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Nature of the photon correlation function for quark-gluon plasma

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The correlation function for intensity interferometry of high energy photons emitted from an expanding quark-gluon plasma is examined. It is reiterated that the correlation function for identical bosons emitted from two point sources is given by $1 + \cos(\Delta k \cdot \Delta x)$ and will oscillate between 0 and 2, depending on the values of the difference in the 4-momenta of the two bosons (Δk) and the difference in their 4-positions (Δx). Integration over smooth, realistic sources, relevant for very high energy nucleus-nucleus collisions, gives a correlation function which essentially decreases monotonically from 2 to 1 as the momentum difference becomes large. The oscillatory behavior seen in a recent work for certain momentum configurations is caused by an approximation which amounts to including only those sources for which the space-time rapidities are identical to the photon rapidities.

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Intensity interferometry of two identical particles is by now routinely employed to obtain the space-time extent of sources both in astronomy [1] and in heavy-ion reactions [2]. A large literature has developed on utilizing pion interferometry to get valuable insight about the reaction zone in heavy-ion reactions. However, since hadrons only appear in the final state of a very high energy nucleus-nucleus collision which may admit a QCD phase transition, their correlations mainly carry information about the late dilute stage of the collision, not about the early dense stage. The usefulness and feasibilty of photon interferometry as a source of information on the history of evolution of an expanding quark-gluon plasma likely to be created in collisions involving ultrarelativistic heavy ions has been examined recently [3–6]. In contrast to hadrons, photons are produced throughout the space-time evolution of the reaction, and suffer essentially no interactions with the surrounding medium once they are produced; this would make them a very valuable probe if photons from hadronic decays can be subtracted out.

In these studies both (1+1) and (3+1) dimensional expansions of the plasma, driven by internal pressures, were considered. The correlation function for (1+1) dimensional expansion generally exhibited stronger oscillatory behavior than it did for (3+1) dimensional expansion. Such oscillations are typical of stellar observations [1]. Here we reexamine the (1+1) dimensional results, and show that this strong oscillatory behavior was caused by an approximation employed to simplify the eight-dimensional integral, which was not done for the (3+1) dimensional case.

However, before proceeding to the approximation, let us discuss briefly the formulation of the correlation function, which will also help us to understand the oscillatory behavior mentioned above. In what follows we shall closely follow the treatment of McLerran [8].

To begin with, let us assume that there are two indentical *point* sources, located at positions x_1 and x_2 with respect to the observer. Let the single particle momentum distribution be dN/d^3k, and the two-particle momentum distribution be $dN/d^3k_1d^3k_2$. Then the two-particle correlation function is defined by

$$C(\mathbf{k}_1,\mathbf{k}_2) = E_1E_2\frac{dN}{d^3k_1d^3k_2}\Bigg/ E_1\frac{dN}{d^3k_1}E_2\frac{dN}{d^3k_2}\,. \tag{1}$$

In the absence of any correlation this ratio should be 1. In order to compute the correlation, the emission amplitude is written as

$$A(k) = \frac{1}{\sqrt{2}}\rho(k)\left[e^{i\phi_1}e^{ikx_1} \pm e^{i\phi_2}e^{ikx_2}\right]\,, \tag{2}$$

where $\rho(k)$ is a real function which characterizes the source strength and ϕ_i are the phases of the sources. The plus sign is for identical bosons and the minus sign is for fermions. The single particle distribution function is

$$\frac{dN}{d^3k} = \rho(k)^2\left[1 \pm \cos(k\Delta x + \phi_1 + \phi_2)\right]\,. \tag{3}$$

Now, for incoherent emission, we have

$$\langle e^{i\phi_1 - i\phi_2}\rangle = 0\,, \tag{4}$$

and thus

$$\frac{dN}{d^3k} = \rho(k)^2\,. \tag{5}$$

The amplitude for two-particle emission is given by

$$A(k_1,k_2) = \frac{1}{\sqrt{2}}\rho(k_1)\rho(k_2)\left[e^{ik_1x_1+ik_2x_2+i\phi_1+i\phi_2}\right. \\ \left. \pm\, e^{ik_2x_1+ik_1x_2+i\phi_1+i\phi_2}\right]\,. \tag{6}$$

The two-particle distribution function is then given by

$$\frac{dN}{d^3k_1 d^3k_2} = \rho(k_1)^2 \rho(k_2)^2 \left[1 \pm \cos(\Delta k \cdot \Delta x) \right], \quad (7)$$

which is independent of ϕ_1 and ϕ_2. Now the correlation function becomes

$$C(\mathbf{k}_1, \mathbf{k}_2) = 1 \pm \cos(\Delta k \cdot \Delta x) \,. \quad (8)$$

We see that the correlation function has a characteristic oscillation scale of order $\Delta k \approx 1/\Delta x$. In this example, the correlation function does not go to 1 at large relative momenta, but oscillates between 0 and 2 indefinitely. The oscillating cosine *may* be thought of as zero for larger relative momenta if one averages it over a sufficiently large momentum interval. For zero relative momentum the correlation function reduces to 1 ± 1.

For sources continuously distributed over space and time we can write the correlation function for photons with the same helicity as

$$C(\mathbf{k}_1, \mathbf{k}_2) = \frac{P(\mathbf{k}_1, \mathbf{k}_2)}{P(\mathbf{k}_1) P(\mathbf{k}_2)} \,, \quad (9)$$

where

$$P(\mathbf{k}) = \int d^4x E \frac{dN(x, \mathbf{k})}{d^4x d^3k} \quad (10)$$

and

$$P(\mathbf{k}_1, \mathbf{k}_2) = \int d^4x_1 d^4x_2 E_1 \frac{dN(x_1, \mathbf{k}_1)}{d^4x_1 d^3k_1} E_2 \frac{dN(x_2, \mathbf{k}_2)}{d^4x_2 d^3k_2} \times [1 + \cos(\Delta k \cdot \Delta x)] \,. \quad (11)$$

$dN(x, \mathbf{k})/d^4x d^3k$ is the rate per unit volume for producing a photon with momentum $\mathbf{k}$ at the space-time point x.

As mentioned earlier, the approximation we want to examine was used only in the (1+1) dimensional Bjorken hydrodynamics [7]. In Bjorken hydrodynamics the local flow velocity of the matter can be expressed in terms of the space-time rapidity η as $u^\mu = (\cosh\eta, 0, 0, \sinh\eta)$. The coordinate time and position of the matter are $x^\mu = (\tau \cosh\eta, r \cos\phi, r \sin\phi, \tau \sinh\eta)$, where τ is the proper time and r and ϕ are the radial coordinate and angle. The 4-momentum of the ith photon is expressed as $k_i^\mu = (k_{iT} \cosh y_i, k_{iT} \cos\psi_i, k_{iT} \sin\psi_i, k_{iT} \sinh y_i)$, where k_T is the transverse momentum, ψ is the azimuthal angle, and y is the rapidity. We shall choose axes such that $|k_{1T} \sin\psi_1 - k_{2T} \sin\psi_2| = 0$. For the thermal emission rate of photons from quark-gluon plasma and hadronic gas we use the results of Ref. [9]:

$$E \frac{dN}{d^4x d^3k} = K T^2 \ln\left(\frac{2.9}{g^2}\frac{E}{T} + 1\right) \exp(-E/T), \quad (12)$$

where E is the photon energy, g is the QCD coupling constant, and K is a constant.

Evaluation of the correlation function will involve the computation of an eight-dimensional integral. The first, exploratory studies [3,4] used Bjorken hydrodynamics and simplified this multidimensional integral by using an approximation suggested in [11]. In the limit that $k_T/T \gg 1$ one makes the replacement

$$e^{-k_T \cosh(y-\eta)/T} \rightarrow e^{-k_T/T} \left[\frac{2\pi T}{k_T}\right]^{1/2} \delta(y - \eta) \,. \quad (13)$$

With this δ function approximation the expressions for $P(\mathbf{k}_1, \mathbf{k}_2)$ and $P(\mathbf{k})$ simplify. The problem reduces to one-dimensional quadrature. $P(\mathbf{k}_1, \mathbf{k}_2) = P_1 P_2 + P_{c1} P_{c2} + P_{s1} P_{s2}$, where

$$P_i = P(\mathbf{k}_i) = \pi R^2 K \int d\tau\, \tau \sqrt{\frac{2\pi T}{k_{iT}}} T^2 \ln\left(\frac{2.9}{g^2}\frac{k_{iT}}{T} + 1\right) \times \exp(-k_{iT}/T) \,, \quad (14)$$

and

$$P_{ci} = \pi R^2 K \int d\tau\, \tau \sqrt{\frac{2\pi T}{k_{iT}}} T^2 \ln\left(\frac{2.9}{g^2}\frac{k_{iT}}{T} + 1\right) \times \exp(-k_{iT}/T) \times \left[\frac{2J_1(q_T R)}{q_T R}\right] \cos[(\Delta E \cosh y_i - q_L \sinh y_i)\tau] \,. \quad (15)$$

Here J_1 is the Bessel function, R is the radius of the identical nuclei undergoing a central collision, and $\Delta E = k_{1T} \cosh y_1 - k_{2T} \cosh y_2$, $q_T = k_{1T} \cos\psi_1 - k_{2T} \cos\psi_2$, $q_L = k_{1T} \sinh y_1 - k_{2T} \sinh y_2$. The P_{si} are the same as the P_{ci} with the substitution of a sine for the cosine. When combining these expressions to obtain $C(\mathbf{k}_1, \mathbf{k}_2)$ it is apparent that the normalization of the rate, K, is irrelevant.

If we are interested in the particular momentum configuration $k_{1T} = k_{2T} = k_T$ and $y_1 = y_2$, which is equivalent to $\Delta E = q_L = 0$, the correlation function can be evaluated exactly and without the need of the δ function approximation to get

$$C(\mathbf{k}_1, \mathbf{k}_2) = 1 + \left[\frac{2J_1(q_T R)}{q_T R}\right]^2 \,. \quad (16)$$

In order to check the validity of the δ function approximation we shall take the time dependence of the temperature and the parameters as used before in [3]. These are $R = 7$ fm, $T_c = 160$ MeV, $T_{\text{initial}} = 532$ MeV at $\tau_{\text{initial}} = 1/3T_i = 0.124$ fm/c [10], and terminate the computation at $T_{\text{final}} = 140$ MeV.

In Fig. 1 we show the correlation function from the plasma for the configuration $k_{1T} = k_{2T}$ and $\psi_1 = \psi_2 = 0$ as a function of q_L, corresponding to Fig. 3(a) of Ref. [4]. The solid curves give the results for numerical integration of the eight-dimensional expression, and the dashed curves give the results with the δ function approximation. We see that the approximation used for Bjorken hydrodynamics in the earlier studies [3,4] overestimates the oscillations. The same applies to the combined contribution from all the stages of evolution of the system,

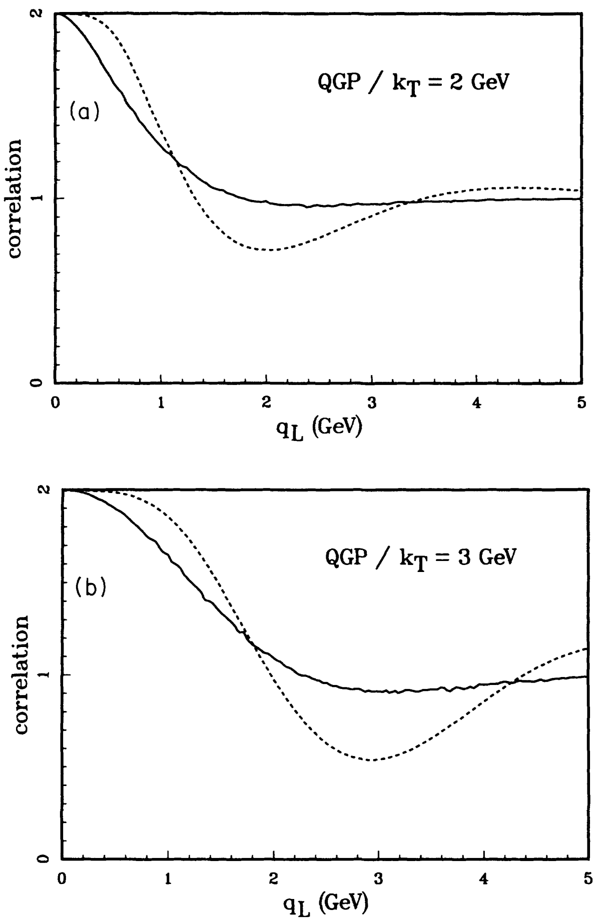

FIG. 1. Correlation function at RHIC energy with parameters as specified in the test. This figure shows the correlation due to the plasma phase only. The dashed curve is the result of the δ function approximation; the solid curve is the result of a numerical evaluation of the eight-dimensional integral. The numbers 2 and 3 indicate the transverse momentum of a single photon.

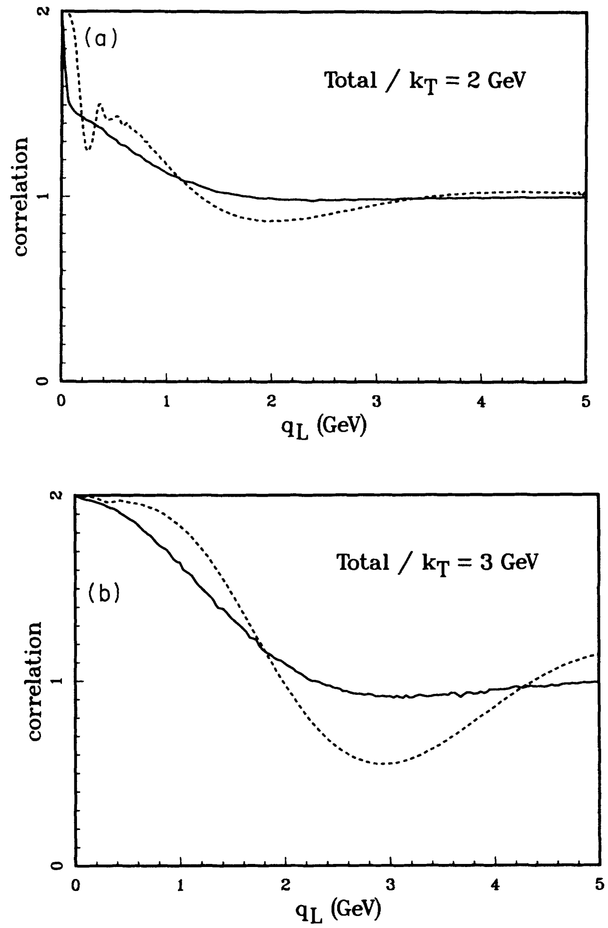

FIG. 2. Correlation function at RHIC energy as in Fig. 1 when contributions from the plasma, mixed, and hadronic phases are all added together.

which we show in Fig. 2. The reason for this enhanced oscillatory behavior is that the δ function approximation requires the space-time rapidity η of the source to be the same as the rapidity y of the photon. This means that the time of emission of the photon t is related uniquely to its longitudinal position of emission z by $z/t = \tanh(y)$ in a fixed frame of reference. Since the rapidity spread of the source function for each photon is set to zero one obtains a correlation function more characteristic of point sources. *A priori* we had expected the δ function approximation to work much better than it does, especially for $k_T = 3$ GeV since the initial value of the ratio k_T/T is 5.64 and gets bigger with time.

Thus far we have assumed that the polarization of each photon is measured. If a polarization average is taken instead, then the factor $1 + \cos(\Delta k \cdot \Delta x)$ in Eq. (3) gets replaced by $1 + \frac{1}{2}\cos(\Delta k \cdot \Delta x)$. This means that $C \to \frac{3}{2}$ at zero relative momentum rather than 2. Otherwise the behavior of the correlation function is unchanged.

The lesson learned is that the δ function approximation is not very good when applied to the photon correlation function used for intensity interferometry of high energy nuclear collisions. The reason is that it includes contributions only from source elements which have the same rapidity as the photon. This causes an enhancement of the oscillatory tendency of the interference term, closer to what one gets with two point sources. This does not diminish the importance of photon interferometry as a probe of quark-gluon dynamics during the early high density stage of the collision. It only means that one generally cannot avoid doing the multidimensional integrals numerically.

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[1] R. Hanbury Brown, *The Intensity Interferometer* (Taylor & Francis, London, 1974).
[2] D. H. Boal, C.-K. Gelbke, and B. K. Jennings, Rev. Mod. Phys. **62**, 553 (1990); W. Bauer, C.-K. Gelbke, and S. Pratt, Ann. Rev. Nucl. Part. Sci. **42**, 77 (1992).
[3] D. K. Srivastava and J. I. Kapusta, Phys. Lett. B **307**, 1 (1993).
[4] D. K. Srivastava and J. I. Kapusta, Phys. Rev. C **48**, 1335 (1993).
[5] D. K. Srivastava, Phys. Rev. D **49**, 4523 (1994).
[6] D. K. Srivastava and C. Gale, Phys. Lett. B **319**, 407 (1994).
[7] J. D. Bjorken, Phys. Rev. D **27**, 140 (1983).
[8] L. McLerran, Invited talk given at *International Conference on Physics and Astrophysics of Quark-Gluon Plasma*, edited by B. Sinha and S. Raha (World Scientific, Singapore, 1988), p. 163.
[9] J. Kapusta, P. Lichard, and D. Seibert, Phys. Rev. D **44**, 2774 (1991); H. Nadeau, J. Kapusta, and P. Lichard, Phys. Rev. C **45**, 3034 (1992).
[10] J. Kapusta, L. McLerran, and D. K. Srivastava, Phys. Lett. B **283**, 145 (1992).
[11] L. D. McLerran and T. Toimela, Phys. Rev. D **31**, 545 (1985).