Elimination of the Landau ghost from chiral solitons

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We show a practical way based on the Källén-Lehmann representation for the two-point functions to eliminate the instability of the vacuum against formation of small size meson configurations in the chiral σ model.

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I. INTRODUCTION

Since QCD is presently intractable at low energies, approximation schemes based on effective models have been introduced [1]. These models in a way incorporate some important features of QCD, like chiral symmetry and its spontaneous breakdown.

Though being approximative, these effective models still cannot be treated exactly. Therefore we have essentially two levels of approximation: (i) when we choose the model itself and (ii) when we specify an approximation scheme in order to perform actual calculations within the model. This implies that the results may depend on the actual approximation scheme. Therefore, a failure of the model could be due to the model itself, due to the approximation scheme or both. So, if we want to test the model, we should be careful about the approximation scheme and keep the dependence on *ad hoc* parameters as small as possible.

One of the effective models of QCD that describes the nucleon is the chiral σ model [2]. The standard procedure to calculate nucleonic properties in the soliton picture is to use the loop expansion up to the one quark loop level, i.e., to integrate out the quark degrees of freedom and use the remaining purely mesonic effective action at the zero loop level in the mesonic fields [3]. One can choose some cutoff procedure to regularize the quark determinant [4], but, in any case, one has to renormalize by introducing appropriate counter terms in order to define the parameters appearing in the original Lagrangian of the model [5]. Eventually one can let the cutoff go to infinity as it is usually done in renormalization theory, or leave it finite. We note that in the σ model, however, neither the cutoff scheme itself nor the size of the cutoff can be determined uniquely.

It is known, however, that the loop expansion for asymptotically nonfree theories without *ad hoc* cutoff parameters displays an unphysical pole in the corresponding propagators, the so-called Landau ghost [6]. Therefore, if one follows the procedure outlined above, one finds that the renormalized σ model is plagued by the Landau ghost, which leads to an instability of the usual translational invariant vacuum [7,8]. In view of our remarks at the beginning of this section, the problem of this Landau ghost can be treated in several ways: (1) The effective model is considered as unphysical and is abandoned. (2) The approximation scheme (loop expansion) is considered appropriate, but the effective model can be used only with a regularization scheme, hereby being restricted to energies far below the unphysical pole. (3) The approximation scheme is improved such that the effective model does not *a priori* yield unphysical results for energies up to the order of the baryon mass.

The second viewpoint implies that vacuum loops are either simply discarded, or calculated with finite cutoff. Concerning the first alternative (drop vacuum loops), we note that in some situations, like in an external field, such a treatment violates conservation laws [9] and cannot be implemented consistently. Besides this, the investigation of the change of the vacuum structure due to the finite density should be an important subject for any relativistic field theory. Concerning the second alternative (finite cutoff), we note that the Landau ghost in hadronic theories occurs at rather low energies of the order of the baryon mass ($\simeq 1$ GeV). This means that its presence is a real problem for effective quark theories, and its avoidance by the introduction of an *ad hoc* parameter is somewhat unsatisfactory. Since the energy scale relevant for effective quark theories is set by hadronic masses, a treatment according to the third viewpoint, which we will adopt in this paper, is a feasible alternative. We consider the Landau ghost as one of the basic infinities in effective field theories which should be brought under control without the introduction of further parameters. In a different context, this viewpoint has been taken already a long time ago by Redmond and Bogoliubov *et al.* [10], who have shown how to construct ghost free propagators in the framework of the loop expansion based on the requirement of the Källén-Lehmann representation. Their propagators have several interesting features: The associated wave-function renormalization constant is finite, and they have an essential singularity at $g^2 = 0$ (g is the coupling constant) which is expected from more intuitive physical arguments [11]. Moreover, since the associated running coupling constant has no singularity at spacelike momentum transfers, the "zero charge problem" is

avoided. Their method has recently been formulated in the language of the effective action [12], which is suitable for effective quark theories, and also applied to infinite systems [12,13]. The main purpose of the present paper is to show a practical way to incorporate the ghost elimination in a finite system like the soliton, and to demonstrate that with this procedure the translational invariant vacuum becomes stable with respect to decay into an array of small-sized configurations. We will *not* construct the fully self-consistent solutions including valence quarks in this paper which is left for a future work. (For another possibility to avoid the Landau ghost by introducing vector bosons, see Ref. [14].)

The σ model has so far not yet been fully exploited to obtain self-consistent solutions including the effect of the Dirac sea. One of our motivations for investigating this model is as follows: In contrast to the Nambu-Jona-Lasinio (NJL) model [15], which has been extensively used to construct solitonic solutions [16–18], it has a ϕ^4 interaction term with variable strength (specified by the σ mass) which controls the deviation of the chiral radius from its vacuum value. The amount such a term would contribute to the ground state energy increases the farther the chiral fields are away from the chiral circle of the vacuum. Since it is known that interaction terms of the form ϕ^4 [19], or of higher order like the 't Hooft determinant interaction [17], can prevent the collapse of the NJL soliton, there is the possibility that stable solitons exist for the linear σ model. The major obstacle to a full investigation of the solitonic sector of the σ model, however, was the instability of the translational invariant vacuum. The fact, to be established in this paper, that the method of Redmond and Bogoliubov *et al.* leads to a stabilization of the translational invariant vacuum is a first step towards self-consistent solutions in the σ model.

The rest of the paper is organized as follows: To be self-contained and to set the formalism we state in Sec. II some results for the ground state energy in the σ model obtained earlier [7], and explain the role of the Landau ghost in a way which seems most transparent to us and which is appropriate for implementing the ghost elimination procedure. In Sec. III we construct a ghost free model and show results for the ground state energy, and in Sec. IV we summarize our results.

II. THE GROUND STATE ENERGY AND THE LANDAU GHOST

We now proceed to exhibit how the Landau ghost contributes to the vacuum instability. The Euclidean Lagrangian of the σ model reads, after introducing the "physical" parameters m_σ and m_π instead of the original ones μ^2 and λ^2 [2],

$$\mathcal{L} = \mathcal{L}_F + \mathcal{L}_M + \mathcal{L}_{SB} \tag{1}$$

with

$$\mathcal{L}_F = \bar\psi D\psi, \quad D = -i\not\partial + MU, \tag{2}$$

$$\mathcal{L}_M = \frac{v^2}{2}[(\partial_\mu U)(\partial_\mu U^+) + \tfrac{1}{4}(m_\sigma^2 - m_\pi^2)(U^+U - 1)^2$$

$$+ m_\pi^2(U^+U - 1)]. \tag{3}$$

Here ψ stands for the u and d quarks, $M = gv$ is the effective quark mass, $U = (1/v)(\sigma + i\gamma_5\boldsymbol{\pi}\cdot\boldsymbol{\tau})$ is the chiral field, and $\mathcal{L}_{SB} = f_\pi m_\pi^2 \sigma$ with f_π the pion decay constant. The model contains the two free parameters g (or M) and m_σ. v is the vacuum expectation value of σ, and in the symmetric limit $(m_\pi = 0)$ we have $v = f_\pi$ [2]. Our Euclidean metric is such that $x_\mu = x^\mu = (\tau, \mathbf{r})$ with $\tau = ix^0$, $0 < \tau < \beta$ with β the upper limit of Euclidean time integration, and $\gamma_\mu = \gamma^\mu = (i\beta_D, \boldsymbol{\gamma})$ with β_D the usual Dirac β matrix. Defining the effective bosonic action Γ from the generating functional Z as usual [3] by $Z = \int DU \exp(-\Gamma)$ we obtain after integrating out the quark fields

$$\Gamma = \Gamma_F + \Gamma_M + \Gamma_{SB} \tag{4}$$

with [7]

$$\Gamma_F = -\tfrac{1}{2}\mathrm{Tr}\ln D^+D + \text{c.t.}$$

$$= -\tfrac{1}{2}\mathrm{Tr}\ln(1 + GV) + \tfrac{1}{2}\mathrm{Tr}GV - \tfrac{1}{4}\mathrm{Tr}G^2V^2, \tag{5}$$

$$\Gamma_M = \int d^4x\, \mathcal{L}_M, \quad \Gamma_{SB} = \int d^4x\, \mathcal{L}_{SB}. \tag{6}$$

Here $\mathrm{Tr}A \equiv N_c \int d^4x\, \mathrm{tr}\langle x|A|x\rangle$ where $N_C = 3$ is the number of colors and tr refers to the Dirac and isospin indices, $G = (-\partial^2 + M^2)^{-1}$, and

$$V = iM\not\partial U + M^2(U^+U - 1). \tag{7}$$

The integrals over the Euclidean time x_0 extend from 0 to β. The counter terms (c.t.) in (5), which include the subtraction of the translational invariant vacuum contribution $(U = 1)$, are determined such that Γ_F gives no terms of the same form as those already present in Γ_M. This corresponds to the mesonic mass and wave function renormalization at the renormalization point $\mu^2 = 0$ [5]. From here on we consider the chiral symmetric case $(m_\pi = 0)$.

The vacuum instability can be seen by expanding the ground state energy $E = \Gamma/\beta = (\Gamma_F + \Gamma_M)/\beta$ in powers of a characteristic length scale R where the classical meson fields are localized. We assume time independent fields $U(\mathbf{r}) = \tilde{U}(\mathbf{x})$ with $\mathbf{x} = \mathbf{r}/R$. If we use $\mathbf{x}$ as the integration variable, every derivative gives rise to a factor $1/R$, and therefore terms involving derivatives will be dominant for small R compared to those without derivatives. Therefore, up to $O(R)$ Γ_M contributes only the kinetic term. To obtain the lowest order contribution from Γ_F we expand the logarithm in (5) and note that the term linear in V is canceled by a counter term. Then for small R the leading term is the one quadratic in the derivatives, i.e.; quadratic in the first term of Eq. (7). From the above we see that the linear and the nonlinear model look the same at small sizes. The complete term of order V^2 is

$$\Gamma_F^{(s.s.)} = \frac{1}{4}\mathrm{Tr}(GVGV - G^2V^2). \tag{8}$$

Let us consider Eq. (8) in a continuous and infinite plane wave (PW) basis $\langle x|k\rangle = (\beta\Omega)^{-1/2}\exp(ikx)$, where Ω is the volume. Then the matrix element $\langle k'|V|k\rangle$ equals $\Omega^{-1}\delta_{q_0,0}V(\mathbf{q})$, i.e.; the Fourier transform of $V(\mathbf{r})$ with $q = k - k'$. Using the dimensionless variables $\mathbf{x} = \mathbf{r}/R$ and $\mathbf{t} = \mathbf{q}R$ and keeping for V only the first term in Eq. (7) we obtain for (8) in the continuum limit [7]

$$\frac{1}{\beta}\Gamma_F^{(s.s.)} = \frac{1}{4}N_C M^2 R \int \frac{d^3t}{(2\pi)^3}\mathrm{tr}|L(\mathbf{t})|^2\phi\left(\left(\frac{\mathbf{t}}{R}\right)^2\right) + O(R^2) \tag{9}$$

with

$$\phi(q^2) = \int \frac{d^4k}{(2\pi)^4}\left(\frac{1}{(k^2 + M^2)[(k+q)^2 + M^2]} - \frac{1}{(k^2 + M^2)^2}\right) \tag{10}$$

and $L(\mathbf{t})$ is the Fourier transform of $[\not{\partial}_x\tilde{U}(\mathbf{x})]$:

$$L(\mathbf{t}) = \int e^{i\mathbf{t}\cdot\mathbf{x}}\not{\partial}_x\tilde{U}(\mathbf{x})d^3x. \tag{11}$$

Adding the kinetic term, we get the leading contribution to the effective action in the small-size expansion as

$$\frac{1}{\beta}\Gamma^{(s.s.)} = \frac{M^2R}{16g^2}\int \frac{d^3t}{(2\pi)^3}\mathrm{tr}|L(\mathbf{t})|^2$$
$$\times\left[1 + 4g^2N_C\phi\left(\left(\frac{\mathbf{t}}{R}\right)^2\right)\right] + O(R^2). \tag{12}$$

Since $\phi(q^2) \to -(1/16\pi^2)\ln(q^2/M^2)$ as $q^2 \to \infty$ it follows that (9) behaves as $\alpha M^2 R\ln(MR)$ for $R \to 0$ with $\alpha > 0$, which overcomes the kinetic term [the 1 in (12)] for R small enough, leading to a negative value for $\Gamma^{(s.s.)}$. This means that for small sizes R the energy of this "non-translational invariant vacuum" can become lower than the energy of the translational invariant vacuum.

To see that the Landau ghost in the meson propagators is responsible for this vacuum instability, we use the fact that the effective action, when expanded around the fields in the translational invariant vacuum ($\sigma = v, \pi = 0$), gives the inverse σ and π propagators in the translational invariant vacuum as the coefficients of the terms of second order in $s \equiv \sigma - v$ and π. If we express V of Eq. (7) in terms of σ, π using $U = (1/v)(\sigma + i\gamma_5\pi\cdot\tau)$, we see that these second order terms are completely contained in (8). Taking also the terms of second order in s, π in the mesonic term in (6), we obtain [13]

$$\frac{1}{\beta}\Gamma^{(2)} = \frac{1}{2}\int \frac{d^3q}{(2\pi)^3}[s(\mathbf{q})s(-\mathbf{q})G_\sigma^{-1}(\mathbf{q}^2)$$
$$+ \pi(\mathbf{q})\cdot\pi(-\mathbf{q})G_\pi^{-1}(\mathbf{q}^2)] \tag{13}$$

with the inverse Euclidean Schwinger-Dyson propagators
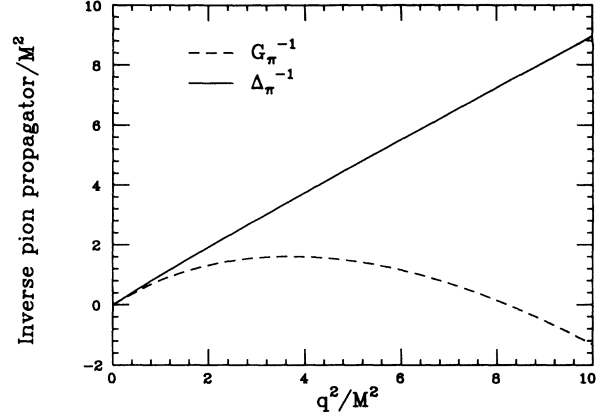

FIG. 1. The inverse pion propagator with $m_\pi = 0, g = 4$ for Euclidean q^2. The dashed line shows the Schwinger-Dyson propagator and the solid line the Källén-Lehmann propagator.

$$G_\pi^{-1}(q^2) = q^2(1 + 4g^2N_C\phi(q^2)), \tag{14}$$

$$G_\sigma^{-1}(q^2) = q^2(1 + 4g^2N_C\phi(q^2)) + m_\sigma^2 + 16g^2N_C M^2\phi(q^2). \tag{15}$$

In Fig. 1 the dashed line shows the inverse pion propagator (14) for $g = 4$. The previously noted behavior of $\phi(q^2)$ leads to the Landau ghost pole at Euclidean q^2, and as a consequence (13) can become more and more negative for large $\mathbf{q}^2$, i.e., for small sized fields. In this limit of small sized fields the leading contribution to (13) comes from the terms $\propto q^2$ in Eqs. (14) and (15), and if expressed in terms of $\mathbf{t} = \mathbf{q}R$ this gives again Eq. (9). [Note that

$$\mathrm{tr}|L(\mathbf{t})|^2 = \frac{8}{v^2}\mathbf{t}^2[\tilde{s}(\mathbf{t})\tilde{s}(-\mathbf{t}) + \tilde{\pi}(\mathbf{t})\cdot\tilde{\pi}(-\mathbf{t})]$$

with $\tilde{s}(\mathbf{t}) = (1/R^3)s(\mathbf{q})$ the Fourier transform of $\tilde{s}(\mathbf{x})$ and similar for $\tilde{\pi}(\mathbf{t})$.]

III. GHOST REMOVAL IN A FINITE SYSTEM

We now address the problem of removing the Landau ghost in a finite system. The procedure is described in Ref. [10] and has been extended to the path integral formalism in Ref. [12]. To calculate the ground state energy we have to first obtain meson propagators with the correct analytical properties. These propagators are then implanted into the effective action which is in turn used to calculate the ground state energy.

The prescription of Ref. [12] to eliminate the Landau ghost from the effective action is to replace the Schwinger-Dyson propagators G_α in the second order term (13) by the Källén-Lehmann (KL) propagators Δ_α. According to Ref. [10], these are constructed from the KL representation using a spectral function obtained from the one-loop meson self-energy, and are free of the Landau ghost. Thus, in this method the loop approxima-

tion is used only for the *imaginary* part, while the real part is calculated from the dispersion relation. Since this method avoids the Landau ghost, it clearly represents an improvement of the straightforward loop approximation for the whole propagator. The new effective action becomes

$$\Gamma_{KL} = \Gamma + \delta\Gamma, \quad \delta\Gamma \equiv \Gamma_{KL}^{(2)} - \Gamma^{(2)}. \tag{16}$$

In order to be consistent with chiral symmetry, however, this ghost subtraction has to be done under the constraint of the Ward identity [2] $\Delta_\sigma(q^2)^{-1} - \Delta_\pi(q^2)^{-1} = ivT(-q; q, 0)$ with $T(-q; q, 0)$ the $\sigma\pi^2$ vertex where one of the external pions has zero momentum. To preserve this identity the difference of the σ and π inverse propagators has to remain invariant under ghost subtraction [13]; i.e., $\Delta_\sigma^{-1} - G_\sigma^{-1} = \Delta_\pi^{-1} - G_\pi^{-1}$, and we obtain from (16) and (17)

$$\frac{1}{\beta}\delta\Gamma = \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} [s(\mathbf{q})s(-\mathbf{q}) + \boldsymbol{\pi}(\mathbf{q}) \cdot \boldsymbol{\pi}(-\mathbf{q})]$$
$$\times [\Delta_\pi^{-1}(\mathbf{q}^2) - G_\pi^{-1}(\mathbf{q}^2)]. \tag{17}$$

It can be shown [10] that the KL propagator can be obtained from the SD propagator by subtracting the ghost pole: $\Delta_\pi(q^2) = G_\pi(q^2) - Z_G/(q^2 - m_G^2)$, where $Z_G < 0$ is the residue of the pole at Euclidean $q^2 = m_G^2$. We show Δ_π^{-1} for $m_\pi = 0$ and $g = 4$ by the solid line in Fig. 1.

The ghost free ground state energy is thus obtained by adding the piece (17) to $(1/\beta)\Gamma$ with Γ given by Eq. (4). The exact evaluation of Γ_F, given by Eq. (5), requires the diagonalization of the Dirac Hamiltonian $h = i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \beta_D MU$, and this is done most conveniently in a basis $|\lambda\rangle$ which diagonalizes the free Dirac Hamiltonian

$$h_0 = h(U = 1) = -i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \beta_D M : \quad h_0|\lambda\rangle = \varepsilon_\lambda^0|\lambda\rangle$$

with $\varepsilon_\lambda^0 = \sqrt{\mathbf{k}_\lambda^2 + M^2}$ in a box of size D with discrete momenta $|\mathbf{k}_\lambda| < k_{\max}$. (Here λ labels all necessary quantum numbers except color.) D and $k_{\max}$ should be taken large enough such that all results are unchanged under further increase. We will call this discrete and finite basis the Kahana-Ripka (KR) basis [20]. On the other hand, the ghost subtraction is formulated most conveniently in the continuous and infinite PW basis, for which the result is given by Eq. (17). Therefore, we proceed in two steps: First, in order to demonstrate that the KR basis gives the same results as the PW basis, provided that $k_{\max}$ and D are taken large enough, we evaluate Γ_F in the small-size approximation both in the KR and the PW basis and compare the results. Then we evaluate Γ_F exactly in the KR basis and add the ghost subtraction term (17).

The expression for Γ_F in the small size approximation using the PW basis has already been given in Eq. (9), and the corresponding result for the KR basis is obtained from Eq. (8) as

$$\frac{1}{\beta}\Gamma_F^{(\text{s.s.})} = \frac{1}{8}N_C \sum_{\lambda'\lambda} |\langle\lambda'|V|\lambda\rangle|^2$$
$$\times \left(\frac{1}{|\varepsilon_{\lambda'}^0||\varepsilon_\lambda^0|(|\varepsilon_{\lambda'}^0| + |\varepsilon_\lambda^0|)} - \frac{1}{2}\frac{1}{|\varepsilon_{\lambda'}^0|^3} \right). \tag{18}$$

Since for the exact calculation one has to diagonalize h, it is convenient here to treat $\langle\lambda'|V|\lambda\rangle$ as $h^2 - h_0^2$, where $h_{\alpha\beta}^2 = h_{\alpha\gamma}h_{\gamma\beta}$ is the square of a finite dimensional matrix. In Fig. 2 we show the small-size approximation to the Fermion loop energy, based on Eq. (9) (solid line) and Eq. (18) (dashed line) in units of MN_C as a function of MR, assuming a Hedgehog profile with the vacuum value for the chiral radius and winding number $n = 1$:

$$U = \exp(i\hat{\mathbf{r}} \cdot \boldsymbol{\tau}\Theta(r)\gamma_5), \quad \Theta(r) = \pi\exp(-r/R). \tag{19}$$

In this case there are no additional $O(R^2)$ terms in (9) due to $U^\dagger U = 1$. Also, Γ_F/M or $\Gamma_F^{(\text{s.s.})}/M$, viewed as a function of MR, is independent of M (or g). When the KR basis is used, the effective action becomes a function of $k_{\max}$, and in order to reach convergence $[\Gamma(k_{\max}) \to \Gamma]$ in the small-size approximation we had to go up as high as $k_{\max} \simeq 40M$ for $MR \simeq 1$. This is in contrast to models with finite cutoff like the NJL model, where $k_{\max} \simeq 10M$ is sufficient for solitons of normal size [17]. The calculations in this paper are performed with values for $(k_{\max}/M, DM)$ ranging from $(70,5.7)$ for small R to $(20,20)$ for large R, such that $k_{\max}D \simeq 400$ is kept constant. Figure 2 demonstrates that the results using the PW and the KR basis agree well if $k_{\max}$ and also the box size D are taken large enough.

Comparison with the full calculation, to be discussed below, shows that the result of Fig. 2 is quite accurate up to $MR \simeq 0.7$. The fact that the ground state energy relative to the translational invariant vacuum is negative for these small R indicates the vacuum instability due to the Landau ghost discussed above.

The total ground state energy in the small-size approximation is shown in Fig. 3. It is obtained from Eq. (12), and for the ghost free model by further adding the piece (17). We see that the ground state energy in the small-size approximation becomes a positive quantity af-
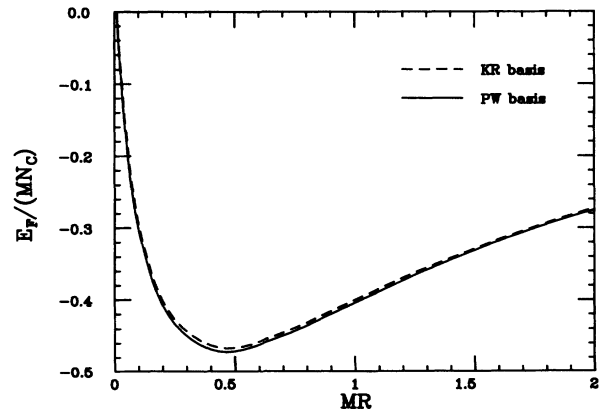

FIG. 2. The Fermion loop energy $E_F = (1/\beta)\Gamma_F$ in units of MN_C as a function of MR in the small size approximation. The solid line shows the result with the PW basis, Eq. (9), and the dashed line shows the result with the KR basis, Eq. (18), with values for $k_{\max}$ and D described in the main text.
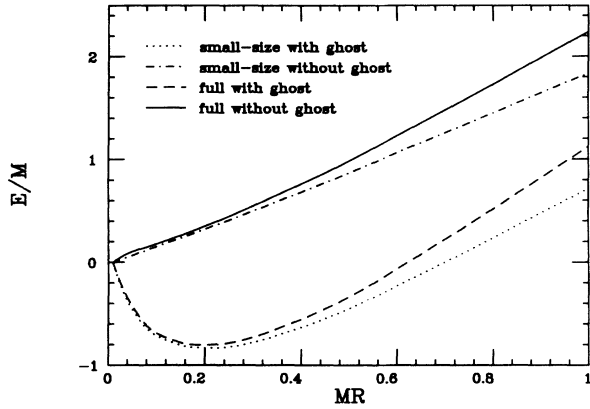
FIG. 3. The total energy $E = (1/\beta)\Gamma$ in units of M as a function of MR. The dotted and dashed-dotted lines refer to the small size expansion before and after the ghost subtraction, respectively, and the dashed and solid lines show the full result before and after the ghost subtraction, respectively.

ter removal of the Landau ghost.

We now return to the full effective action, Eqs. (5) and (6). In the KR basis Γ_F can be written in the form [7]

$$\frac{1}{\beta}\Gamma_F = -\frac{N_C}{2}\sum_\lambda \left(|\epsilon_\lambda| - |\epsilon_\lambda^0|\right) + \frac{N_C}{4}\sum_\lambda \frac{\langle\lambda|V|\lambda\rangle}{|\epsilon_\lambda^0|}$$
$$-\frac{N_C}{16}\sum_\lambda \frac{\langle\lambda|V^2|\lambda\rangle}{|\epsilon_\lambda^0|^3} \qquad (20)$$

with $V = h^2 - h_0^2$ as before, and ϵ_λ are the eigenvalues of h. Since we have already demonstrated for the small-size expansion that for $k_{\max}$ and D large enough the results with the KR basis agree well with those of the PW basis, it is feasible to use the expression in the PW basis, Eq. (17), for the ghost subtraction term $\delta\Gamma$.

Figure 3 shows the total ground state energy $(1/\beta)\Gamma$ including the ghost (dashed line) and $(1/\beta)\Gamma_{KL}$ without the ghost (solid line) in units of M as a function of RM for the Hedgehog meson profile (19). After ghost subtraction, the total ground state energy relative to the translational invariant vacuum is positive, which demonstrates that amending the two-point functions such that they satisfy the KL representation is sufficient to stabilize the vacuum against the formation of arrays of strongly localized meson field configurations.

IV. SUMMARY

In effective quark-meson theories, which have been designed to model QCD in the energy region of hadron masses, the Landau ghost appears in the meson propagators at rather low energies of $\simeq 1$ GeV. Its presence is a real problem for these theories and leads to an instability of the translational invariant vacuum. In this paper we have taken the viewpoint that the Landau ghost should be brought under control without the *ad hoc* introduction of further parameters. Based on the work of Redmond and Bogoliubov *et al.* [10], who have shown how one can improve the loop expansion such that the propagators have the correct analytical properties, we have shown a practical way to eliminate the Landau ghost from chiral solitons in the σ model. We have demonstrated that the ghost free effective action formalism of Ref. [12] can be applied successfully to finite systems. In particular, since the ghost subtraction term is expressed most conveniently in a continuous and infinite plane wave basis, we have investigated in detail the conditions under which it is feasible to add this term to the Fermion loop term which is evaluated in a discrete and finite basis. By considering vacuum configurations characterized by various sizes of the meson profiles we have shown that in our ghost free model the translational invariant vacuum has the lowest energy and is therefore stable with respect to decay into small-sized configurations.

The method given in this paper can form the base for further investigations on self-consistent soliton solutions in the σ model. In particular, as mentioned in the Introduction, it would be interesting to see whether self consistent solutions can be obtained with a reasonable strength of the ϕ^4 term.

The problem of the Landau ghost and the associated vacuum instability occurs also in relativistic meson-nucleon theories for nuclear structure [21]. For example, if a finite nucleus is described in the relativistic Hartree approximation including the effect of the Dirac sea, one has to apply the same "overall ghost subtraction" as performed in this paper [see Eq. (16)]. In more sophisticated approximations like the "$1/N$" expansion [22] or the "modified loop expansion" [23] the Landau ghost appears also in the subgraphs (even for infinite systems), and the ghost subtraction must be applied to these subgraphs. Thus, although we used the language of effective quark theories in this paper, the method is applicable to relativistic nuclear structure physics as well.

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