the system causes the critical temperature of the deformed-to-spherical shape transition to shift downward by about 0.7 MeV. The density of states of the compressed system is also decreased considerably. In the present work we present the results at different volumes for fixed temperatures of 1.0 and 2.6 MeV and for different temperatures at a fixed volume $V = 1.025V_0$ where V_0 is the unconstrained volume. The general formalism is given in Sec. II. We then present and discuss our results in Sec. III. Finally, in Sec. IV, we draw our conclusions.

II. THE VOLUME-CONSTRAINED FINITE TEMPERATURE HARTREE-FOCK APPROXIMATION

We expand the Hartree-Fock (HF) orbitals in a harmonic oscillator basis with oscillator frequency $\hbar\omega = 14$ MeV, consisting of the $0s, 0p, 1s-0d$ shells [28]. A realistic effective Hamiltonian, including folded diagram corrections, was used [29–31]. Since we enforce degeneracy of the neutron and proton orbitals our solutions automatically have isospin zero.

Minimizing the free energy,

$$F = \langle \hat{H} \rangle - TS, \quad (1)$$

subject to following constraints,

$$\sum_i d_i^{\alpha *} d_i^{\beta} = \delta_{\alpha\beta}, \quad (2)$$

$$\langle N \rangle = \sum_\alpha f_\alpha = N, \quad (3)$$

$$\langle V \rangle = \frac{4\pi}{3} \langle R \rangle^3 = V_s \quad (4)$$

with

$$\langle R \rangle = \left[\frac{1}{N} \sum_{\alpha,i,j} \langle i | r^2 | j \rangle f_\alpha d_i^{\alpha *} d_j^{\alpha} \right]^{\frac{1}{2}}, \quad (5)$$

where T is the temperature, $S = -\sum_\alpha [f_\alpha \ln f_\alpha + (1 - f_\alpha)\ln(1 - f_\alpha)]$ is the entropy, f_α is the thermal occupation probability in the αth HF orbit, and d_i^α is the expansion coefficient of the αth orbit in the ith oscillator state, yields the FTHF equations. The volume constraint here is actually simply a constraint on the mean squared radius of the system and has been introduced in this way so that we can extract the "effective pressure" from the Lagrange multiplier of this constraint. Furthermore we approximate the volume of the system at all temperatures with the spherical form and V_s is the prescribed volume for the system. This approximation is reasonable in the vicinity of the shape transition since here the deformations are in any case small. Even in the region far away from the critical points, the maximum deformation of $\beta \sim 0.3$ only introduces a maximum error of $\sim 2\%$ [32]. In our calculation we have taken V_s to lie between 0.975 to 1.026 times V_0, where V_0 $(= \frac{4\pi}{3}\langle R \rangle^3 = 90.58$ fm^3) is the volume of the system at zero temperature. The FTHF equations are given by

$$\sum_j h_{ij}^{\mathrm{HF}} \, d_j^{\lambda} = \epsilon_\lambda \, d_i^{\lambda}, \quad (6)$$

where

$$h_{ij}^{\mathrm{HF}} = \sum_{\alpha,k,l} f_\alpha d_k^{\alpha *} d_l^{\alpha} \langle ik | \hat{H} | jl \rangle_A + \frac{2\pi P \langle R \rangle}{N} \langle i | r^2 | j \rangle, \quad (7)$$

and

$$f_\alpha = \left[1 + \exp\left(\frac{\epsilon_\alpha - \mu}{T} \right) \right]^{-1}. \quad (8)$$

ϵ_λ, μ, and P are Lagrange multipliers associated with the equations of constraint and are determined self-consistently. If we had simply constrained the mean squared radius, the Lagrange multiplier Λ would have been given by

$$\Lambda = 2\pi P \langle R \rangle.$$

The subscript A in Eq. (7) indicates the two-body state $|jl\rangle$ is antisymmetrized. Note that the HF Hamiltonian h_{ij}^{HF} possesses an extra term due to the volume constraint.

III. RESULTS AND DISCUSSIONS

A. Results at fixed temperature

The Hill-Wheeler deformation parameters [33] β and γ as functions of V obtained at $T = 1.0$ and 2.6 MeV are given in Figs. 1 and 2. At $T = 2.6$ MeV the system becomes spherical at $V_1 = 88.4$ and $V_2 = 92.8$ fm^3. The system reaches its maximal deformation at $V_0(2.6) = 90.89$ fm^3, its unconstrained ($P = 0$) volume. As soon as we begin to compress or to expand the system, it becomes less deformed. Between V_1 and V_2, the system remains prolate. At $T = 1.0$ MeV, the system also reaches its maximal deformation at its unconstrained volume $V_0(1.0) = 90.60$ fm^3. It is very interesting to note that the system remains triaxial for most of the volumes considered, except at $V_c = 92.84$ fm^3, when it suddenly becomes oblate. Generally speaking the system becomes less deformed at all volumes with increasing temperature. The discontinuities in β and γ as well as other properties of the system, as we shall show, seem to suggest that at $T = 1.0$ MeV the ^{24}Mg system undergoes a first-order transition at V_c.

The ensemble averages of the energy at both temperatures as functions of volume V are shown in Fig. 3. It is clearly seen that both the curves exhibit minima at the unconstrained volumes $V_0(1.0)$ and $V_0(2.6)$. For $T = 2.6$ MeV, a change of slope at V_1 and V_2 occurs when the system changes its shape from prolate to spherical. At $T = 1.0$ MeV, a discontinuity at V_c occurs as the system suddenly changes its shape from triaxial to oblate.

In the FT mean field approximation the many-body level density is given by [1,2,34]

$$\rho = \frac{\exp(S)}{\left[2\pi \sum_\alpha (\epsilon_\alpha - \mu)^2 f_\alpha (1 - f_\alpha)\right]^{1/2}}. \quad (9)$$

The result of these level density calculations are given in

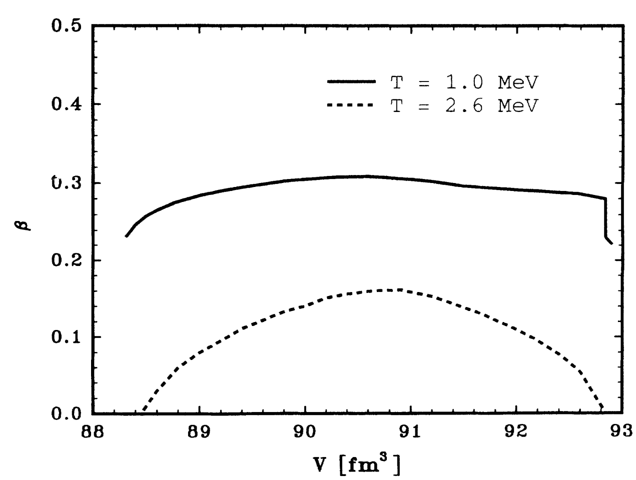

FIG. 1. The Hill-Wheeler parameter β of ^{24}Mg systems as a function of V at $T = 1.0$ and 2.6 MeV.

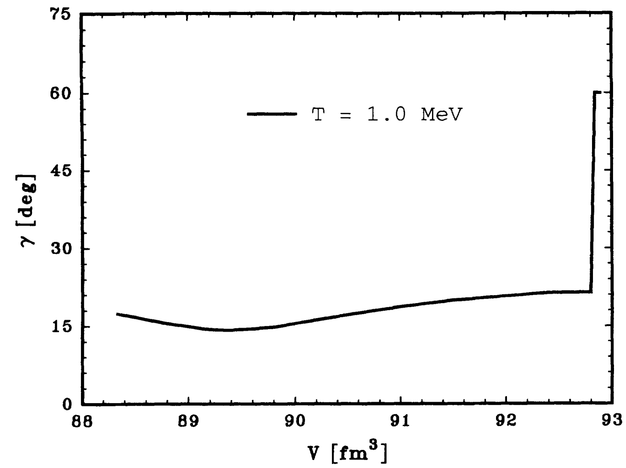

FIG. 2. The Hill-Wheeler parameter γ of ^{24}Mg systems as a function of V at $T = 1.0$. The system has a prolate shape, *i.e.* $\gamma = 0°$ at $T = 2.6$ MeV for $V_1 < V < V_2$.

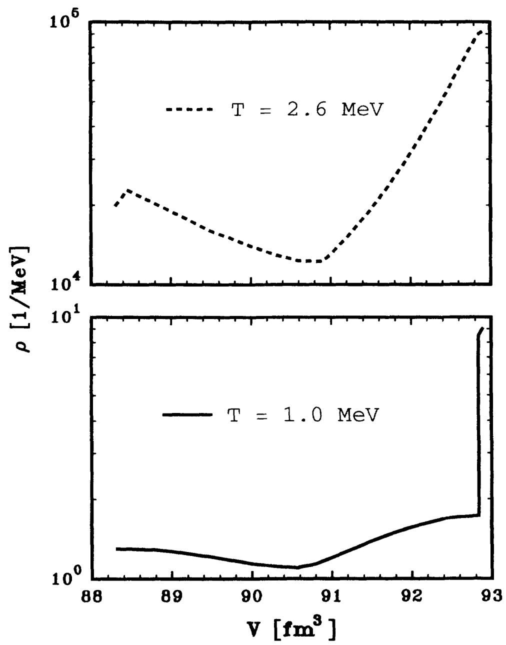

FIG. 4. The many-body level density ρ of ^{24}Mg systems as a function of V at $T = 1.0$ and 2.6 MeV. Note that the scale for ρ is logarithmic.

Fig. 4. Again, the level density at each temperature has a minimum at the volume corresponding to the unconstrained volume. For $T = 2.6$ MeV, two discontinuities in the slope of ρ can be seen where shape transformations occur. For $T = 1.0$ MeV the discontinuity of ρ at V_c is also an indication of a first-order shape transition of the system from triaxial to oblate.

Finally in Fig. 5, we give the "effective pressure" as a function of V. At both temperatures the pressure decreases with increasing volume. For $V < 89.6$ fm^3, the pressure of system decreases with increasing temperature as we have shown in our previous work [27]. The trend reverses for $V > 89.8$ fm^3.

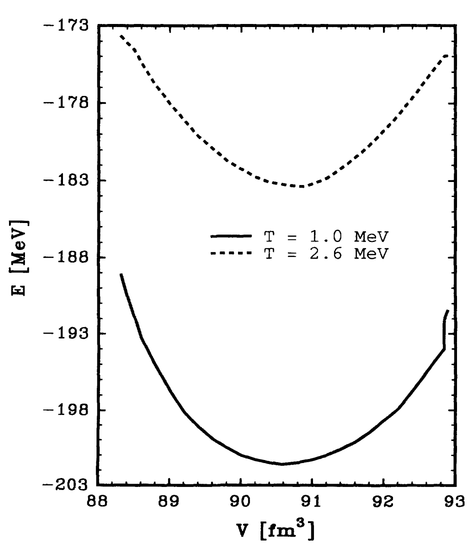

FIG. 3. The ensemble average of the energy E of ^{24}Mg systems as a function of V at $T = 1.0$ and 2.6 MeV.

B. Results at fixed volume

The results for a compressed volume have been given previously [27]. Here we would like to show the result of an expansion of the system. The Hill-Wheeler deformation parameters β and γ as functions of T at $V = 1.025\,V_0$ are shown in Fig. 6. The value of β jumps from 0.21 to 0.29 at around $T = 1.0$ MeV, then decreases monotonically to 0 at $T = 2.6$ MeV when the system becomes spherical. γ also shows a discontinuity at $T = 1.0$ MeV where its value drops suddenly to 21° from 60°. It then

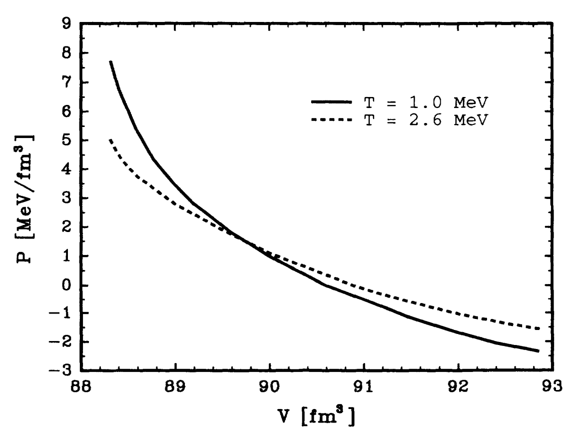

FIG. 5. The "effective pressure" P of ^{24}Mg systems as a function of V at $T = 1.0$ and 2.6 MeV.

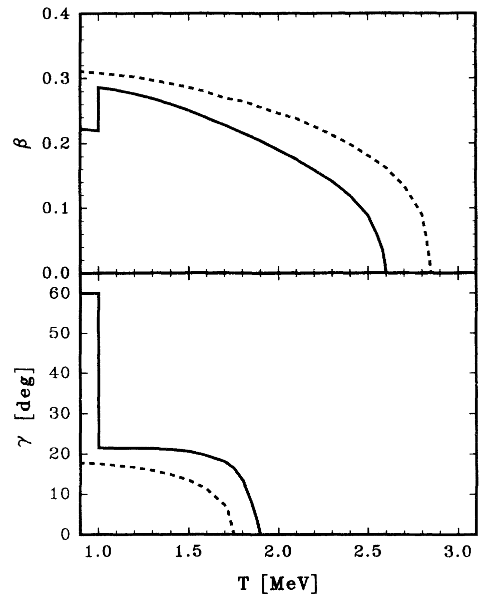

FIG. 6. The Hill-Wheeler parameters β and γ of ^{24}Mg systems as a function of T at $V = 1.025\,V_0$ (solid curves). The dotted curves designate the unconstrained results.

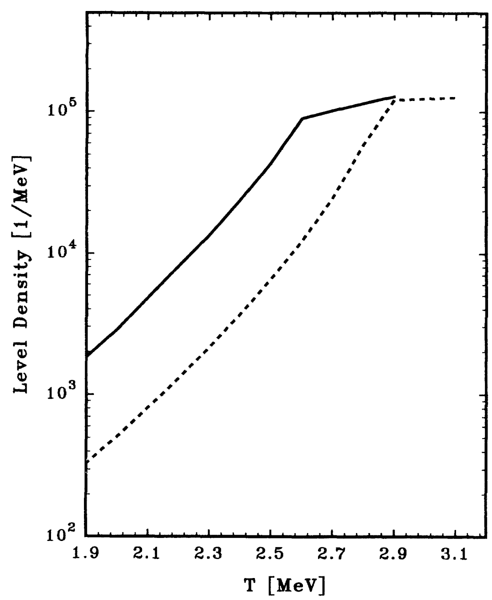

FIG. 8. The many-body level density ρ of ^{24}Mg systems as a function of T at $V = 1.025\,V_0$ (solid curve). The dotted curve represents the unconstrained results.

decreases monotonically to 0° at $T = 1.9$ MeV. Again the first-order transition from oblate shape to triaxial at $T = 1.0$ MeV and at $V = 1.025\,V_0 = V_c$ is clearly seen. When the system expands the critical temperature of the triaxial-to-prolate shape transition increases from 1.8 to 1.9 MeV, while it decreases to 1.6 MeV for the compressed case ($V = 0.975\,V_0$). The critical temperature of the prolate-to-spherical shape transition decreases from

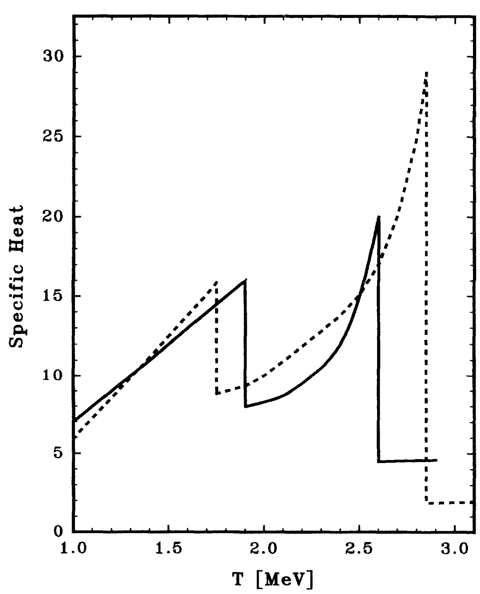

FIG. 7. The specific heat of ^{24}Mg systems as a function of T at $V = 1.025\,V_0$ (solid curve). The dotted curve denotes the unconstrained results.

2.9 to 2.6 MeV when the system expands by 2.5%. For the same amount of compression the critical temperature decreases to 2.2 MeV. This is somewhat unexpected, but not too surprising, as the system starts with the minimal deformation at low temperatures in the compressed case.

In Fig. 7 the specific heat as a function of T is given. The two peaks in the specific heat at $T = 1.9$ and 2.6 MeV indicate the aforementioned triaxial-to-prolate and prolate-to-spherical transitions, respectively. Finally, we give the many-body level density ρ as a function of T in Fig. 8. The expansion in volume appears to cause ρ to increase slightly from the unconstrained case, while a similar compression drastically reduces its value.

IV. CONCLUSIONS

From these results it is easily seen that the critical temperature for the deformed-to-spherical shape transition responds in a very sensitive manner to the small changes in the nuclear volume. The deformation parameters β and γ are very sensitive to changes in the nuclear volume. A first-order shape transition in ^{24}Mg takes place at the temperature of 1 MeV and at $V = V_c$. Previously we have shown that a 2.5% compression of the system yields a downward shift in the critical temperature of 0.7 MeV [27]. Similarly a 2.5% expansion of the system also yields a decrease in the critical temperature of 0.3 MeV. As we have already noted the critical temperature is determined remarkably well in the FTMF approximation in the uncompressed system. Varying the amount of compression or expansion will yield a corresponding shift in the critical temperature. A compression of the system leads to a corresponding reduction in the magnitude of the level density, while an expansion increases its value only slightly.

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