Spontaneous breakdown of isospin symmetry in nuclei and isobaric analog states

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We present a self-consistent analysis of SU(2)-isospin symmetry breakdown in nuclei. We derive the energy difference of nearby nuclei (Z and $Z-1$) with the same A and splitting in the dispersion relations of protons and neutrons and show that the isobaric analog state is the massive Goldstone boson associated with the isospin polarization of the vacuum. The key relation in this analysis is the partially conserving isobaric current relation (PCIC), this is analogous to the PCAC relation with pion being the massive Goldstone boson.

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Low-lying bosonic excitations in nuclei have been the subject of intensive recent investigations in the nuclear many-body problem [1–3], mainly because of the possibility that their appearance may signify the presence of certain boson modes associated with the breakdown of certain symmetries of the effective nuclear Hamiltonian.

It is usually believed that the isospin symmetry of the effective nucleon-nucleon interaction in nuclei is only explicitly broken by electromagnetic and weak interactions [4]. In this context the isobaric analog states [5] are believed to be evidence of exact isospin symmetry, which appears to hold even in heavy nuclei, where the Coulomb repulsion is comparable to the effective nuclear interaction.

However, the real situation seems to require a deeper consideration. It frequently happens that ground states of nearby nuclei, one with (A, Z) and another with $(A, Z-1)$ with large A, appear to be nearly degenerate. Since the Coulomb interaction is expected to break such a degeneracy, the observed degeneracy suggests that there is another mechanism which restores the degeneracy. A plausible agent for this restoration is the breakdown of the isospin symmetry in the ground state (vacuum); the vacuum has nonvanishing isospin polarization due to the electromagnetic effect. The situation resembles the ferromagnetlike state in a paramagnetic metal under a uniform external magnetic field. In the latter case the electron spins try to point in the direction of the external magnetic field. In such a system there usually appears a magnon mode which is caused by a spin oscillation around the stationary direction. This magnon mode has a finite energy gap, because the oscillation needs an energy larger than the threshold in order to overcome the effect of the external field which tries to align the spin directions. The situation is also similar to the case of chiral symmetry with the pion being the Goldstone field. In this case the pion has a finite mass because the symmetry is explicitly broken; the axial current is partially conserved [partial conservation of axial current (PCAC)] and the pion acts as the massive Goldstone particle. In the present case the isospin symmetry is explicitly broken, giving rise to the partially conserved isospin current (PCIC). The broken isosymmetry in the vacuum creates a Goldstone mode with an energy gap. This mode will be called an isomagnon.

We thus feel that we need a self-consistent analysis of the isospin symmetry in large nuclei by taking into consideration the broken symmetry nature (isobaric polarization) of the vacuum. Then it becomes reasonable to assume that the isobaric analog state is the Goldstone mode, because the difference between the energy levels of the isobaric analog state with $(A, Z-1)$ and that of the parent nuclei with (A, Z) is about equal to the Coulomb energy shift; this energy difference should vanish if the Coulomb interaction were not present in the same way as the energy gap of the Goldstone boson vanishes when there is no symmetry-breaking interaction. Isospin current conservation is violated by the Coulomb effect; we have a partially conserved isospin current (PCIC). We therefore call this approach the PCIC approach. Employing the formalism of dynamical symmetry rearrangement [6], which has proved very fruitful in the study of spontaneously broken symmetries in solid state and particle physics, we show that, indeed, there are strong reasons to believe that this conjecture is correct.[1]

We consider a schematic contact model, for simplicity, of the effective nuclear interactions and the Coulomb interaction with dynamically rearranged symmetry of the

[1]Recently Khanna, He, and Umezawa [7] have conjectured that the isobaric analog states may be the Goldstone bosons of the spontaneous breakdown of the SU(2)-isospin symmetry in nuclei, arguing that a field theoretical approach may not only reproduce the well-known facts about them, but may also throw light into future investigations. Engelbrecht and Lemmer [8] noticed as early as 1970 that the isobaric analog states bear resemblance to Goldstone bosons.

ground state. Employing a mean field approximation we solve the gap equation for the order parameter and calculate the proton-neutron energy difference and also the ground state energy difference of nearby nuclei. The result shows how the Coulomb energy shift of the ground states and the effect of the broken symmetry vacuum partially compensate among them to produce the degeneracy in nearby nuclei. Thus we are able to calculate the coefficient of the symmetry energy term in the Bethe-Weizsäcker semiempirical mass formula. The result is in qualitative agreement with experiment. Further we calculate the energy gap of the Goldstone boson due to the Coulomb interaction in order to show that the isobaric analog states are Goldstone bosons. We estimate the coherence length of the proton-particle neutron-hole Cooper pairs and find out that it is of the order of the nuclear size. This is consistent with our expectations that the Goldstone boson is a collective mode which is involved in maintaining the order inside the nucleus.

The symmetry energy term in the Bethe-Weizsäcker semiempirical mass formula provides the splitting in the dispersion relations of protons and neutrons in heavy nuclei [10]. This splitting can be described as protons and neutrons "feeling" different effective nuclear potentials as shown in Fig. 1.

When the Coulomb potential (dashed line denoted by V_c) is accounted for in the overall proton potential, the obtained (nuclear) proton potential (the other dashed line) is different from the neutron's potential. In our analysis, the isospin polarization of the vacuum (broken symmetry vacuum) plays the key role. Self-consistent analysis of the effects of the polarized vacuum follows the method used in analysis of spontaneous breakdown of symmetries [6,11,12].

Now we try to formulate the PCIC approach applied to heavy nuclei. Consider a nonrelativistic field theory of interacting nucleons with an isospin-invariant contact interaction and the Coulomb interaction which is not isoinvariant. Thus the Hamiltonian is $H = H_0 + H_I$, where H_0 and H_I are the free and interaction Hamiltonians respectively:

$$H_0 = -\int d^3x\, \psi^\dagger(x) \left(\frac{\nabla^2}{2m} + \epsilon_F \right) \psi(x), \tag{1}$$

$$H_I = \frac{\lambda}{2} \int d^3x \left(\psi^\dagger(x) \frac{\tau}{2} \psi(x) \right) \left(\psi^\dagger(x) \frac{\tau}{2} \psi(x) \right) + H_c. \tag{2}$$

Here H_c is the Coulomb interaction which is

$$H_c(t) = \frac{e^2}{2} \int d^3x \int d^3y\, \rho_p(\mathbf{x},t) \frac{1}{|\mathbf{x}-\mathbf{y}|} \rho_p(\mathbf{y},t). \tag{3}$$

Here $\rho_p \equiv \psi_p^\dagger \psi_p = \psi^\dagger[(1-\tau_3)/2]\psi$. The $-\psi^\dagger(\nabla^2/2m)\psi$ term will be called the kinetic energy term. The vacuum is an isospin-symmetry broken state with the nonvanishing order parameter which is the isospin polarization:

$$I \equiv \langle 0|\psi^\dagger \frac{\tau_3}{2} \psi|0\rangle \neq 0. \tag{4}$$

This gives

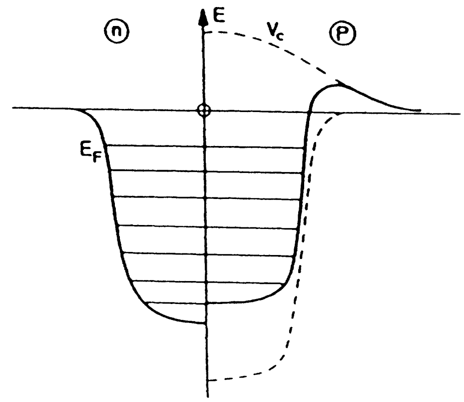

FIG. 1. Splitting in the dispersion relations of protons and neutrons in nuclei [9].

$$I = \frac{n_+ - n_-}{2}, \tag{5}$$

where the $+(-)$ suffix means neutron (proton); $n_+ = n_n$ and $n_- = n_p$ where the neutron and proton densities are denoted by n_n and n_p, respectively. In the following consideration it is convenient to introduce the dimensionless order parameter Δ:

$$\Delta = \frac{n_+ - n_-}{n} = \frac{N-Z}{A} = \frac{A-2Z}{A}, \tag{6}$$

where $n \equiv n_+ + n_-$ is the nucleon number density, and N, Z, A are, respectively, the neutron, proton, and nucleon number. We have the relation $\Delta \equiv 2I/n$. Note that, although the isospin SU(2) symmetry is broken, the cylindrical symmetry around the third axis is not. Thus the third component of the isospin is a good quantum number.

In the mean field approximation H becomes

$$H = W(A,Z) + \frac{\lambda I}{2} :\psi^\dagger \tau_3 \psi: + H_c^m. \tag{7}$$

Here the normal-order product symbol is used. The ground state energy $W(A,Z) \equiv \langle 0|H|0\rangle$ is

$$W(A,Z) = -\epsilon_F A + K(A,Z) + \frac{\lambda}{2} I^2 V + W_c, \tag{8}$$

where the kinetic energy is denoted by $K(A,Z)$, V is the nuclear volume, and W_c is given by

$$W_c = e^2 \frac{n_p^2}{2} \int d^3y F(r_y) = 4\pi e^2 \frac{n_p^2}{2} \int_0^R F(r_y) r_y^2\, dr_y \tag{9}$$

$$= e^2 \frac{3}{5} \frac{Z^2}{R}, \tag{10}$$

which is a well-known result. Here, R is the nuclear radius. We take the center of the nucleus for the origin of our coordinate and $r_y^2 \equiv |\mathbf{y}|^2$. The function $F(r_y)$ is defined by

$$F(r_y) \equiv \int d^3x \frac{1}{|\mathbf{x}-\mathbf{y}|} \tag{11}$$

$$= 2\pi R^2 - \frac{2\pi}{3} r_y^2. \tag{12}$$

We can rewrite $W(A,Z)$ as

$$W(A,Z) = -\epsilon_F A + K(A,Z) + \frac{1}{8}\lambda n A \Delta^2 + e^2 \frac{3}{5}\frac{Z^2}{R}. \tag{13}$$

Note that we have used $I = (n/2)\Delta$ and that $(\lambda n A)$ does not depend on Z. Therefore, the energy difference of the nearby nuclei is

$$\delta W \equiv W(A,Z) - W(A,Z-1) \sim \delta K - \frac{1}{2}\lambda n \Delta + \delta W_c \tag{14}$$

$$= \delta K + \delta W_c - \lambda I \tag{15}$$

for large A. Here

$$\delta W_c \equiv e^2 \frac{6}{5}\frac{Z}{R} \tag{16}$$

and

$$\delta K = -\frac{2}{9\pi}\left(\frac{9\pi}{4}\right)^{5/3}\frac{1}{mR^2}\left[N^{2/3} - Z^{2/3}\right]. \tag{17}$$

We have also used the relation $\delta\Delta = -2/A$. The near nuclear degeneracy means that δW is small.

The Coulomb term H_c^m is given by

$$H_c^m = e^2 n_p F(r_y) \, : \rho_p(\mathbf{y}) := \frac{e^2}{2}\left(3 - \frac{r_y^2}{R^2}\right)\frac{Z}{R} \, : \rho_p(\mathbf{y}) : . \tag{18}$$

In a precise analysis based on quantum field theory we should treat a nucleus as a system with a self-consistently created boundary (like a bag model). When the nucleus is large, most of the inner domain except the vicinity of the surface boundary is practically uniform. The situation in the vicinity of the boundary has infinite degrees of freedom, reflecting the infinite degrees of freedom of quantum fields. This is the reason why ordered states can be created in the nucleus. However, this precise approach is very hard to follow. Here, we crudely approximate H_c^m as

$$H_c^m \sim E_c \rho_p, \tag{19}$$

with

$$E_c = \frac{e^2 n_p}{V}\int d^3y\, F(r_y) = e^2 \frac{6}{5}\frac{Z}{R}. \tag{20}$$

To calculate E_c it is convenient to introduce the constant α by

$$mR = \alpha A^{1/3}. \tag{21}$$

Crudely speaking $\alpha = 6$. Since $A = [(4\pi)/3]R^3 n$, we obtain

$$E_c = \frac{3e^2}{5\alpha}(1-\Delta)A^{2/3}m. \tag{22}$$

We now calculate the proton-neutron energy difference. From now on we use field theory for the infinite volume. The proton acquires an energy $E_c - (\lambda I/2)$, while the neutron has the energy $(\lambda I/2)$. E_c is the Coulomb energy shift while $(\lambda I/2)$ is due to the isobaric polarization of the vacuum. Thus the neutron energy ϵ_+ and the proton energy ϵ_- are

$$\epsilon_\pm = \frac{p^2}{2m} - \epsilon_F + \frac{E_c}{2} \pm \frac{1}{2}(\lambda I - E_c), \tag{23}$$

where m is the nucleon mass. Since $(E_c/2)$ renormalizes the chemical potential as $\bar\epsilon_F = \epsilon_F - (E_c/2)$, we can write

$$\epsilon_\pm = \frac{p^2}{2m} - \bar\epsilon_F \pm \frac{1}{2}(\lambda I - E_c). \tag{24}$$

The proton-neutron energy difference $\epsilon_- - \epsilon_+$ is

$$\delta E = E_c - \lambda I. \tag{25}$$

Comparing this with (15), we find

$$\delta W = \delta K + \delta E + \delta W_c - E_c. \tag{26}$$

Thus, the proton-neutron energy difference is not equal to the energy difference of the ground states of nearby nuclei. This is not surprising, because the nucleons considered here are the asymptotic particles (quasiparticles), while the ground state energy is controlled by nucleons bound to a nucleus.

In order to calculate δE, we now derive the self-consistent equation for the order parameter I. The neutron density $n_n = n_+$ and proton density $n_p = n_-$ are given by

$$n_\pm \equiv 2\int \frac{d^3p}{(2\pi)^3}\Theta(-\epsilon_\pm), \tag{27}$$

which gives

$$n_\pm = \frac{\bar n}{2\bar\epsilon_F^{3/2}}\left[\bar\epsilon_F \mp \frac{1}{2}(\lambda I - E_c)\right]^{3/2}, \tag{28}$$

with $\bar n \equiv \frac{2}{3\pi^2}(2m\bar\epsilon_F)^{3/2}$. Note that the Coulomb energy makes the proton number smaller, while the isobaric polarization of the vacuum tries to increase the proton number.

It follows from Eq. (28) that

$$\frac{E_c - \lambda I}{\bar\epsilon_F} = \left[\left(\frac{2n_+}{n}\right)^{2/3} - \left(\frac{2n_-}{n}\right)^{2/3}\right]\left(\frac{n}{\bar n}\right)^{2/3}. \tag{29}$$

This determines the proton-neutron energy difference as

$$\delta E = E_c - \lambda I = \bar\Delta\left(\frac{n}{\bar n}\right)^{2/3}\bar\epsilon_F, \tag{30}$$

where $\bar\Delta$ is defined by

$$\bar\Delta \equiv (1+\Delta)^{2/3} - (1-\Delta)^{2/3}. \tag{31}$$

Equation (30) is the self-consistent equation which determines the isospin polarization I [13]. To calculate $\bar\epsilon_F$, we recall the relation $\bar n \equiv \frac{2}{3\pi^2}(2m\bar\epsilon_F)^{2/3}$ which gives

$$\bar{\epsilon}_F = \frac{(9\pi)^{2/3}}{8mR^2}\left(\frac{\bar{n}}{n}\right)^{2/3} A^{2/3} = \frac{(9\pi)^{2/3}}{8\alpha^2}\left(\frac{\bar{n}}{n}\right)^{2/3} m. \quad (32)$$

Now Eq. (30) gives

$$\delta E = \frac{(9\pi)^{2/3}}{8}\frac{\bar{\Delta}}{\alpha^2}m = 1.16\frac{\bar{\Delta}}{\alpha^2}m. \quad (33)$$

Thus we have obtained the proton-neutron energy difference. This equation gives

$$I = \frac{1}{\lambda}\left(E_c - 1.16\frac{\bar{\Delta}}{\alpha^2}m\right). \quad (34)$$

This is the isospin polarization density.

Let us now turn to the difference δW of the nearby ground states. According to (26) we have $\delta W = \delta K + \delta E + \delta W_c - E_c$. Considering (16) and (20), we find

$$\delta W_c - E_c = 0. \quad (35)$$

The order parameter Δ should be related to A. Such a relation follows from (34) when we recall $I = n\Delta/2$ and use $n = (3A/4\pi R^3)$, which gives $n = (3/4\pi\alpha^3)m^3$. Thus A determines Δ when the coupling constant λ is given. According to the experiment we have $\Delta = 0.21$ for $A = 208$. This determines λ, which gives Δ as a function of A. The result shows that Δ saturates beyond $A = 208$.

We choose $\Delta = 0.21$ which corresponds to $A = 208$. We then have $\bar{\Delta} = 0.283$. With these values and $m = 939$ MeV we obtain $\delta E = 9$ MeV. δK is determined by Eq. (17). We then find that δK and δE nearly compensate each other: *The energy difference δW of the nearby nuclei turns out to be close to zero.* We also find that $\bar{\epsilon}_F$ is about equal to 30 MeV.

Having calculated the energy gap between protons and neutrons we proceed to estimate the coherence length ξ_0 of the Cooper pair. The maximum distance a Cooper pair can travel by the Heisenberg uncertainty principle is

$$\xi_0 \sim V_F \delta t \sim \frac{V_F}{\delta E} = \left(\frac{2\bar{\epsilon}_F}{m}\right)^{1/2}\frac{1}{\delta E}. \quad (36)$$

When the above values for m, $\bar{\epsilon}_F$, and δE are used, this coherence length is ~ 5.5 fm which is a reasonable value, i.e., smaller than the diameter of a large nucleus.

Let us now study the massive Goldstone boson associated with the isobaric polarization in the vacuum. To obtain a precise expression for the energy and wave function of the Goldstone boson, we must solve the Bethe-Salpeter equation for the Bethe-Salpeter amplitude of the Goldstone boson. However, we are interested only in the energy gap of the Goldstone boson. This energy gap can easily be obtained from the PCIC relation. Since the cylindrical symmetry around the third iso-axes is not broken, we have two components of the Goldstone bosons which behave as components of a vector in the two-dimensional plane orthogonal to the third axis. They can be given by two boson fields $\chi_\pm$ in such a way that χ_- is the Hermitian conjugate of χ_+. It can be shown that χ_+ and χ_- are the annihilation and creation operators of the isomagnon. We do not enter into detail, because this is well known in the theory of ferromagnetism where there appear spin magnons (instead of isospin magnons) as a result of the spontaneous breakdown of the spin-SU(2) symmetry. Our Hilbert space is the space of three kinds of particles which are the physical neutron, physical proton, and χ bosons. The action of any operators acting on a vector in this Hilbert space is specified by their expression in terms of normal products of creation-annihilation operators of these three kinds of particles [6]. Such an expression is called the dynamical map which is the same as the Haag expansion when these particles are asymptotic particles. Since the cylindrical symmetry around the third axis is not broken, the dynamical map of the T_3 generator does not have a linear χ field. The linear χ fields appear in the dynamical map of $\mathbf{j}_\pm$ and $\rho_\pm$, which are the spatial and fourth components of the four-dimensional isobaric currents $j_{\pm,\mu}$. For example, ρ_- is $\psi^\dagger\tau_-\psi$ and $\mathbf{j}_-$ is the corresponding current. Their dynamical maps are [6]

$$\rho_- = Z^{1/2}(\nabla^2)\chi_- + \cdots, \quad (37)$$

$$\mathbf{j}_- = Z^{1/2}(\nabla^2)v_-(\nabla^2)\nabla\chi_- + \cdots, \quad (38)$$

where the ellipses stand for higher-order normal products. The $Z^{1/2}(\nabla^2)$ is the normalization factor which contains ∇^2 and $v_-(\nabla^2)$ controls the velocity of the χ_- boson. The field equation of the χ boson is obtained from the PCIC relation [6]

$$\partial^\mu j_\mu = \delta L, \quad (39)$$

where δL is a change of the Lagrangian under the isospin transformation. Let us consider a transformation generated by T_-. This changes the proton density ρ_p into ρ_-,

$$\delta\rho_p = -i\rho_-, \quad (40)$$

because of the relation $[\tau_3, \tau_-] = -2\tau_-$. Using H_c^m in (19) for the Coulomb interaction, we have

$$\delta L = -\delta H_c = -E_c\delta\rho_p = iE_c\rho_- \quad (41)$$

$$= iZ^{1/2}(\nabla^2)E_c\chi_- + \cdots. \quad (42)$$

Feeding the dynamical maps of currents in the PCIC relation (39) and inspecting the linear χ terms, we obtain the wave equation of the Goldstone boson:

$$\left[\frac{\partial}{\partial t} + iv_-(\nabla^2)\nabla^2\right]\chi_- = iE_c\chi_-. \quad (43)$$

Note that the normalization factors $Z^{1/2}$ compensate among themselves. Since χ_- creates the isomagnon, its Fourier amplitude depends on energy and time through $\exp(i\omega t)$. Thus the Goldstone boson energy is

$$\omega_k = v_-(-k^2)k^2 + E_c, \quad (44)$$

which implies that the energy gap of the Goldstone boson is

$$\omega_c = E_c. \quad (45)$$

To calculate the velocity v_-, we need the Bethe-Salpeter equation. However, the energy gap ω_c is simply obtained from the PCIC relation.

It is remarkable that the gap energy is the same as the Coulomb shift of the proton energy E_c. When we have a nucleus with (A, Z), the T_- acting on the ground state $|A, Z\rangle$ of this nucleus excites the Goldstone field as $\chi_-|A, Z\rangle$ with $(A, Z-1)$.[2] The energy difference between the parent nucleus and this excited state is E_c, implying that, when we would subtract E_c from the excited energy level, the excited level would be degenerate with the ground state of the parent nucleus. This is expected because the Goldstone boson energy is gapless when there is no Coulomb interaction. However, this makes it very reasonable to assume that the massive Goldstone state is the isobaric analog state.

We have shown that the PCIC approach can reasonably describe the relations among nearby states of heavy nuclei and isobaric analog states. Being encouraged by the results, we are planning to develop a deeper analysis along this approach. The breakdown of the isospin symmetry in nuclei seems to be the simplest example of more general features in nuclei. We have made a preliminary analysis of the spontaneous breakdown of the SU(3) (Elliot) and SU(4) (spin-isospin) symmetries [7,3] in nuclei. Further analysis in this direction would open a new and unified picture of nuclei.

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[2]When a symmetry is broken, the generators become not meaningful in a strict sense. To be more careful we must replace these generators by those in which the space integrations are smeared out with square integrable functions. We omit here any detailed considerations. See Ref. [6].

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