Equation of state of homogeneous nuclear matter and the symmetry coefficient

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For different Skyrme-type forces we investigate the equation of state of homogeneous nuclear matter under the conditions appropriate to a collapsing star. We find that the stiffness of the equation of state increases significantly as the symmetry coefficent J of nuclear matter increases over the range of its experimental uncertainty. We present analytic expressions for the adiabatic index Γ permitting the elimination of all numerical derivatives.

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I. INTRODUCTION

We are involved in a program to develop a microscopic theory of nuclear systems applicable to the wide variety of situations encountered at subnuclear and nuclear densities during stellar collapse and type-II supernova explosions. The main achievement so far has been the development for the first time of a mass formula based entirely on microscopic forces, the ETFSI-1 mass formula [1–5]. The astrophysical interest of such a mass formula lies in the fact that the r process of nucleosynthesis depends crucially on the binding energies of nuclei that are so neutron-rich that there is no hope of being able to measure them in the laboratory. It is thus of the greatest importance to be able to make reliable extrapolations of masses away from the known region, relatively close to the stability line, out towards the neutron-drip line.

The ETFSI method is essentially a high-speed approximation to the Hartree-Fock (HF) method, with the macroscopic part treated by the extended Thomas-Fermi (ETF) method [6,7], and shell corrections calculated by the so-called Strutinsky-integral (SI) method [1]. Pairing is handled in the BCS approximation with a δ-function force. Although this is strictly a microscopic-macroscopic mass formula, there is a much greater coherence between the two parts than is the case with mass formulas based on the drop(-let) model, since the same Skyrme force underlies both parts. In fact, it has been shown [1,2] that the ETFSI method is equivalent to the HF method in the sense that when the two methods fit the same *form* of Skyrme force to the mass data they give essentially the same extrapolation. This presumably accounts for the fact that with just nine parameters the ETFSI mass formula fits the 1492 mass data for $A \geq 36$ with a rms error of only 0.736 MeV [4].

In obtaining this fit particular attention was paid to the nuclear-matter symmetry coefficient J, since the mass formula is intended to be applied especially in situations of large neutron excess [4]. The optimal fit to absolute masses that we have obtained has $J = 27.0$ MeV (parametrization SkSC4), and the mass table [4,5] is based on this value, but, as will be discussed in Sec. II, the data are not very decisive on this point, and it is hard to exclude any value between 26 and 32 MeV.

The main objective of this paper is to study the extent to which this ambiguity in J is relevant to the equation of state during the collapse stage that precedes a supernova explosion. As has been described many times (see, for example, the review of Bethe [8]), when the iron core of a massive star ($\mathcal{M} \geq 8\mathcal{M}_\odot$) starts to collapse, nuclei that are stable under normal terrestrial conditions will begin to capture electrons, forming thereby nuclei that are highly neutron-rich and yet nevertheless stable under the conditions of rising density. Eventually, as the nuclei come closer and closer together, bridges will form between them, and the entire core of the star will resemble a Swiss cheese, the holes being filled with a neutron vapor. Finally, when the density is close to that of ordinary nuclei, $\rho \simeq 0.16$ fm^{-3} $= 2.6 \times 10^{14}$ g cm^{-3}, this so-called bubble phase goes over into the homogeneous phase of nuclear matter. (In this stellar situation the protons in the nuclear matter are rigorously neutralized by the electrons that are present: this neutralized nuclear matter, which can exist in reality, is to be distinguished from the hypothetical charge-free nuclear matter in which the Coulomb forces are imagined to be switched off.)

The electron concentration per nucleon Y_e in this final stage of the collapse, immediately prior to bounce, is much higher than it would be but for the onset of neutrino trapping, which begins when the density has risen to about 6×10^{11} g cm^{-3}, at which point Y_e has fallen from its original value of 0.464 (corresponding to ^{56}Fe) to about 0.4. Thereafter the total lepton concentration Y_ℓ remains constant, and in fact electron capture soon comes to a halt. In this paper we are interested only in what happens after this point has been reached, and will suppose a constant electron fraction of $Y_e = 0.33$ and a constant neutrino fraction per nucleon of $Y_\nu = 0.07$. Furthermore, once neutrino trapping has set in the collapse will be essentially adiabatic, and thus isentropic, insofar as thermodynamic equilibrium is maintained. For the constant value of the entropy per nucleon during this stage of the collapse we shall take the fairly typical value of $s = 1.0$ (with temperature measured in energy units, entropy is dimensionless).

Of crucial importance for the collapse is the adiabatic index, defined by

$$\Gamma = \left(\frac{\partial \ln P}{\partial \ln \rho}\right)_{S,Y_e}, \tag{1.1}$$

with the derivative evaluated under the conditions of constant entropy and electron fraction that we assume to prevail. The stability of a star depends on the value of Γ in the core being larger than $\frac{4}{3}$, collapse beginning when Γ falls below $\frac{4}{3}$. However, as nuclear densities are approached in the core Γ will rise above $\frac{4}{3}$ again, with the result that the collapse will come rapidly to a halt, and be reversed into a bounce that may lead to a supernova explosion.

For two reasons we shall confine ourselves in the present paper to the final stage in which the core consists of homogeneous nuclear matter. (i) It is in this stage of the collapse that the highest densities are encountered, and therefore where the greatest sensitivity to the symmetry coefficient (and to other properties of the nuclear force) may be expected. (Note, however, that we do not consider the supernuclear densities that may be briefly encountered immediately before bounce, since our forces are quite inappropriate for this regime.) (ii) The calculations are simplest in the homogeneous phase, and in fact can be performed analytically almost to the end (note that the ETF and HF formalisms are identical in this situation). Because of the double differentiation that is involved in the computation of the adiabatic index Γ this should lead to a considerable enhancement of the accuracy.

In Sec. II we present the forces that are used in this study, while Sec. III is devoted to the derivation of the analytical expression for the pressure and adiabatic index in the homogeneous phase. The results and conclusions will be found in Sec. IV.

II. THE FORCES

We limit ourselves to Skyrme-type forces, the most general form considered here being

$$v_{ij} = t_0(1+x_0P_\sigma)\delta(\mathbf{r}_{ij}) + t_1(1+x_1P_\sigma)\frac{1}{2\hbar^2}\{p_{ij}^2\delta(\mathbf{r}_{ij}) + \text{H.c.}\} + t_2(1+x_2P_\sigma)\frac{1}{\hbar^2}\mathbf{p}_{ij}\cdot\delta(\mathbf{r}_{ij})\mathbf{p}_{ij}$$
$$+\frac{1}{6}t_3(1+x_3P_\sigma)\{a(\rho_{q_i}+\rho_{q_j})^\alpha + b\rho^\alpha\}\delta(\mathbf{r}_{ij}) + \frac{i}{\hbar^2}W_0(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\cdot\mathbf{p}_{ij}\times\delta(\mathbf{r}_{ij})\mathbf{p}_{ij}\,. \tag{2.1}$$

For the parameters a and b appearing in the density-dependent term the usual choice is $a = 0$, $b = 1$, but for the SkSC forces used in the ETFSI project (see Table I) we have taken rather $a = 1$, $b = 0$. This latter choice is more physically reasonable, since it implies that the density dependence of the effective interaction between two protons, for example, depends only on the proton density (see Ref. [1] for a fuller discussion). The subscript q in these terms denotes n or p, according to whether the term in question relates to neutrons or protons.

TABLE I. Parameters of the ETFSI Skyrme forces.

	SkSC4	SkSC5	SkSC6	SkSC10
t_0 (MeV fm^3)	-1789.42	-1788.17	-1792.47	-1795.12
t_1 (MeV fm^5)	283.467	281.931	291.964	298.950
t_2 (MeV fm^5)	-283.467	-281.931	-291.964	-298.950
t_3 (MeV fm^4)	12782.3	12771.9	12805.7	12827.7
x_0	0.790000	0.980000	0.370038	0.159124
x_1	-0.5	-0.5	-0.5	-0.5
x_2	-0.5	-0.5	-0.5	-0.5
x_3	1.13871	1.38526	0.581085	0.292918
W_0 (MeV fm^5)	124.877	126.219	126.014	127.137
α	0.333333	0.333333	0.333333	0.333333
a	1	1	1	1
b	0	0	0	0
ϵ(mass) (MeV)	0.736	0.762	0.794	0.893
$\epsilon(S_n)$	0.524	0.535	0.518	0.525
$\epsilon(Q_\beta)$	0.683	0.699	0.686	0.702

The last term in Eq. (2.1), the two-body spin-orbit force, gives no contribution to homogeneous nuclear matter, and thus has no role in this paper. For the same reason we do not present the pairing forces with which each of the Skyrme forces considered here has to be supplemented for finite-nucleus calculations.

The forces of principal interest in this work are labeled SkSC4, 5, 6, and 10, all of which have been obtained within the ETFSI framework by fitting to the 1492 mass data for $A \geq 36$. The first of these forces has already been published [4], but not the other three, so for completeness we list the parameters for all four forces in Table I (the pairing force, not shown here, is the same for all four parameter sets, and thus is as in [4]). The parameters t_1, t_2, x_1, and x_2 are related in such a way that the effective nucleon mass M^* is equal to the real nucleon mass M for all four forces, a choice which not only simplifies the ETF formalism for finite nuclei but is also close to optimal for fitting to nuclear masses. The last three lines of Table I give the rms error in the fit to the data on (i) the absolute masses, (ii) the neutron-separation energies S_n, and (iii) the beta-decay energies Q_β, respectively.

The only essential difference between these four forces is that in fitting them to the mass data they are constrained to different fixed values of the symmetry coefficient J, shown in Table II. We see from Table I that the absolute-mass fit is optimal for $J = 27$ MeV (set SkSC4), but that acceptable fits to the absolute masses are found for J as small as 26 MeV (SkSC5), and as large as 30 MeV (SkSC6); outside this range of J the fits to the ab-

TABLE II. Nuclear-matter parameters of forces used in this paper.

	J (MeV)	M_s^*/M	K_v (MeV)	$K_{\rm sym}$ (MeV)
SkSC4	27.0	1.0	234.7	-334.9
SkSC5	26.0	1.0	234.4	-392.9
SkSC6	30.0	1.0	235.4	-203.5
SkSC10	32.0	1.0	235.8	-136.6
SkM*	30.0	0.79	216.7	-155.9
RATP	29.3	0.67	239.6	-191.2

solute masses deteriorate. However, we note that even for $J = 32$ MeV (force SkSC10) the fits to the astrophysically relevant quantities S_n and Q_β are as good as for the other forces, so this force is not to be rejected *a priori*, and we shall in fact find it to be of some interest.

These four forces are thus well suited to our study of the sensitivity of the equation of state to the symmetry coefficient J. However, it is of interest to compare these forces not only with each other, but also with some of the other Skyrme forces that have been widely used for some years. We will consider just two: the very popular SkM* [9,10] (note that the former paper refers to this force as "SkM modified"), and the astrophysically motivated RATP [11] (both of these forces have $a = 0$, $b = 1$).

In Table II we list for all six of these forces some of the relevant nuclear-matter parameters. M_s^* is the effective nucleon mass in the charge-symmetric case, $\eta = 0$, where

$$\eta = 1 - 2Y_e. \tag{2.2}$$

We have also written

$$K(\eta) = K_v + K_{\rm sym}\eta^2, \tag{2.3}$$

TABLE III. Error in calculated masses of doubly closed-shell nuclei ($M_{\rm calc} - M_{\rm expt}$).

	SkSC4	SkSC5	SkSC6	SkSC10	SkM*	RATP
^{16}O	-	-	-	-	0.1	0.4
^{40}Ca	-1.8	-2.0	-2.5	-3.1	-1.0	2.5
^{48}Ca	-1.0	-1.2	-1.0	-1.2	-4.1	-1.7
^{132}Sn	0.4	0.6	0.7	0.9	-8.0	-4.7
^{208}Pb	1.1	1.5	0.8	0.6	-0.1	-0.2

where $K(\eta)$ is the incompressibility of nuclear matter with fractional neutron excess η, calculated at the equilibrium density of symmetric nuclear matter. Thus K_v refers to the symmetric case, and is what is usually referred to simply as the incompressibility coefficient; $K_{\rm sym}$ then gives a measure of how the incompressibility varies as a function of neutron excess.

In Table III we show for all six forces the errors in the calculated masses of the known doubly closed-shell nuclei (the SkSC forces are calculated with the ETFSI method and the other two with the HF method). The most notable feature of these results is the fact that force SkM* performs significantly worse than the other forces for highly neutron-rich nuclei.

While our main concern is with the neutralized nuclear matter encountered during stellar collapse, it is of some interest to set the stage by considering a somewhat different astrophysical situation: a neutron star. Since in this case there has been sufficient time for neutrinos to escape and for beta-decay equilibrium to be established, much larger neutron excesses will prevail, and we may expect a significant dependence on the symmetry coefficient J of the force. Figure 1 shows for each of our forces except SkSC5 ($J = 26$ MeV) the energy per nucleon e in

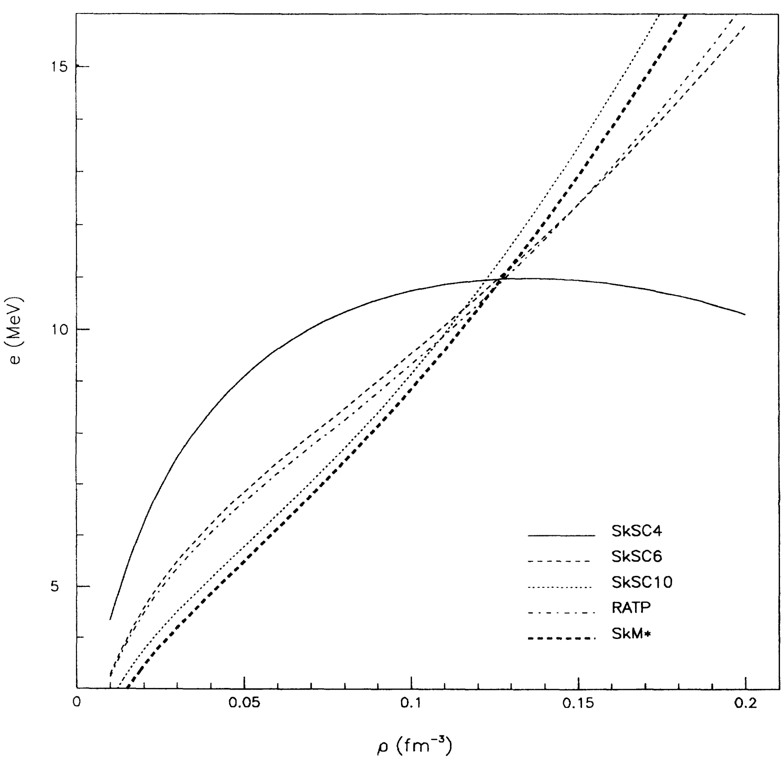

FIG. 1. Energy per nucleon e as a function of density ρ in neutron-star matter.

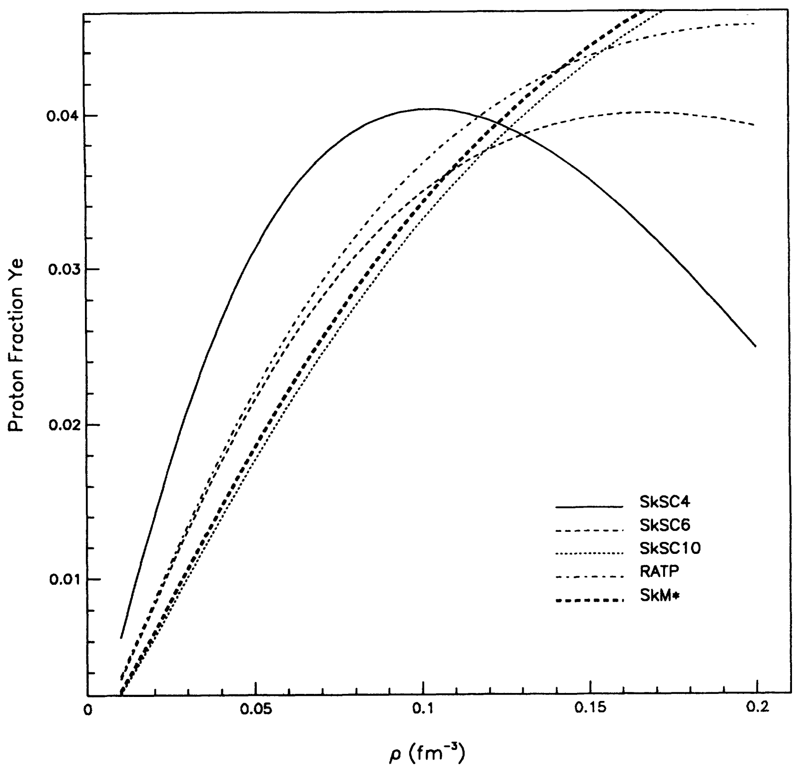

FIG. 2. Proton-electron concentration Y_e as a function of density ρ in neutron-star matter.

neutron-star matter at zero temperature as a function of the density ρ, while in Fig. 2 we give the corresponding values of the proton-electron fraction Y_e. A strong dependence on J is apparent in Fig. 1, and in fact these curves bear a close resemblance to those found for pure neutron matter in Ref. [3], where it was pointed out that a comparison with neutron-matter calculations based on realistic nucleon-nucleon forces [12] favors a value of J significantly higher than the 27 MeV that optimizes the mass fit. Moreover, we see from Fig. 1 that force SkSC4 ($J = 27$ MeV) implies a nonphysical collapse of neutron stars (we thank J. M. Lattimer for this remark), while forces SkSC6 and 10 are quite acceptable from this point of view. In this paper we leave the value of J open, but since we found that SkSC5 has an even stronger collapse than SkSC4 in neutron stars we discard it in the following.

III. FORMALISM

A. Equation of state

To determine the total pressure P_{tot} of homogeneous neutralized nuclear matter as a function of the temperature T and the density ρ one must first determine the total energy density

$$\mathcal{E}_{\text{tot}} = \mathcal{E}_{\text{nuc}} + \mathcal{E}_{\text{el}} + \mathcal{E}_{\text{ce}} + \mathcal{E}_{\nu}. \tag{3.1}$$

The first term here is the contribution associated with the Skyrme force (2.1),

$$\mathcal{E}_{\text{nuc}} = \sum_q \frac{\hbar^2}{2M_q^*}\tau_q + \frac{1}{2}t_0\left\{\left(1+\frac{1}{2}x_0\right)\rho^2 - \left(\frac{1}{2}+x_0\right)\sum_q \rho_q^2\right\}$$
$$+\frac{1}{6}at_3\left\{\left(1+\frac{1}{2}x_3\right)\rho^\alpha\rho_n\rho_p + \frac{1}{16}(1-x_3)\sum_q(2\rho_q)^{\alpha+2}\right\} + \frac{1}{12}bt_3\left\{\left(1+\frac{1}{2}x_3\right)\rho^2 - \left(\frac{1}{2}+x_3\right)\sum_q\rho_q^2\right\}\rho^\alpha, \tag{3.2}$$

where the effective mass M_q^* is given by

$$\frac{\hbar^2}{2M_q^*} = \frac{\hbar^2}{2M_q} + \frac{1}{4}\left\{t_1\left(1+\frac{1}{2}x_1\right) + t_2\left(1+\frac{1}{2}x_2\right)\right\}\rho$$
$$+\frac{1}{4}\left\{t_2\left(\frac{1}{2}+x_2\right) - t_1\left(\frac{1}{2}+x_1\right)\right\}\rho_q, \tag{3.3}$$

and the kinetic-energy density (divided by $\hbar^2/2M_q$) by

$$\tau_q = \frac{1}{2\pi^2}\left(\frac{2M_q^*}{\hbar^2}\right)^{\frac{5}{2}} T^{\frac{5}{2}} I_{\frac{3}{2}}(\nu_q). \tag{3.4}$$

In the latter equation we have introduced the Fermi integral

$$I_\sigma(\nu_q) = \int_0^\infty \frac{x^\sigma}{1+\exp(x-\nu_q)}dx \tag{3.5}$$

($\sigma > -1$), where ν_q is determined by the density according to

$$\rho_q = \frac{1}{2\pi^2}\left(\frac{2M_q^*}{\hbar^2}\right)^{\frac{3}{2}} T^{\frac{3}{2}} I_{\frac{1}{2}}(\nu_q). \tag{3.6}$$

With

$$\rho_q = \frac{1}{2}\rho(1+q\eta), \tag{3.7}$$

where $q = \pm 1$ for n and p, respectively, Eq. (3.2) becomes

$$\begin{aligned}\mathcal{E}_{\rm nuc} = \sum_q \frac{\hbar^2}{2M_q^*}\tau_q + \frac{1}{8}t_0\{3-(2x_0+1)\eta^2\}\rho^2 \\ +\frac{1}{48}at_3\{(2+x_3)(1-\eta^2)+(1-x_3)f_{\alpha+2}(\eta)\}\rho^{\alpha+2} \\ +\frac{1}{48}bt_3\{3-(2x_3+1)\eta^2\}\rho^{\alpha+2},\end{aligned} \tag{3.8}$$

where

$$f_x(\eta) = \frac{1}{2}\{(1+\eta)^x + (1-\eta)^x\}. \tag{3.9}$$

Also Eq. (3.3) becomes

$$\begin{aligned}\frac{\hbar^2}{2M_q^*} = \frac{\hbar^2}{2M_q} + \frac{1}{16}[3t_1 + t_2(5+4x_2) \\ +q\eta\{t_2(1+2x_2) - t_1(1+2x_1)\}]\rho.\end{aligned} \tag{3.10}$$

The second term in Eq. (3.1) is the electron kinetic-energy density, given by

$$\mathcal{E}_{e\ell} \simeq \frac{1}{4\pi^2}\frac{\mu_{0e}^4}{(\hbar c)^3}\left(1+\frac{2}{3}\frac{\pi^2T^2}{\mu_{0e}^2}\right), \tag{3.11}$$

where

$$\mu_{0e} = \hbar c(3\pi^2 Y_e\rho)^{\frac{1}{3}} \tag{3.12}$$

is the electron Fermi energy, and we have neglected terms $O(m_e^2c^4/\mu_{0_e}^2)$ and $O(\pi^4T^4/\mu_{0_e}^4)$. The corrections associated with the nonzero rest mass of the electron amount to less than 0.001% over the range of densities that interest us, while even for temperatures as high as 20 MeV the higher-order temperature corrections do not exceed 0.5% (see, for example, Appendix C of Ref. [13]). With nuclear matter always being electrically neutral in stellar conditions there will, of course, be no direct Coulomb term, but there will be an exchange term given by

$$\begin{aligned}\mathcal{E}_{\rm ce} &= -\frac{3}{4}\left(\frac{3}{\pi}\right)^{\frac{1}{3}} e^2(\rho_p^{\frac{4}{3}} + \rho_e^{\frac{4}{3}}) \\ &= -\frac{3}{2}\left(\frac{3}{\pi}\right)^{\frac{1}{3}} e^2(Y_e\rho)^{\frac{4}{3}}\end{aligned} \tag{3.13}$$

(actually, this is negligible within the approximations already made). Finally, corresponding to Eq. (3.11) we have for the neutrino kinetic-energy density

$$\mathcal{E}_\nu \simeq \frac{1}{8\pi^2}\frac{\mu_{0\nu}^4}{(\hbar c)^3}\left(1+\frac{2}{3}\frac{\pi^2T^2}{\mu_{0\nu}^2}\right) \tag{3.14}$$

in which

$$\mu_{0\nu} = \hbar c(6\pi^2 Y_\nu\rho)^{\frac{1}{3}}. \tag{3.15}$$

(Note that the spin degeneracy of the electron is 2 and that of the neutrino is 1.)

Turning now to the entropy density, since an independent-particle picture is being adopted for the particles we can write

$$\mathcal{S} = \sum_q \mathcal{S}_q + \mathcal{S}_e + \mathcal{S}_\nu. \tag{3.16}$$

Here we have for the entropy density of each type of nucleon

$$\mathcal{S}_q = \frac{5}{3T}\frac{\hbar^2}{2M_q^*}\tau_q - \nu_q\rho_q. \tag{3.17}$$

This, taken with Eq. (3.6), means that the only way in which the nuclear entropy depends on the force is through the effective mass. In other words, to decouple the temperature from the density in adiabatic processes it is necessary to take a force with a different effective mass. For the entropy density of the leptons we have

$$\mathcal{S}_e = \frac{1}{3}\frac{\mu_{0e}^2}{(\hbar c)^3}T \tag{3.18}$$

and

$$\mathcal{S}_\nu = \frac{1}{6}\frac{\mu_{0\nu}^2}{(\hbar c)^3}T. \tag{3.19}$$

With the density of the Helmholtz free energy given by

$$\begin{aligned}\mathcal{F}_{\rm tot} &\equiv \mathcal{E}_{\rm tot} - T\mathcal{S}_{\rm tot} \\ &= \mathcal{F}_{\rm nuc} + \mathcal{F}_e + \mathcal{F}_\nu + \mathcal{E}_{\rm ce},\end{aligned} \tag{3.20}$$

the pressure will be given by

$$\begin{aligned}P_{\rm tot} &= \rho^2\left(\frac{\partial(\mathcal{F}_{\rm tot}/\rho)}{\partial\rho}\right)_{T,\eta} \\ &= P_{\rm nuc} + P_e + P_\nu + P_{\rm ce},\end{aligned} \tag{3.21}$$

in an obvious notation (because of the neutron-proton interaction it is not possible to define independent partial pressures for neutrons and protons separately).

Using Eqs. (3.2) and (3.17) we have for the nuclear pressure

$$P_{\rm nuc} = -\frac{2}{3}\rho^2\left(\frac{\partial}{\partial\rho}\right)_{T,\eta}\left(\frac{1}{\rho}\sum_q \frac{\hbar^2}{2M_q^*}\tau_q\right) + T\rho^2\left(\frac{\partial}{\partial\rho}\right)_{T,\eta}\left(\frac{1}{\rho}\sum_q \nu_q\rho_q\right) + \frac{t_0}{8}\left\{3-(2x_0+1)\eta^2\right\}\rho^2$$
$$+\frac{1}{48}at_3(\alpha+1)\left\{(2+x_3)(1-\eta^2)+(1-x_3)f_{\alpha+2}(\eta)\right\}\rho^{\alpha+2} + \frac{1}{48}bt_3(\alpha+1)\left\{3-(2x_3+1)\eta^2\right\}\rho^{\alpha+2}. \quad (3.22)$$

The first two terms here become, using Eq. (3.4),

$$\frac{2}{3}\sum_q \frac{\hbar^2}{2M_q^*}\tau_q - \frac{2}{3}\rho\sum_q\left(\frac{\partial}{\partial\rho}\right)_{T,\eta}\left(\frac{\hbar^2}{2M_q^*}\tau_q\right) - T\sum_q \nu_q\rho_q + T\rho\left(\frac{\partial}{\partial\rho}\right)_{T,\eta}\sum_q \nu_q\rho_q$$

$$= \frac{2}{3}\sum_q \frac{\hbar^2}{2M_q^*}\tau_q - \frac{1}{3\pi^2}\rho T^{\frac{5}{2}}\sum_q\left\{I_{\frac{3}{2}}(\nu_q)\left(\frac{\partial}{\partial\rho}\right)_{T,\eta}\left(\frac{2M_q^*}{\hbar^2}\right)^{\frac{3}{2}} + \left(\frac{2M_q^*}{\hbar^2}\right)^{\frac{3}{2}}\left(\frac{\partial I_{\frac{3}{2}}(\nu_q)}{\partial\rho}\right)_{T,\eta}\right\} + T\rho\sum_q \rho_q\left(\frac{\partial\nu_q}{\partial\rho}\right)_{T,\eta}$$
$$(3.23)$$

(note that M_q^*, and thus ν_q, depend on both ρ_n and ρ_p). Now using a well-known property of the Fermi integrals (see, for example, Eq. (6.26) of [10]) we have

$$\left(\frac{\partial I_{\frac{3}{2}}(\nu_q)}{\partial\rho}\right)_{T,\eta} = \frac{3}{2}I_{\frac{1}{2}}(\nu_q)\left(\frac{\partial\nu_q}{\partial\rho}\right)_{T,\eta}, \quad (3.24)$$

whence, in view of Eq. (3.6), a strong cancellation occurs in Eq. (3.23), and we are left with

$$P_{\rm nuc} = \frac{2}{3}\sum_q \frac{\hbar^2}{2M_q^*}\tau_q + \rho\sum_q \tau_q X_q + \frac{t_0}{8}\left\{3-(2x_0+1)\eta^2\right\}\rho^2$$
$$+\frac{1}{48}at_3(\alpha+1)\left\{(2+x_3)(1-\eta^2)+(1-x_3)f_{\alpha+2}(\eta)\right\}\rho^{\alpha+2} + \frac{1}{48}bt_3(\alpha+1)\left\{3-(2x_3+1)\eta^2\right\}\rho^{\alpha+2}, \quad (3.25)$$

in which, with Eq. (3.10), we have

$$X_q \equiv \left(\frac{\partial}{\partial\rho}\right)_{T,\eta}\left(\frac{\hbar^2}{2M_q^*}\right) = \frac{1}{\rho}\left(\frac{\hbar^2}{2M_q^*} - \frac{\hbar^2}{2M_q}\right). \quad (3.26)$$

Then

$$P_{\rm nuc} = \sum_q\left(\frac{5}{3}\frac{\hbar^2}{2M_q^*} - \frac{\hbar^2}{2M_q}\right)\tau_q + \frac{t_0}{8}\left\{3-(2x_0+1)\eta^2\right\}\rho^2$$
$$+\frac{1}{48}at_3(\alpha+1)\left\{(2+x_3)(1-\eta^2)+(1-x_3)f_{\alpha+2}(\eta)\right\}\rho^{\alpha+2} + \frac{1}{48}bt_3(\alpha+1)\left\{3-(2x_3+1)\eta^2\right\}\rho^{\alpha+2}. \quad (3.27)$$

The two lepton pressures are

$$P_e = \frac{1}{12\pi^2}\frac{\mu_{0e}^4}{(\hbar c)^3}\left(1+\frac{2}{3}\frac{\pi^2T^2}{\mu_{0e}^2}\right) \quad (3.28)$$

and

$$P_\nu = \frac{1}{24\pi^2}\frac{\mu_{0\nu}^4}{(\hbar c)^3}\left(1+\frac{2}{3}\frac{\pi^2T^2}{\mu_{0\nu}^2}\right). \quad (3.29)$$

The Coulomb-exchange pressure is

$$P_{\rm ce} = -\frac{1}{2}\left(\frac{3}{\pi}\right)^{\frac{1}{3}}e^2(Y_e\rho)^{\frac{4}{3}}. \quad (3.30)$$

B. The adiabatic index

The differentiation of the equation of state that is implicit in definition (1.1) of the adiabatic index can be performed analytically in the case of homogeneous systems. Using standard properties of partial derivatives we have

$$\Gamma \equiv \left(\frac{\partial \ln P_{\rm tot}}{\partial \ln\rho}\right)_{s_{\rm tot},\eta} = \frac{\rho}{P_{\rm tot}}\left(\frac{\partial P_{\rm tot}}{\partial\rho}\right)_{s_{\rm tot},\eta}$$
$$= \frac{\rho}{P_{\rm tot}}\left\{\left(\frac{\partial P_{\rm tot}}{\partial\rho}\right)_T - \left(\frac{\partial P_{\rm tot}}{\partial T}\right)_\rho\left(\frac{\partial s_{\rm tot}}{\partial\rho}\right)_T\Big/\left(\frac{\partial s_{\rm tot}}{\partial T}\right)_\rho\right\}_\eta, \quad (3.31)$$

where $s_{\rm tot}$ denotes the total entropy per nucleon. The four derivatives appearing here can be obtained quite straightforwardly from the preceding results.

The following two intermediate results occur in the case of the nucleonic terms. First, using Eq. (3.24) we find

$$\left(\frac{\partial \tau_q}{\partial \rho}\right)_{T,\eta} = \frac{1}{2}\left(\frac{2M_q^*}{\hbar^2}\right)\left\{3\rho_q T\left(\frac{\partial \nu_q}{\partial \rho}\right)_{T,\eta} - 5\tau_q X_q\right\}, \tag{3.32}$$

into which we substitute Eq. (3.26) and

$$\left(\frac{\partial \nu_q}{\partial \rho}\right)_{T,\eta} = \left(\frac{\partial I_{\frac{1}{2}}(\nu_q)}{\partial \rho}\right)_{T,\eta} \Big/ \frac{dI_{\frac{1}{2}}(\nu_q)}{d\nu_q} = \frac{4\pi^2}{T^{\frac{3}{2}} I_{-\frac{1}{2}}(\nu_q)}\frac{\rho_q}{\rho}\left(\frac{\hbar^2}{2M_q^*}\right)^{\frac{3}{2}} \times\left\{1+\frac{3}{2}\left(\frac{2M_q^*}{\hbar^2}\right)X_q\rho\right\}. \tag{3.33}$$

Second

$$\left(\frac{\partial \tau_q}{\partial T}\right)_{\rho,\eta} = \frac{5}{2}\frac{\tau_q}{T} + \frac{3}{2}\left(\frac{2M_q^*}{\hbar^2}\right)T\rho_q\left(\frac{\partial \nu_q}{\partial T}\right)_{\rho,\eta}, \tag{3.34}$$

in which

$$\left(\frac{\partial \nu_q}{\partial T}\right)_{\rho,\eta} = -\frac{6\pi^2}{T^{\frac{5}{2}} I_{-\frac{1}{2}}(\nu_q)}\left(\frac{\hbar^2}{2M_q^*}\right)^{\frac{3}{2}}\rho_q. \tag{3.35}$$

Then for the nuclear term in the pressure we have

$$\left(\frac{\partial P_{\rm nuc}}{\partial \rho}\right)_{T,\eta} = \sum_q\left\{\frac{5}{2}\left(\frac{M_q^*}{M_q}-1\right)\tau_q X_q + \left(\frac{5}{2}-\frac{3}{2}\frac{M_q^*}{M_q}\right)\rho_q T\left(\frac{\partial \nu_q}{\partial \rho}\right)_{T,\eta}\right\} + \frac{1}{4}t_0\left\{3-(2x_0+1)\eta^2\right\}\rho + \frac{1}{48}at_3(\alpha+1)(\alpha+2)\left\{(2+x_3)(1-\eta^2)+(1-x_3)f_{\alpha+2}(\eta)\right\}\rho^{\alpha+1} + \frac{1}{48}bt_3(\alpha+1)(\alpha+2)\left\{3-(2x_3+1)\eta^2\right\}\rho^{\alpha+1} \tag{3.36}$$

and

$$\left(\frac{\partial P_{\rm nuc}}{\partial T}\right)_{\rho,\eta} = \sum_q\left\{\frac{5}{2T}\left(\frac{5}{3}\frac{\hbar^2}{2M_q^*} - \frac{\hbar^2}{2M_q}\right)\tau_q + \left(\frac{5}{2}-\frac{3}{2}\frac{M_q^*}{M_q}\right)\rho_q T\left(\frac{\partial \nu_q}{\partial T}\right)_{\rho,\eta}\right\}. \tag{3.37}$$

As for the nuclear term in the entropy per nucleon we find

$$\left(\frac{\partial s_{\rm nuc}}{\partial \rho}\right)_{T,\eta} = \frac{1}{\rho}\sum_q\left\{-\frac{5}{6}\frac{\tau_q}{\rho T}\left(5\frac{\hbar^2}{2M_q^*} - 3\frac{\hbar^2}{2M_q}\right) + \frac{3}{2}\rho_q\left(\frac{\partial \nu_q}{\partial \rho}\right)_{T,\eta}\right\} \tag{3.38}$$

and

$$\left(\frac{\partial s_{\rm nuc}}{\partial T}\right)_{\rho,\eta} = \frac{1}{2\rho}\sum_q\left\{\frac{5}{T^2}\frac{\hbar^2}{2M_q^*}\tau_q + 3\rho_q\left(\frac{\partial \nu_q}{\partial T}\right)_{\rho,\eta}\right\}. \tag{3.39}$$

For the electron terms we have

$$\left(\frac{\partial P_e}{\partial \rho}\right)_{T,\eta} = \frac{1}{9\pi^2}\frac{\mu_{0e}^4}{(\hbar c)^3\rho}\left(1+\frac{1}{3}\frac{\pi^2T^2}{\mu_{0e}^2}\right), \tag{3.40}$$

$$\left(\frac{\partial P_e}{\partial T}\right)_{\rho,\eta} = \frac{\mu_{0e}^2}{9(\hbar c)^3}T, \tag{3.41}$$

$$\left(\frac{\partial s_e}{\partial \rho}\right)_{T,\eta} = -\frac{\mu_{0e}^2}{9(\hbar c)^3}\frac{T}{\rho^2}, \tag{3.42}$$

$$\left(\frac{\partial s_e}{\partial T}\right)_{\rho,\eta} = \frac{\mu_{0e}^2}{3(\hbar c)^3}\frac{1}{\rho}, \tag{3.43}$$

while for the neutrino terms

$$\left(\frac{\partial P_\nu}{\partial \rho}\right)_{T,\eta} = \frac{1}{18\pi^2}\frac{\mu_{0\nu}^4}{(\hbar c)^3\rho}\left(1+\frac{1}{3}\frac{\pi^2T^2}{\mu_{0\nu}^2}\right), \tag{3.44}$$

$$\left(\frac{\partial P_\nu}{\partial T}\right)_{\rho,\eta} = \frac{\mu_{0\nu}^2}{18(\hbar c)^3}T, \tag{3.45}$$

$$\left(\frac{\partial s_\nu}{\partial \rho}\right)_{T,\eta} = -\frac{\mu_{0\nu}^2}{18(\hbar c)^3}\frac{T}{\rho^2}, \tag{3.46}$$

$$\left(\frac{\partial s_\nu}{\partial T}\right)_{\rho,\eta} = \frac{\mu_{0\nu}^2}{6(\hbar c)^3}\frac{1}{\rho}. \tag{3.47}$$

Finally, for the Coulomb-exchange pressure

$$\left(\frac{\partial P_{\rm ce}}{\partial \rho}\right)_{T,\eta} = \frac{4}{3}\frac{P_{\rm ce}}{\rho}, \tag{3.48}$$

$$\left(\frac{\partial P_{\rm ce}}{\partial T}\right)_{\rho,\eta} = 0. \tag{3.49}$$

IV. RESULTS AND DISCUSSION

Using the formalism of Sec. III we compute for all of our forces the adiabatic index Γ of homogeneous nuclear matter with an entropy $s = 1.0$ per nucleon, $Y_e = 0.33$, and $Y_\nu = 0.07$. Figure 3 shows our results over the range of densities for which nuclear matter can be expected to be homogeneous and the Skyrme-type forces appropriate: towards the lower end of the range there will be a transition to the bubble phase, while at the upper end of the range the densities are supernuclear.

Confining our attention at first to the forces SkSC4, 6, and 10 (J = 27.0, 30.0, and 32.0 MeV, respectively), we see that there is a significant correlation between Γ and J, in that the greater the value of J the greater the value

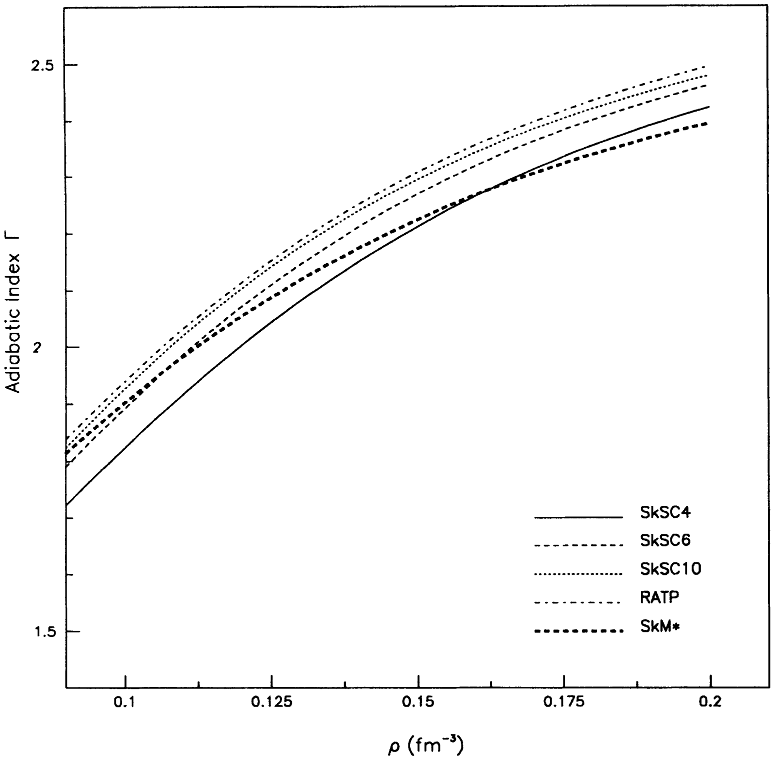

FIG. 3. Adiabatic index Γ of homogeneous neutralized nuclear matter (s = 1.0, Y_e = 0.33, Y_ν = 0.07), expressed as a function of density ρ.

of Γ at a given density. This can easily be understood in terms of the correlation between J and $K_{\rm sym}$ that is apparent in Table II: since K_v is the same for all three of these forces the incompressibility $K(\eta)$, defined by Eq. (2.3), will be greater the greater the value of J.

Turning now to the force RATP, one finds in this case that Γ lies much higher than for the force SkSC6, even though the symmetry coefficients and incompressibilities of the two forces are almost the same. What we are seeing here is the role of the effective mass M^*, already discussed in Ref. [14]. The point is that the lower the value of M^* the higher will be the temperature for a given entropy and density, as can be seen very simply from the quasidegenerate limit of Eq. (3.17),

$$\mathcal{S}_q = \frac{M_q^*}{3\hbar^2}(3\pi^2\rho_q)^{1/3}T. \quad (4.1)$$

We see, in fact, that the RATP adiabat runs consistently hotter than those of any of the SkSC forces, all of which have $M^* = M$, and as a result its equation of state is stiffer.

Finally, it will be seen that the force SkM* does not conform to the systematics that we have established for the other forces. However, we note that while all the other forces have about the same value of K_v, that of SkM* is different. Moreover, the fit of force SkM* to highly neutron-rich finite nuclei is conspicuously worse than that given by the other forces (Table III). Thus we do not regard the anomalous behavior of force SkM* as significant.

To summarize, we have shown that even if the sensitivity to J is not as strong as in the case of neutron-star matter (Figs. 1 and 2), the equation of state of the nuclear matter of a collapsing star does change appreciably as J is varied over the range of its experimental uncertainty, as determined by the fit to the nuclear binding energies. In the present context this question would seem to be as important as the correct choice of the effective mass. From the forces considered here it appears that an increase in the symmetry coefficient leads to a stiffening of the equation of state. To eliminate this significant ambiguity in the equation of state it will be necessary to find some way to fix J with greater precision than is possible from a fit to the nuclear masses alone. Nevertheless, fitting the force to nuclear masses remains a *necessary* condition for a reliable equation of state describing the collapse phase.

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