Reaction $pp \to n\Delta^{++}$: Observables and model predictions

F. Sammarruca
Department of Physics, University of Idaho, Moscow, Idaho 83843
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A recent analysis of exclusive pion production in proton-proton scattering allows for the direct comparison of theory with $pp \to n\Delta^{++}$ observables. We have extracted $NN \to N\Delta$ transition amplitudes from the Bonn meson-exchange model and calculated differential cross section and asymmetry for this reaction. In particular, we establish correlations between the asymmetry and the phases of the major inelastic transition amplitudes. We find evidence that the phases of the transition amplitude ${}^3F_3(NN) \to {}^5P_3(N\Delta)$ are responsible for the large discrepancy between theory and experiment in the asymmetry around 800 MeV incident laboratory kinetic energy. We also suggest how the theory can be improved.

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I. INTRODUCTION

For several years, it has become increasingly clear that a quantitative description of the pion production channel in nucleon-nucleon (NN) scattering is problematic. Moreover, different models seem to have rather similar trends, regardless the details of the physical input. Typically, one observes underestimation of the inelasticity in some NN partial waves, especially ${}^3F_3$ and, to a lesser extent, ${}^1D_2$, even though the qualitative features of these waves are usually reproduced.

All the theories we are referring to have in common that they involve only known mesons and baryons together with the usual effective meson-baryon interactions. Essentially, we can distinguish between two basic frameworks, namely, coupled two-body channels [1–5] and three-body equations [6–14]. Coupled two-body channels seem to do better in the nucleon-nucleon elastic sector, especially at low energy. On the other hand, three-body models, which concentrate on having control over the πN input, tend to provide less quality in the description of the low NN partial waves, and therefore appear more peripheral in nature. For instance, a realistic description of the P_{11} πN partial wave seems to imply a deterioration of the NN P waves, which become overly attractive [15].

Assessing the quality of a model is then somewhat dependent on the specific aspects of the dynamics which are being probed, with no model up to now being able to claim good control over the entire intermediate energy range (incident kinetic laboratory energy $T_{\rm lab} \leq 1$ GeV).

Within this energy regime, the nucleon-nucleon inelasticity is almost entirely provided by the $N\Delta$ intermediate state. Therefore, the theoretical model for the NN-$N\Delta$ transition amplitude will be crucial in describing pion production. For a better understanding of the microscopic mechanism of pion production, it is then best to look at those observables which "amplify" the Δ signal, like exclusive observables (the exclusive kinematics actually singles out $NN \to N\Delta$ amplitudes with a definite final momentum). A recent experimental analysis of exclusive pion production data [16] for the reaction $pp \to pn\pi^+$ actually allows for a direct comparison of theory with $pp \to n\Delta^{++}$ observables, which is clearly a very stringent test of the (off-shell) $N\Delta$ amplitudes.

With this objective, we have extracted the NN-$N\Delta$ amplitudes from the field-theoretic Bonn model [17] and applied them to the $NN \to N\Delta$ reaction. In the next section, we discuss some technical aspects; we also take a close look at the extracted $N\Delta$ amplitudes and perform a model dependence study of the major inelastic transition amplitudes. We present results in Sec. III, while conclusions and outlook are contained in Sec. IV.

II. $N\Delta$ AMPLITUDES

A model like the one developed by the Bonn group [17], with very few phenomenological parameters, allows for a considerable control over the input, and makes it easy to understand the results in terms of field-theoretic diagrams.

We use here the coupled channel Bonn model [17] which includes isobar degrees of freedom and uses relativistic NN-$N\Delta$ and NN-$\Delta\Delta$ transition potentials, with π and ρ exchanges. In this model, interactions within the $N\Delta$ channels are neglected, as well as $\Delta\Delta$ vertices.

For the purpose of comparison, we have also used a modified version of the Bonn model ("peripheral model" [18]). This version provides less short-range repulsion by suppressing the ρ exchange in the $NN \to N\Delta$ and $NN \to \Delta\Delta$ transition potentials, and applying a weaker ω coupling ($g_\omega^2/4\pi = 10$ instead of 23). As a consequence of this, the S and P waves become too attractive, while the more peripheral waves improve over the entire energy range [18]. In particular, the critical ${}^3F_3$ shows a much more resonant structure in the real phases, and there is some improvement in the ${}^3F_3$ inelasticity parameter ρ, which however still remains below the empirical one; see

Fig. 1. For instance, at $T_{\rm lab} \approx 800$ MeV, the peripheral model yields a value of 18.9° for ρ, while the standard Bonn parametrization predicts a value of 13.5° (the empirical value at 800 MeV is 25.9° [19]). However, a corresponding overestimation in the $^1D_2$ partial wave seems hard to avoid (cf. Fig. 1). Our phase shifts and inelasticity parameters are defined according to the Arndt-Roper convention [20].

By comparing the predictions by the two models, we hope to get insight into how the description of specific partial waves (central or peripheral) is reflected in the observables.

Technically, when the scattering integral equation is solved on the real axis, the half-off-shell $T(NN \to N\Delta; p)$ transition amplitudes, with p the $N\Delta$ relative momentum, can be easily extracted. In the intermediate energy range, the dominant $NN \to N\Delta$ amplitudes are $^1D_2 \to {}^5S_2$ and $^3F_3 \to {}^5P_3$, due to the fact that they couple to very central $N\Delta$ states. To begin with, we have looked at these amplitudes, as functions of the relative $N\Delta$ momentum (in the $N\Delta$ center of mass frame). Even though this information may not be directly extracted from the experimental data, it is interesting to relate the model dependence of the complex amplitudes (modulus and phase) to the observables. For instance, a strong phase difference between two models, even just in a particular partial wave, could explain a large model dependence of the phase-sensitive asymmetry.

In Figs. 2 and 3 we show the magnitude and phase of the transition amplitudes $^1D_2(NN) \to {}^5S_2(N\Delta)$ and $^3F_3(NN) \to {}^5P_3(N\Delta)$ plotted versus the relative $N\Delta$ momentum p, at $T_{\rm lab}$= 800 MeV. The solid and dashed lines refer to the Bonn model of Ref. [17] and the more peripheral version [18], respectively. In the following, we

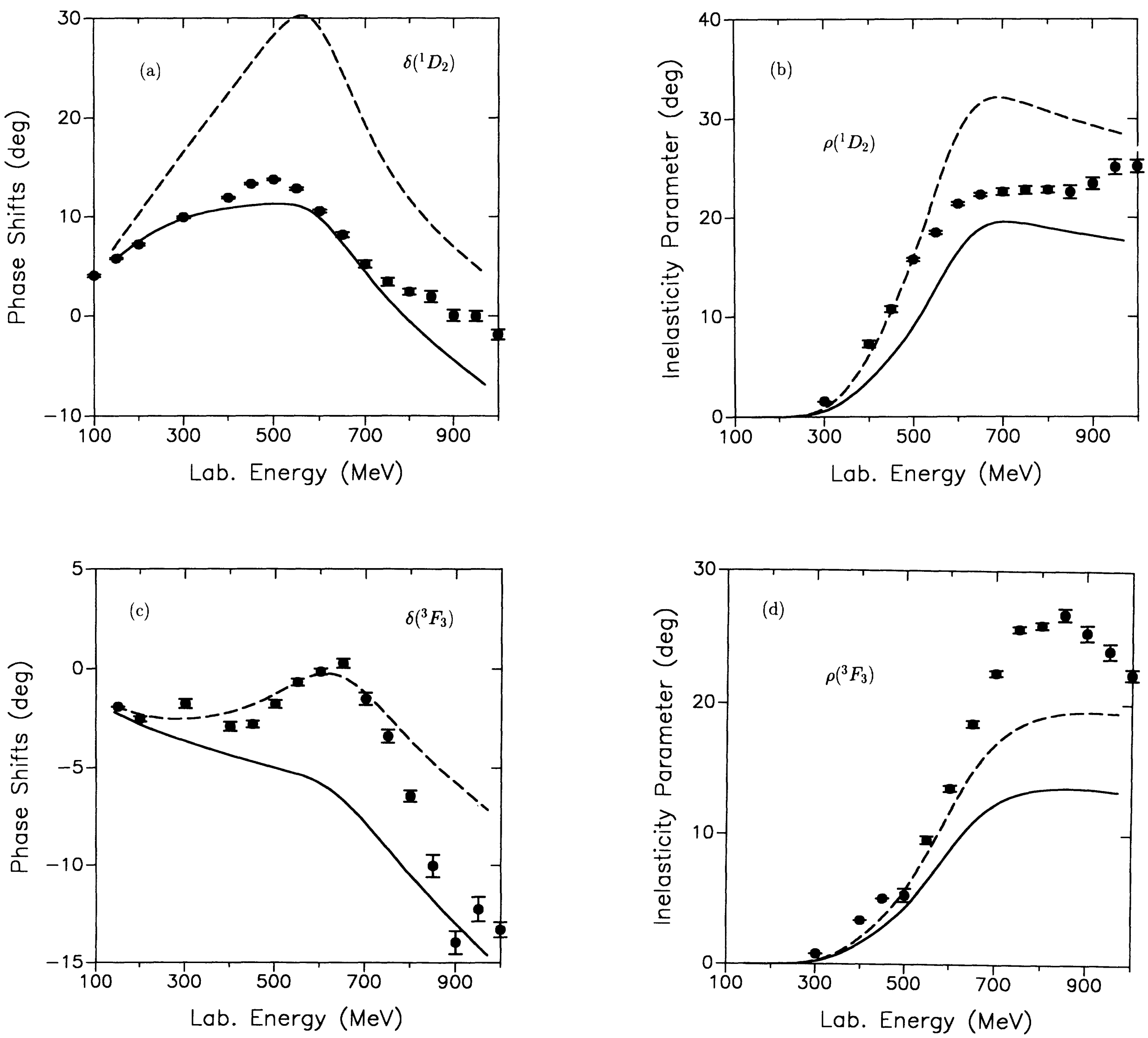

FIG. 1. Phase shifts and inelasticity parameters for the $^1D_2$ (a and b) and $^3F_3$ (c and d) NN partial waves. Predictions by the Bonn (solid line) and the Bonn peripheral (BP) (dashed line) models are shown. The solid dots represent the analysis of Ref. [19].

will refer to those models as Bonn and Bonn peripheral (BP), respectively.

In the reaction $pp \to pn\pi^+$ at T_{lab}= 800 MeV, the interesting area of large outgoing proton momentum corresponds to p values in the range of about 250–500 MeV/c (lower end of the Δ mass range).

The model dependence of the phases is striking: In the $J = 2$ case, the phases with the BP model are always positive, and much larger. A similar correlation was found in a recent work by Kloet and Lomon [21] (see Figs. 5 and 6 therein): They found the phases to be positive in the Kloet-Silbar three-body model [10], where a large attractive $N\Delta$-$N\Delta$ transition potential is involved, and strongly negative in a coupled channel isobar model [22]. In our case, the extra attraction introduced in the peripheral model produces positive ${}^1D_2 \to {}^5S_2$ inelastic phases.

For the triplet, on the other hand, the phases are negative in the Bonn model, and remain mostly negative in the BP version, even though the attraction in the BP version brings the phases to less negative values.

In the model study of Ref. [21], it was found that the coupled channel isobar model [22], where short-range interactions are parametrized as a boundary condition on the wave function, predicts strongly positive (up to 50°) triplet inelastic phases. On the other hand, the Kloet-Silbar three-body model predicts very small and negative triplet phases [very much like our BP curve in Fig. 2(b)]. By replacing selected partial waves (e.g., triplet only or singlet only), the authors of Ref. [21] conclude that the best description of the $pp \to pn\pi^+$ analyzing power at T_{lab}= 800 MeV is obtained with a strongly positive ${}^3F_3(NN) \to {}^5P_3(N\Delta)$ phase and a small positive ${}^1D_2(NN) \to {}^5S_2(N\Delta)$ phase. Although one must be very careful when interpreting the result of mixing

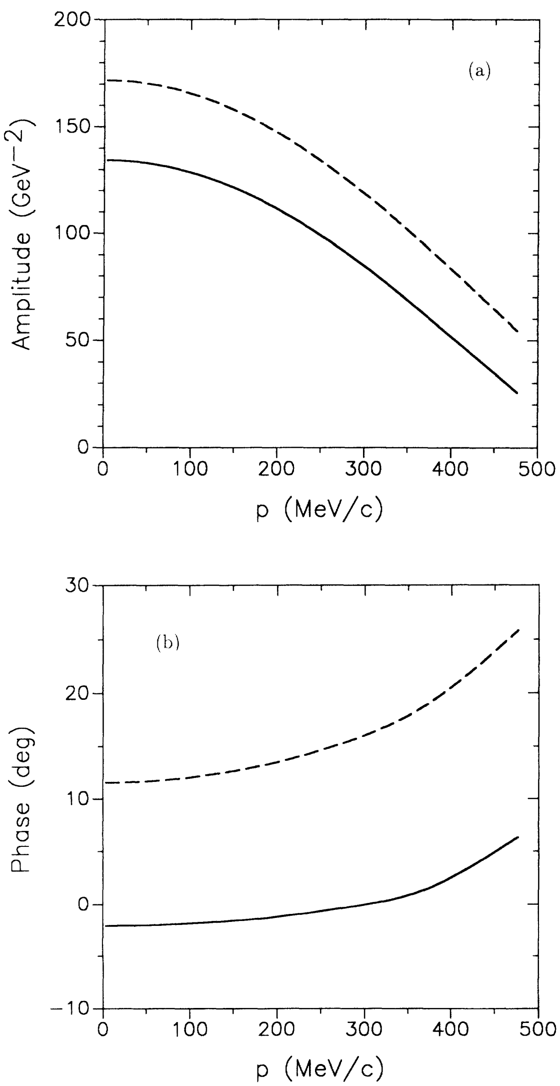

FIG. 2. Magnitude (a) and phase (b) of the ${}^1D_2(NN) \to {}^5S_2(N\Delta)$ amplitude at T_{lab}=800 MeV, as a function of the relative $N\Delta$ momentum p. The solid curve is obtained with the Bonn model, the dashed with the BP model.

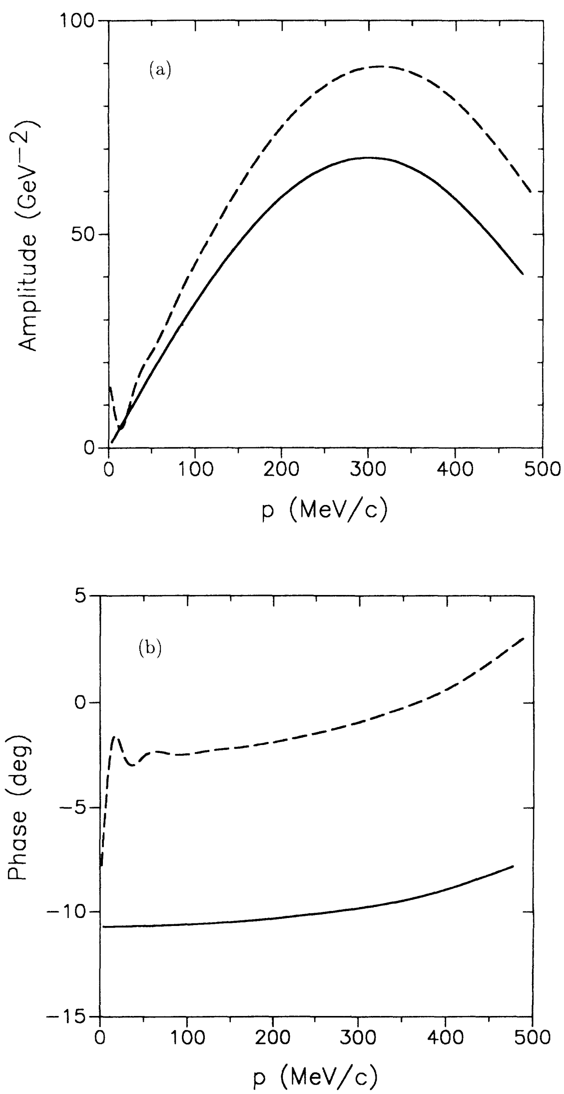

FIG. 3. Magnitude (a) and phase (b) of the ${}^3F_3(NN) \to {}^5P_3(N\Delta)$ amplitude at T_{lab}=800 MeV. Curves as in Fig. 2.

different models, one can certainly draw general and interesting conclusions from this type of study: First of all, getting the asymmetry right is a delicate matter of improving selected partial waves (while it is much easier to improve the scale of the cross section, which depends on the magnitude of the amplitudes). Also, one can understand the model dependence of observables much better than when looking at the elastic phase shifts only, and actually narrow it down to a particular partial wave. In fact, two models with strikingly different inelastic phases may still have rather similar *elastic* phases [21].

In Figs. 4 and 5, we show the same quantities as in Figs. 2 and 3, but at $T_{\rm lab}$= 570 MeV, since we are going to look at observables at that energy as well.

After this preliminary study, we shall see next what the implications are for the cross section and polarization in the reaction $pp \to n\Delta^{++}$.

III. RESULTS

We will be looking at the differential cross section and beam asymmetry A_{N0} for the reaction $pp \to n\Delta^{++}$ at

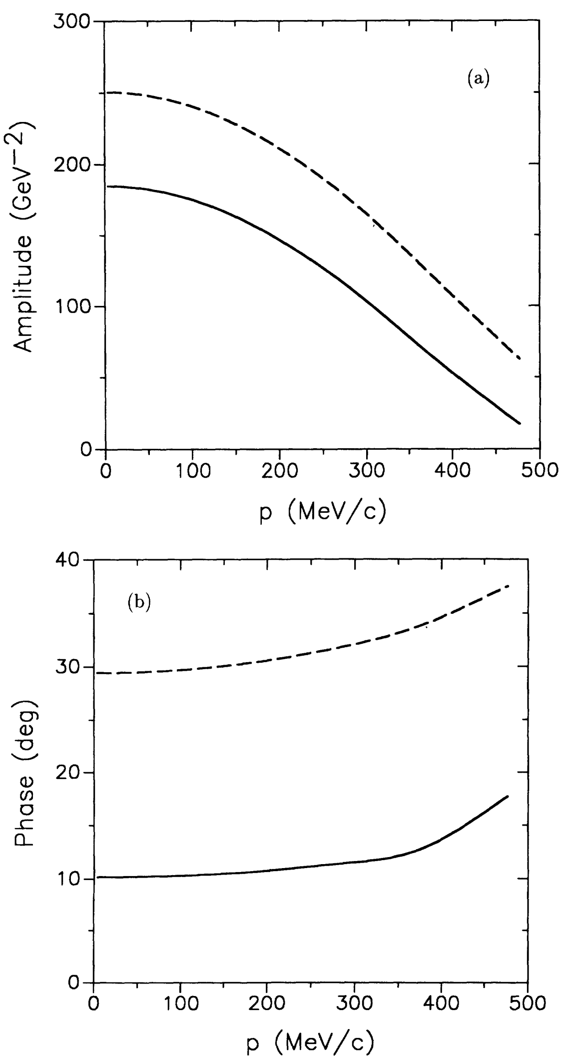

FIG. 4. Same as Fig. 2, but at $T_{\rm lab}$=570 MeV.

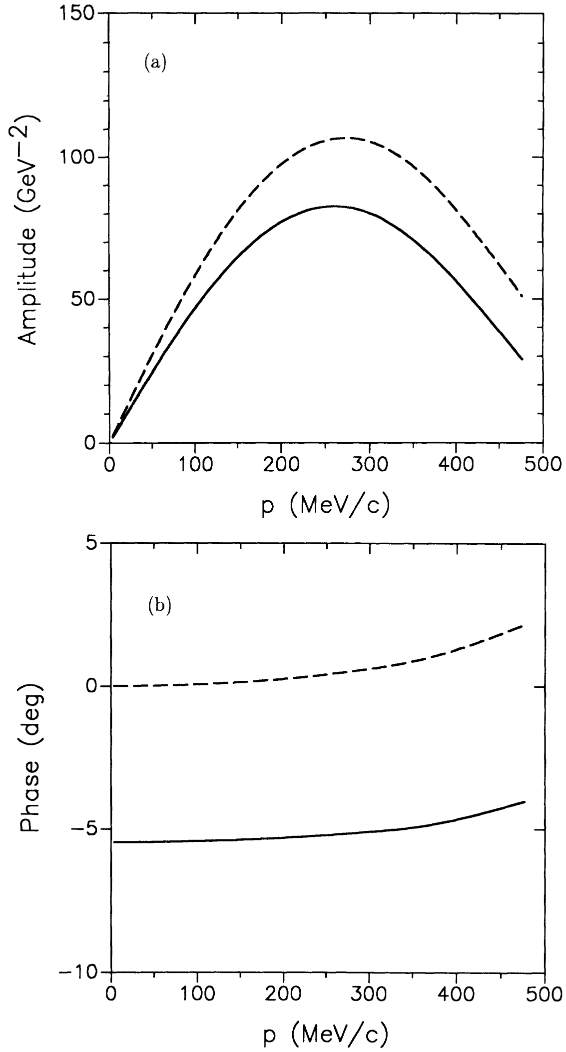

FIG. 5. Same as Fig. 3, but at $T_{\rm lab}$=570 MeV.

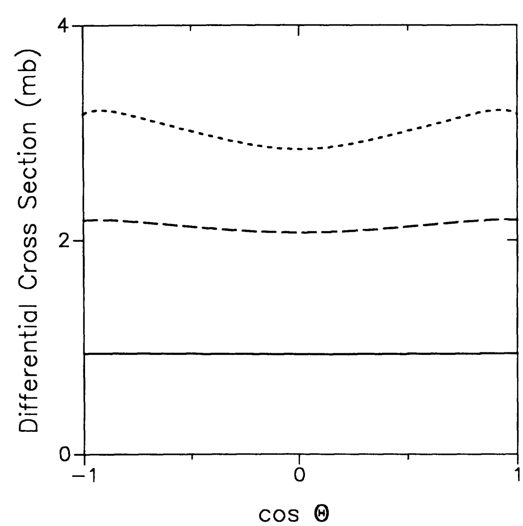

FIG. 6. Differential cross section for the reaction $pp \to n\Delta^{++}$ at $T_{\rm lab}$=570 MeV, at three different values of the Δ invariant mass: M_Δ = 1.20 GeV, solid line; M_Δ = 1.18 GeV, dashed line; M_Δ = 1.16 GeV, dotted line. The Bonn model is used. Throughout this work, angular momentum states up to J = 9 are included.

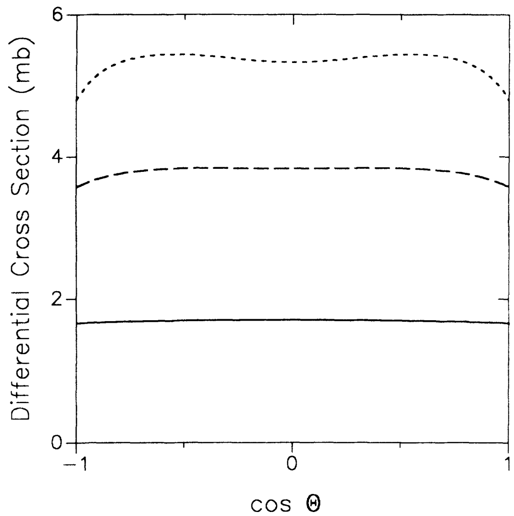

FIG. 7. Same as Fig. 6, but with the BP model.

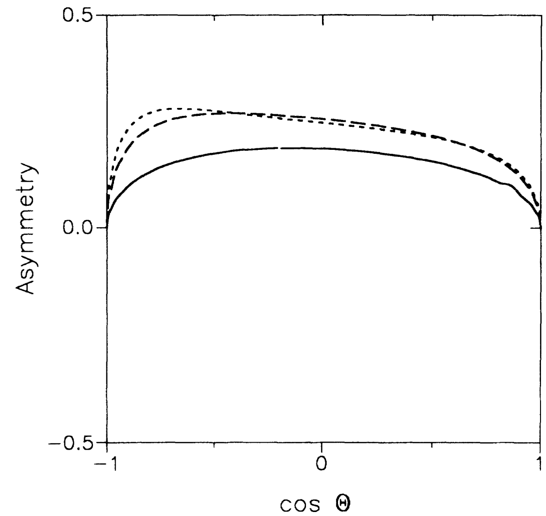

FIG. 9. Beam asymmetry for $pp \to N\Delta^{++}$ at $T_{\rm lab}$=570 MeV for three values of the Δ mass. Values of M_Δ as in Fig. 6. The Bonn model is used.

$T_{\rm lab}$= 570 and 810 MeV, as provided by the analysis by Wicklund *et al.* [16].

In Fig. 6, we show the differential cross section $\frac{d\sigma}{d\cos\theta}$ at $T_{\rm lab}$=570 MeV, with θ the scattering angle in the $n\Delta^{++}$ center of mass, for three values of the Δ invariant mass. Those curves are obtained with the standard Bonn model. The same is shown in Fig. 7, except that the BP model is used. In Fig. 8, we perform a comparison between theory and experiment. Note that the data are integrated over the experimental Δ^{++} band, namely, between 1.16 and 1.20 GeV. Therefore, the theoretical curves in Fig. 8 are an average over that band.

In Figs. 9, 10, and 11, we perform a similar study for the beam asymmetry A_{N0} at $T_{\rm lab}$=570 MeV. Figures 9 and 10 show the dependence of A_{N0} on the Δ mass, while the data in Fig. 8 are integrated over values of the Δ mass from 1.16 GeV to 1.20 GeV. Notice that A_{N0} is not as sensitive to the Δ mass as the cross section (cf. Figs. 6, 7).

We also present results at $T_{\rm lab}$= 810 MeV for the cross section in Figs. 12, 13, and 14 and for the asymmetry in Figs. 15, 16, and 17. In this case, the experimental range of the Δ mass goes from 1.18 GeV to 1.28 GeV.

At 570 MeV, the magnitude of the cross section is reasonable with the Bonn model, and too large for the BP model (Fig. 8). This is probably due to the dominant role of the singlet at this energy, which is too strong in the peripheral model (cf. Fig. 1).

For the asymmetry at 570 MeV there is agreement between the BP model and the data (Fig. 11). This indicates a good phase relation among the amplitudes. In the momentum region of interest for Figs. 9–11, the phases of the major singlet are large and positive (about 10°

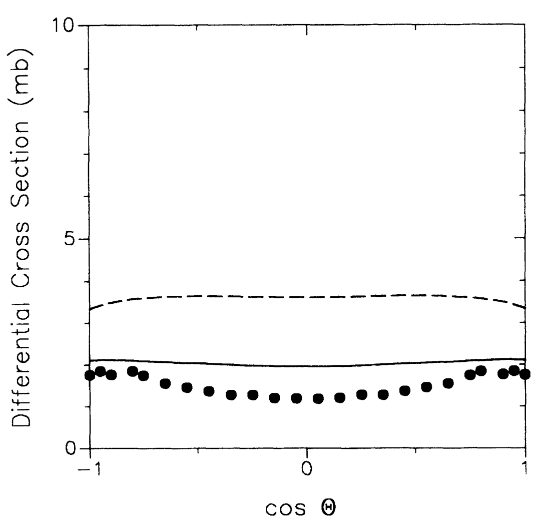

FIG. 8. Average value of the $pp \to n\Delta^{++}$ differential cross section at $T_{\rm lab}$=570 MeV, for M_Δ between 1.16 GeV and 1.20 GeV. The solid curve is obtained with the Bonn model, the dashed with the BP model. Data from Ref. [16].

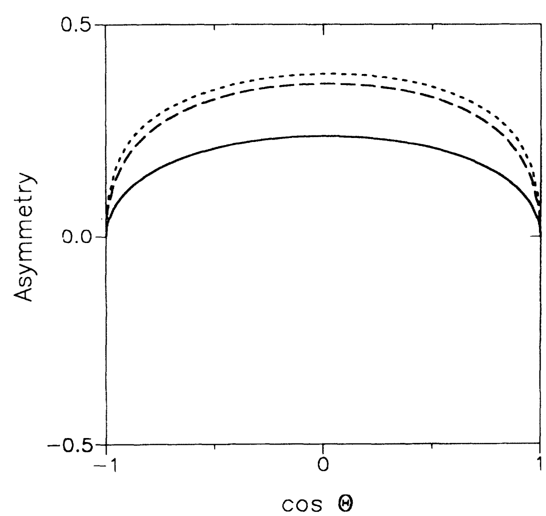

FIG. 10. Same as Fig. 9, but with the BP model.

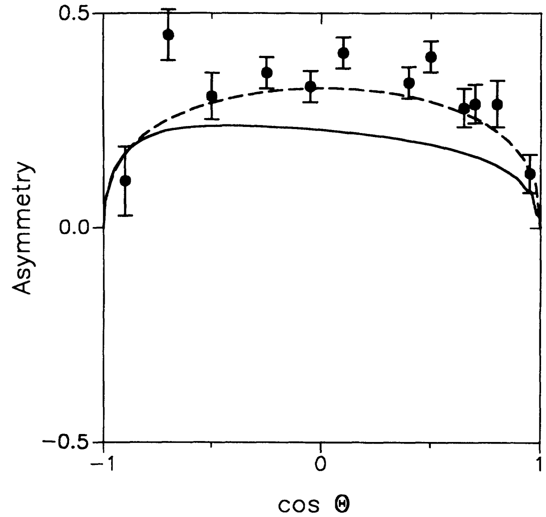

FIG. 11. Average value of the $pp \to n\Delta^{++}$ beam asymmetry at $T_{\rm lab}$=570 MeV, for M_Δ between 1.16 GeV and 1.20 GeV. The solid curve is obtained with the Bonn model, the dashed with the BP model.

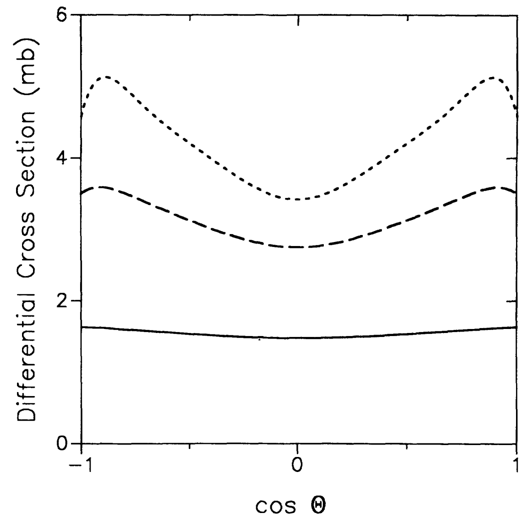

FIG. 13. Same as in Fig. 12, but with the BP model.

with the Bonn model and up to 30° with the BP model; see Fig. 4), while the phases of the largest triplet are negative (about −5°) with the Bonn model, and go up to very small positive values (about zero) with the BP model; see Fig. 5. Clearly, strongly positive phases in the major singlet $NN \to N\Delta$ transition amplitude, together with very small triplet phases, are consistent with the data.

Also, one may notice how the large experimental values of A_{N0} suggest large interference and hence a nontrivial phase structure for the Δ production waves [16]. Another remarkable feature of A_{N0} is the approximately symmetric structure in $\cos\theta$ at all energies. This suggests [16] a predominance of singlet-triplet interference contributions.

At 810 MeV, the situation is much more problematic. The cross section is far too small, also for the BP model (Fig. 14). From the previous section, we know that the BP parametrization enhances the triplet, but not drastically (the inelasticity in this wave is still insufficient; cf. Fig. 1). This explains why the cross section remains poor, especially noticing that, at this energy, the triplet starts to play a dominant role [15].

Concerning the asymmetry, both models fail to reproduce the dip structure about 90° (Fig. 17). This is not surprising, since the more peripheral model does not change the singlet-triplet phase relation, but it rather applies an overall boost to all phases. Note, however, that

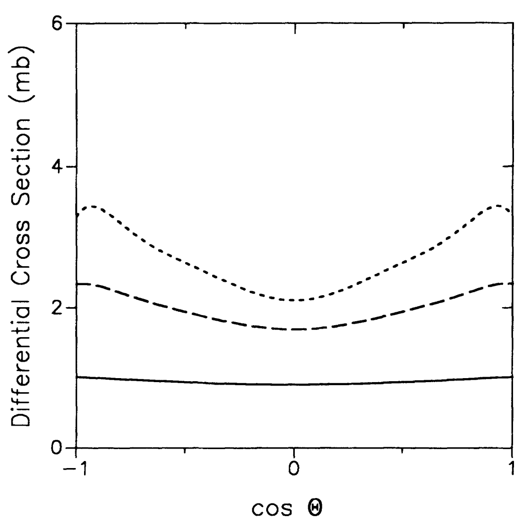

FIG. 12. Dependence of the $pp \to n\Delta^{++}$ differential cross section at 810 MeV on the Δ mass: M_Δ = 1.28 GeV, solid line; M_Δ = 1.24 GeV, dashed line; M_Δ = 1.20 GeV, dotted line. The Bonn model is used.

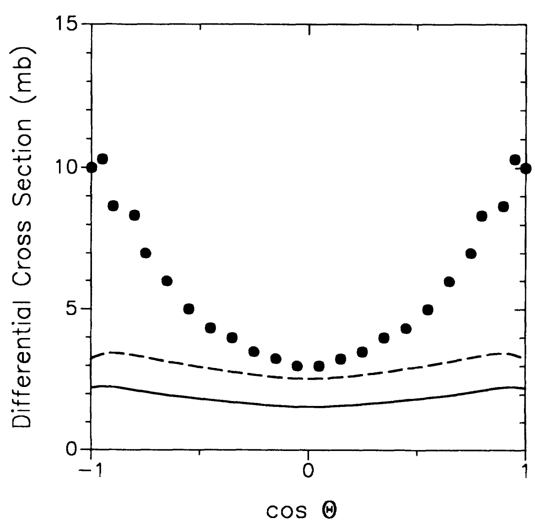

FIG. 14. Average value of the $pp \to n\Delta^{++}$ differential cross section at 810 MeV, for M_Δ between 1.20 GeV and 1.28 GeV. The solid curve is obtained with the Bonn model, and the dashed with the BP model. Data from Ref. [16].

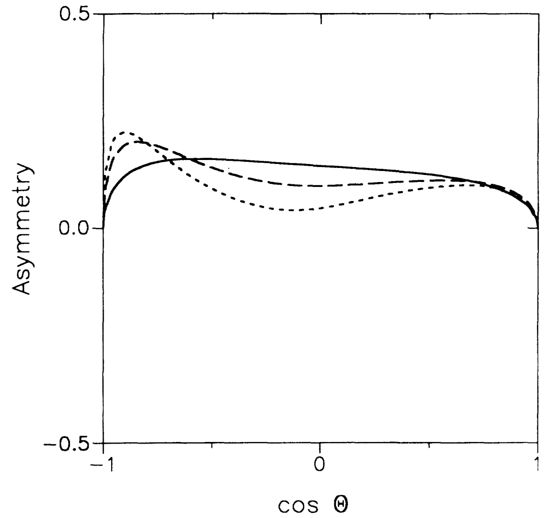

FIG. 15. Dependence on the Δ mass of the $pp \to n\Delta^{++}$ beam asymmetry at 810 MeV. Values of M_Δ as in Fig. 12. The Bonn model is used.

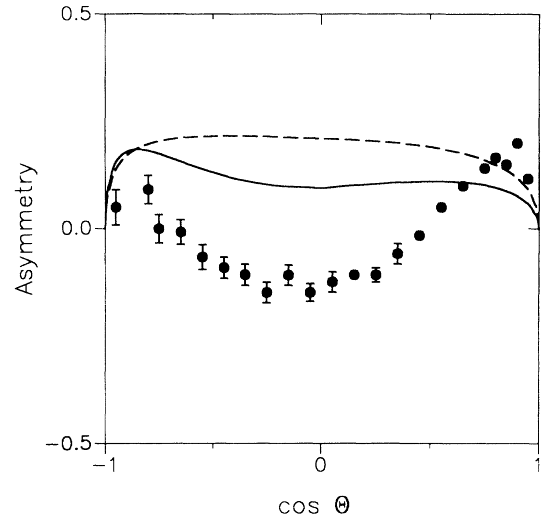

FIG. 17. Average value of the $pp \to n\Delta^{++}$ beam asymmetry at 810 MeV, for M_Δ between 1.20 GeV and 1.28 GeV. Solid curve, Bonn model; dashed curve, BP model. Data from Ref. [16].

the Bonn model has a slight indication of a central dip; this may be attributed to the inclusion of ρ exchange in the $NN \to N\Delta$ transition potential, which is omitted in the BP model.

To get a feeling of what controls the structure of the asymmetry at this energy, we have performed the following test: We have taken the asymmetry at a particular value of the Δ mass, namely, a specific value of p (this enables us to identify the corresponding inelastic phase of the $N\Delta$ transition amplitude). In Fig. 18, the value of the Δ mass is 1.2 GeV, and the corresponding momentum is 335 MeV/c. Around T_{lab}=800 MeV, for p between 300 and 400 MeV/c, the phase of the ${}^3F_3(NN) \to {}^5P_3(N\Delta)$ amplitude (with Bonn) is about −10°; see Fig. 3. (We have checked the phases at 810 MeV, and they are very similar to those at 800 MeV, shown in Fig. 3.) We have then arbitrarily multiplied these phases (triplet only), by a factor −2, making them large and positive. The corresponding structure of the asymmetry changes dramatically (Fig. 18).

This is of course just phenomenology, but it gives a good idea of the sensitivity of these observables to the phases at specific values of the kinematics.

Finally, we compare some of our results for the cross

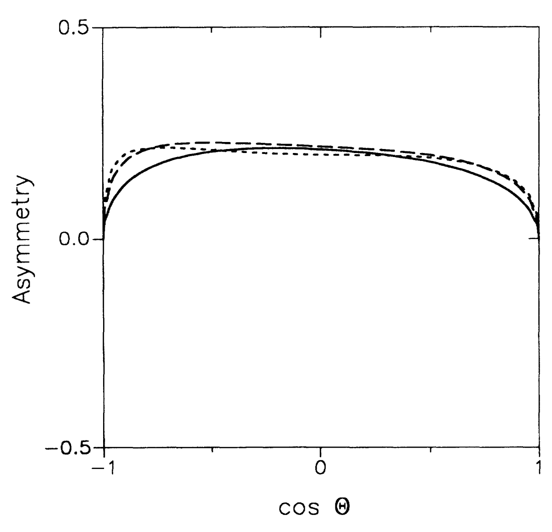

FIG. 16. Same as in Fig. 15, but with the BP model.

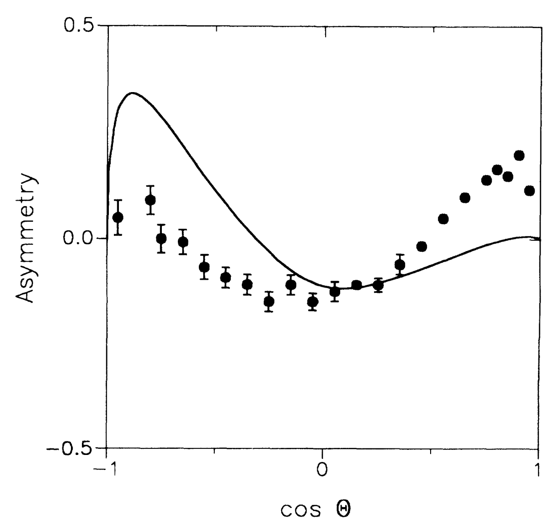

FIG. 18. Beam asymmetry at 810 MeV when the phases of the transition amplitude ${}^3F_3(NN) \to {}^5P_3(N\Delta)$ are multiplied by a factor −2. The Bonn model is used. Data from Ref. [16].

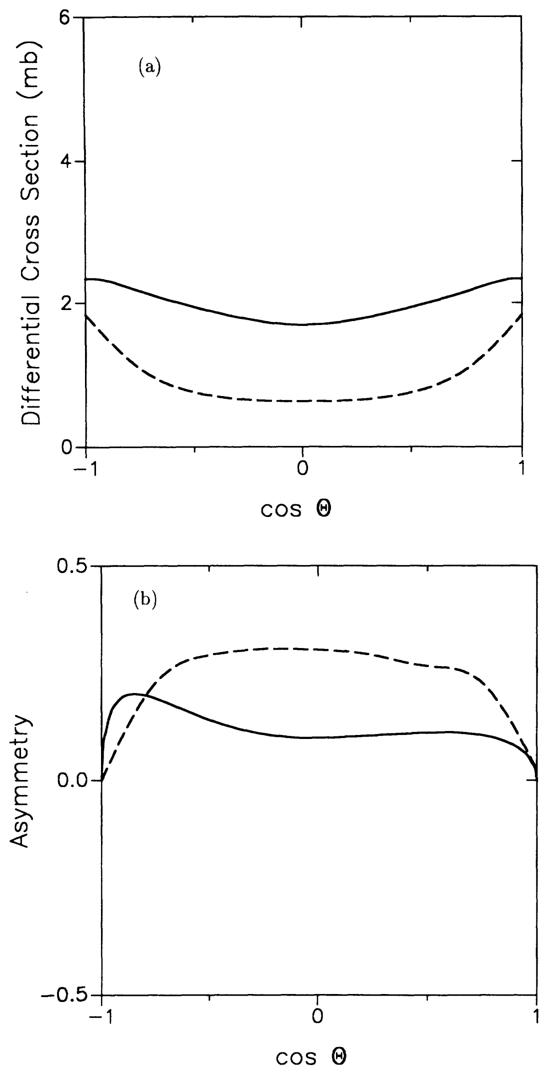

FIG. 19. Cross section (a) and beam asymmetry (b) at $T_{\rm lab}$=800 MeV for M_Δ near 1.24 GeV. The solid curve is obtained with the Bonn model; the dashed curve is from Ref. [23].

section and the asymmetry with those of Ref. [23], where the three-body model of Ref. [24] is used; see Fig. 19. We do the comparison for one particular value of the Δ mass, using the curves from Fig. 1 of Ref. [23]. We notice qualitative agreement in the cross section. In the asymmetry, the three-body model seems to provide less structure as compared with the Bonn model. In fact, the three-body model result is very similar to what we obtain with the peripheral version (see Fig. 16). As we mentioned above, the ρ exchange in the $NN \to N\Delta$ transition could be responsible of the different structure in the asymmetry.

IV. CONCLUSION

We have applied a relativistic isobar model to the $pp \to n\Delta^{++}$ reaction. We have performed tests to get insight into the partial wave content of the reaction. From this study, it is clear that only a careful, energy-dependent modification of selected partial waves will improve the description of this channel. The problems at $T_{\rm lab}$=810 MeV are most likely due to the $NN \to N\Delta$ transitions in the triplet state. These need improvement in the strength (modulus), as seen from the cross section, as well as the phases, as seen from the asymmetry. Even though a large part of the inelasticity may come from the $N\Delta$ box diagram, it is likely that interactions within this channel (which are omitted in the present Bonn model) may play a non-negligible role. Recently some evidence has been presented [25] indicating that the diagonal pion exchange contribution to the $N\Delta$ interaction consistently improves the critical NN partial waves. Considering the dominant role played by the Δ, it seems possible that a recoupling to all orders of all the $N\Delta$ channels may provide, at least to some extent, the energy- and partial-wave-dependent improvement we are seeking. Using the tools we have developed in this work, we should be able to realize whether we are on the right track by a quick check of the inelastic phases.

In conclusion, few-body microscopic calculations are quite laborious, and it is tempting to give them up and invoke exotic degrees of freedom, but still a lot can be done within the conventional approach to refine models like the present one. Separable approximations make a coupled channel treatment much easier to handle, but only a true coupled channel with a nonseparable realistic potential will actually reveal the limits of meson theory.

Given the excellent quality of the Bonn model at low energy [26], it is worthwhile to push it to its limits at intermediate energies, thus exploring the range of validity of effective meson-baryon Lagrangians within a field-theoretic approach.

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