Analytic extension of the nuclear algebraic potential

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An analytic extension of the nuclear algebraic potential in the complex energy and angular momentum planes is discussed and an approximation for the algebraic potential in agreement with the known analytic properties of the S matrix is proposed. The invariance of the energy spectrum of the Coulomb part of the interaction is established. The results are applied to the Regge pole analysis of the $^{12}C + {}^{24}Mg$ elastic collision at $E_{\rm lab} = 23.0$ MeV.

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The algebraic scattering theory (AST) of Alhassid and Iachello [1] built upon the dynamical symmetry SO(3,1) has been applied, with success, in analyses of heavy ion scattering [2–6]. In this theory the S matrix has the form of a ratio of two gamma functions with arguments as in the case of pure Coulomb scattering but modified by adding nuclear algebraic potentials $v_\ell(E)$, which are complex and both energy and ℓ dependent. Of particular interest to us is the application of the AST to analyze the scattering data from collisions of heavy ions at energies near to the Coulomb barrier. Such studies have been made previously by Lepine-Szily *et al.* [2,3]. In a previous paper [6], using a numerical procedure that calculates exactly the algebraic potential for a given S matrix, we were able to propose a three-parameter shape for $v_\ell(E)$ that qualitatively reproduces the S matrix of a regular Woods-Saxon (WS) potential in the Schrödinger equation. Such a shape was able to fit both the diffractive behavior of the elastic channel as well as, by its derivative, the coupling among channels [7]. The interest in low energy heavy ion collisions stems from the importance in the scattering processes not only of diffractive phenomena but also of effects that reflect in resonances and Regge poles attributes in the scattering matrix. In the complex energy plane, resonances of the (now) S function are described by pairs of poles and zeros with the poles placed in the lower half plane [8]. In contrast, Regge poles [9,10], intimately connected to resonances, correspond to poles and zeros associated also in pairs, but with the poles located in the first quadrant of the complex angular momentum plane. In this paper we investigate the consequences of these analytic structures of the S matrix on the algebraic potential proposed in Ref. [1].

One important feature of the algebraic potential is to leave unmodified the original Coulomb energy spectrum, irrespective of the number of Regge poles or resonances added to it. This invariant property shows that the original poles of the symmetry do not disturb the poles introduced by the algebraic potential.

The WS shape commonly used for $v_\ell(E)$ is not in agreement with the known analytic properties of the S matrix. The WS shape has poles in the ℓ plane, and these simple poles in the algebraic potential produce essential singularities in the S matrix that are spurious to its analytic representation. In this paper we propose a new shape for the algebraic potential which is an integral function of ℓ in the right side of the complex plane. Such a shape eliminates the singularities of the S matrix introduced by the WS shape. It also has the correct asymptotic behavior for large values of ℓ to guarantee the

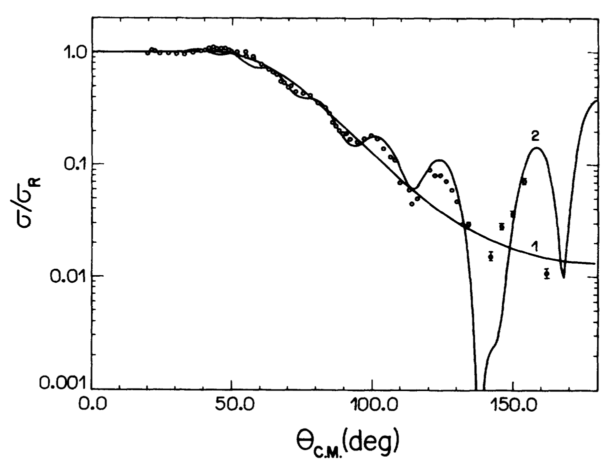

FIG. 1. The analysis of the elastic angular distribution of $^{12}C+{}^{24}Mg$ at $E_{\rm lab} = 23.0$ MeV. Curve 1 is the result of the analysis with only the background potential whose shape is given by Eq. (8). The parameters used were $\lambda_0 = 7.0$, $\alpha = 0.5$, $V = 2.0$, and $W = 2.0$. Curve 2 represents the result of adding a pole to the background potential as given by Eq. (7). The fitting of the data was obtained with the pole at $\lambda_p = 9.75$, partial width $D = 0.10$, total width $\Gamma = 0.15$, and mixing phase $\phi = 0.3$ rad.

applicability of the Regge-Watson transform [9]. We use this shape as the background potential in the Regge pole analysis of the elastic angular distribution of the ^{12}C + ^{24}Mg system at $E_{\rm lab} = 23.0$ MeV.

Let us consider the collision of two spinless ions in the AST framework. The algebraic potential $v_\ell(E)$ is related to the nuclear S matrix S_ℓ by the well-known equation [1]

$$\frac{\Gamma(\ell+1+i\eta+iv_\ell)}{\Gamma(\ell+1-i\eta-iv_\ell)}\cdot\frac{\Gamma(\ell+1-i\eta)}{\Gamma(\ell+1+i\eta)} = S_\ell. \tag{1}$$

This form for the S matrix was also obtained, from a totally different approach, by Müller and Schilcher [11] for the case of a superposition of Yukawa potentials in the Schrödinger equation. This remarkable result shows that although this form for S_ℓ does not exhibit the exact SO(3,1) symmetry, it does constitute a natural basis for the analysis of the strong interaction contribution in heavy ion collisions and so is the starting point of our analysis.

Extending the S matrix into the complex angular momentum plane by using the extension $\ell \to \lambda - 1/2$ where λ is the complex angular momentum variable, we define (twice the phase shift)

$$z = -i\ln[S(\lambda)], \tag{2}$$

and with Eq. (1) now specifying the S function, on differentiating with respect to z, we find

$$\frac{\partial v(\lambda,z)}{\partial z} = \frac{1}{\psi(\lambda+1/2+i\eta+iv(\lambda,z))+\psi(\lambda+1/2-i\eta-iv(\lambda,z))}, \tag{3}$$

where $\psi(z)$ is the digamma function defined in Ref. [12]. In the above equation λ and η are considered to be two fixed parameters and we are concerned mainly with the z dependence of v. For this reason we will omit from now on the argument λ in v. Actually, we define $v(z)$ as the solution of Eq. (3) with the initial condition $v(0) = 0$. We will now consider some global properties of the algebraic potential.

First we observe that $v \equiv 0$ for

$$\lambda + 1/2 \pm i\eta = -\nu, \tag{4}$$

where ν is a non-negative integer. This results from the fact that $\psi(\lambda + 1/2 \pm i\eta) = \infty$ and thus, for $z = 0$, we have both $v(0) = 0$ and $f(0) = 0$. Since Eq. (3) is of first order in z, the solution for $v(0) = 0$ and $dv(0)/dz = 0$ is $v \equiv 0$. The consequence of this fact is that the bound states of the Coulomb part of the interaction determined by Eq. (4) are invariant with respect to the nuclear interaction; i.e., v preserves the original singularities of the Coulomb S matrix.

Expanding $v(z)$ in the neighborhood of z_0, we obtain from Eq. (3)

$$v(z) = v^0 + \frac{z-z_0}{\psi(\lambda+1/2+i\eta+iv^0)+\psi(\lambda+1/2-i\eta-iv^0)}(1+\epsilon), \tag{5}$$

with $v^0 = v(z_0)$ and ϵ, the relative error of the linear term given by

$$\epsilon \approx \frac{(z-z_0)/(2i)}{[\psi(\lambda+1/2+i\eta+iv^0)+\psi(\lambda+1/2-i\eta-iv^0)]^3}[\psi\prime(\lambda+1/2+i\eta+iv^0)-\psi\prime(\lambda+1/2-i\eta-iv^0)].$$

The presence of trigamma function $\psi\prime(z) = d\psi(z)/dz$ in the numerator of the above equation makes ϵ small for applications to heavy ion collisions near the Coulomb barrier. We mention that Eq. (5) was the starting point for obtaining the iterative numerical procedure of Ref. [6].

Let us introduce

$$z^{(n+1)} = 2\pi + z^{(n)},$$

where n is an index to indicate on which sheet of $\ln(S)$ z is being considered. Defining $v^{(n)} \equiv v(z^{(n)})$ and $w^n \equiv v^{(n+1)} - v^{(n)}$ we have

$$w^{(n)} = \frac{2\pi}{\psi(\lambda+1/2+i\eta+iv^{(n)}(z))+\psi(\lambda+1/2-i\eta-iv^{(n)}(z))},$$

within the approximation involved in deriving Eq. (5).

From these considerations we see that the logarithmic nature of z leads to an enumerate family $\{v^{(n)}\}$ of ambiguous algebraic potentials, all corresponding to the same S. In particular, whenever v varies with energy (angular momentum) from the neighborhood of one member of the family to another with larger (smaller) index we may assume the occurrence of resonance (Regge pole) in the S matrix.

Now, the branch points of z occur whenever $S = 0$

or $S = \infty$, with the poles corresponding to resonances in the complex E plane and to Regge poles in the complex λ plane. Thus, the singularities of v are similar to the logarithmic ones, with the branch points associated with resonances or Regge poles according to whether we consider the energy or the angular momentum complex planes, respectively.

The above results are easily applied to Regge pole analysis of elastic differential cross sections. We suppose that the S matrix representation in the complex λ plane takes the form [13,14]

$$S(\lambda) = S^0(\lambda)\left(1 + \frac{iD\exp(2i\phi)}{\lambda - \lambda_p - i\Gamma/2}\right), \tag{6}$$

where $S^0(\lambda)$ is the background S matrix and the factor in brackets represents a pole and its associated zero. The phase ϕ is the so-called mixing phase.

From Eq. (5) we obtain

$$v(\lambda) = v^0(\lambda) + \frac{-i\ln(1 + \frac{iD\exp(2i\phi)}{\lambda-\lambda_p-i\Gamma/2})}{\chi}, \tag{7}$$

where

$$\chi = \psi(\lambda + 1/2 + i\eta + iv^0(\lambda)) + \psi(\lambda + 1/2 - i\eta - iv^0(\lambda))$$

is obtained from Eq. (5). Equation (7) can be extended naturally to include any number of Regge poles. The background potential $v^0(\lambda)$ deserves some consideration. The traditional WS shape [1] contains poles for $\lambda = \lambda_0 + i(2n+1)\pi\Delta$ where n is an integer. These poles, in the argument of the gamma functions in Eq. (1), produce essential singularities in the finite complex λ plane which, besides being spurious to the analytic representation of $S(\lambda)$, make problematic the determination of the true poles of the S matrix. We propose the following shape based on three factors:

$$v^0(\lambda) = (V + iW)f(\lambda),$$

with

$$f(\lambda) = \frac{\Gamma(\frac{\lambda_0}{\Delta}+1, \frac{\lambda}{\Delta})}{\Gamma(\frac{\lambda_0}{\Delta}+1)}\left(1 + \frac{\Delta}{\lambda_0^2}\lambda\right)^{\frac{-\lambda_0}{\Delta}}[\psi(\lambda + 1/2 + i\eta) + \psi(\lambda + 1/2 - i\eta)]^{-1}, \tag{8}$$

where $\gamma(n, z)$ is the incomplete gamma function [12]. The first factor in $f(\lambda)$ corresponds to the traditional WS shape but, being written in terms of the incomplete gamma function, it does not introduce spurious singularities in the S matrix. The second factor corrects the asymptotic behavior of the first and makes the form factor $f(\lambda)$ similar to the one proposed in Ref. [6]. The third factor guarantees the vanishing of $v^0(\lambda)$ for the condition given by Eq. (4). The parameters λ_0 and Δ have the physical meaning of grazing angular momentum and diffuseness, respectively.

Figure 1 exhibits an analysis of the elastic angular distribution for the $^{12}C+^{24}Mg$ system at $E_{lab} = 23.0$ MeV [15]. Curve 1 is the result of the analysis using only the background potential. We observe that it describes well the overall pattern of the elastic cross section but has no oscillating behavior at backward angles. Curve 2 represents the result of adding a pole to the previous background potential as given by Eq. (7). The absence of oscillations in the backward angles from the background potential makes the determination of the pole parameters quite reliable. We would like to emphasize that the analysis proposed here differs in a substantial aspect from previous ones in that the number and the positions of the poles present in the AST S matrix are easily controlled. This is not the case in the Schrödinger approach with space-dependent potentials where the presence of poles in the S matrix is difficult to control [16]. Our approach can be applied to other data and the variation of the pole position with energy will furnish the necessary information for the determination of possible shape resonances of the system.

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