Effect of memory time on the agitation of unstable modes in nuclear matter

Sakir Ayik
Department of Physics, Tennessee Technological University, Cookeville, Tennessee 38505

Jørgen Randrup
Nuclear Science Division, Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720
(Received 13 July 1994)

The spontaneous agitation of collective modes in unstable nuclear matter is addressed with an extended Boltzmann-Langevin (BL) theory that incorporates a memory time in the stochastic force. The growth of the modes is then governed by effective diffusion coefficients which are renormalized by time-dependent factors, relative to the standard treatment. These correction factors deviate significantly from unity in the most unstable domain of the density-temperature phase plane, indicating the importance of including a memory time in numerical BL simulations of nuclear dynamics.

PACS number(s): 24.60.Ky, 21.30.+y, 21.60.Ev, 21.65.+f

I. INTRODUCTION

In the course of the past few years, the nuclear Boltzmann-Langevin model [1] has emerged as a promising microscopic model for nuclear dynamics at intermediate energies. This model describes the evolution of the one-body phase-space density for nucleons (and any other hadrons present), as it evolves in the self-consistent effective one-body field while subjected to the effect of occasional Pauli-suppressed two-body collisions. When the two-body collisions are ignored, the resulting Vlasov model [2] describes the self-consistent one-body evolution of the system and is the semiclassical analogue to the time-dependent Hartree-Fock (TDHF) model [3]. The inclusion of the average effect of the two-body collisions, along the lines pioneered by Nordheim [4,5], leads to the much employed Boltzmann-Uehling-Uhlenbeck (BUU) model which has been very successful in accounting for a variety of phenomena in intermediate-energy nuclear collisions, such as collective flow and particle production [6,7]. When the fluctuating part of the two-body collisions is included as well, the Boltzmann-Langevin (BL) model emerges [8], and it is then possible to address processes exhibiting spontaneous symmetry breaking and catastrophic transformations, such as compound nuclear fission or multifragmentation [9].

In all of these treatments, the collision term is assumed to be local in both space and time, in accordance with Boltzmann's original treatment. This simplification is usually justified by the fact that the interaction range, as measured by the residual scattering cross section $\sigma_{NN} \approx 4$ fm^2, is relatively small on the scale of a typical nuclear system, and the duration of a two-body collision is short on the time scale characteristic of the macroscopic evolution of the system. The resulting collective motion has then a classical character, as is the case also in TDHF. However, when the system possesses fast collective modes, with characteristic energies that are not small in comparison with the temperature, then quantum-statistical effects are important and the standard treatment is inadequate.

In order to improve the one-body transport treatment, a memory-dependent collision kernel was previously introduced in the extended TDHF model [10]. Subsequently, the semiclassical version of this model was applied to the damping of collective vibrations in nuclei [11]. More recently, Ayik extended this approach to the BL model by employing a finite memory time for the correlation function characterizing the stochastic collision term [12]. In the present work we adapt this extended treatment to nuclear matter in the spinodal zone where unstable modes exist.

The agitation of unstable modes in nuclear matter has been addressed within the standard BL model by Colonna, Chomaz, and Randrup [13], who showed that the dynamics of the collective modes is governed by a simple transport equation. The modes are agitated by a source term arising from the fluctuating part of the collision term and amplified exponentially by the unstable self-consistent effective field. The characteristic amplification time t_k corresponding to a given wave number k is determined by the associated dispersion relation. As it turns out [14,15], the fastest-growing collective modes, which are those that will become predominant, have fairly high characteristic energies $E_k = \hbar/t_k$. For example, for densities $\rho \approx 0.3\rho_0$ and wavelengths of 7–8 fm, for which the fastest amplification occurs, we have $E_k \approx 8$ MeV. This is clearly not small in comparison with the temperature in the system, which is $T \approx 4$ MeV, typically. Consequently, one must expect quantum-statistical effects to be important in the dynamics of the collective modes and the standard BL treatment may therefore be inadequate, since it treats the collective modes as classical.

We therefore reexamine the problem considered in Ref. [13], employing the extended theory of Ref. [12]. First, in Sec. II, we briefly review how the standard BL model can be modified to incorporate a memory time. We then, in Sec. III, turn to the specific problem of collective modes in unstable nuclear matter and show that, while the general evolution of the modes retains the features described in Ref. [13], the source terms are modified by time-dependent correction factors that can deviate significantly from unity, and the growth of the unstable modes is then correspondingly affected. Finally, we discuss our results and outline the consequences in Sec. IV.

The dynamics of statistical fluctuations in nuclear matter was also recently considered by Kiderlen and Hofmann [16], who deduced the properties of the stochastic forces from the quantal fluctuation-dissipation theorem, suitably generalized to unstable modes. They found sizable quantum effects both inside and outside the spinodal regime, a result consistent with our present investigation.

II. INCLUSION OF MEMORY TIME

In the Boltzmann-Langevin model the evolution of the phase-space density $f(\mathbf{r},\mathbf{p})$ is determined by a stochastic transport equation

$$\frac{d}{dt}f \equiv \frac{\partial}{\partial t}f - \{h[f],f\} = K[f] = \bar{K}[f] + \delta K[f] \ , \quad (1)$$

where $h[f](\mathbf{r},\mathbf{p})$, and $K[f]$ denotes the effect of the residual two-body collisions. It can be decomposed into its average part $\bar{K}[f]$ and the fluctuating remainder $\delta K[f]$. Only the former part is retained in the ordinary Boltzmann-Uehling-Uhlenbeck equation, which is then devoid of stochasticity.

The fluctuating part of the collision term vanishes on the average by definition, $\prec \delta K(\mathbf{r},\mathbf{p},t) \succ = 0$. The average $\prec \cdots \succ$ is taken with respect to the entire ensemble of possible binary collisions resulting from the specified phase-space density $f(\mathbf{r},\mathbf{p})$, and the functional dependence of $\delta K(\mathbf{r},\mathbf{p},t)$ on f has been suppressed in order not to clutter the notation.[1] The associated correlation function is of the form

$$\prec \delta K(\mathbf{r},\mathbf{p},t)^{*}\delta K(\mathbf{r}',\mathbf{p}',t') \succ = C(\mathbf{p},\mathbf{p}';t-t')\delta(\mathbf{r}-\mathbf{r}') \ . \quad (2)$$

In the standard Boltzmann-Langevin model, the collisions are assumed to be instantaneous and so $\mathring{C}(\mathbf{p},\mathbf{p}';t-t') = \hat{C}(\mathbf{p},\mathbf{p}')\delta(t-t')$, where the circle over the quantity indicates that it is the one associated with the standard BL model. The dependence of the correlation function C on the position $\mathbf{r}=\mathbf{r}'$ has not been exhibited, since it is in fact absent in the case of uniform matter which is the object of the present study.

The finite duration of a two-body collision modulates the corresponding frequency spectrum and it is convenient to perform a Fourier analysis of the correlation function (2),

$$C(\mathbf{p},\mathbf{p}';t-t') = \int \frac{d\omega}{2\pi} e^{-i\omega(t-t')}\tilde{C}(\mathbf{p},\mathbf{p}';\omega) \ . \quad (3)$$

In accordance with Ref. [12], the kernel entering in (3) is given by

$$\tilde{C}(\mathbf{p}_a,\mathbf{p}_b;\omega) = h^D\delta(\mathbf{p}_a-\mathbf{p}_b)\int\frac{d\mathbf{p}_2}{h^D}\int\frac{d\mathbf{p}_3}{h^D}\int\frac{d\mathbf{p}_4}{h^D}w(a2;34;\omega)F(a2;34) + \int\frac{d\mathbf{p}_3}{h^D}\int\frac{d\mathbf{p}_4}{h^D}[w(ab;34;\omega)F(ab;34) - 2w(a3;b4;\omega)F(a3;b4)] \ , \quad (4)$$

where $F(12;34) \equiv f_1f_2\bar{f}_3\bar{f}_4 + \bar{f}_1\bar{f}_2f_3f_4$ with $f_i \equiv f(\mathbf{p}_i)$ denoting the phase-space occupancy factors and the corresponding vacancy factors being $\bar{f}_i \equiv 1-f_i$. The first term arises from the diagonal part of the Langevin noise affecting only a single phase-space location, while the terms in the second line express the correlations between the fluctuations at different locations in momentum space. The frequency-dependent transition rate is given by

$$w(12;34;\omega) = \tfrac{1}{2}[\delta(\Delta\epsilon-\hbar\omega)+\delta(\Delta\epsilon+\hbar\omega)]W_\omega(12;34) \ , \quad (5)$$

where $\Delta\epsilon \equiv \epsilon_3+\epsilon_4-\epsilon_1-\epsilon_2$ is the energy change experienced by the colliding pair of nucleons, equal to the amount taken up by the collective mode.

The elementary transition rate $w(12;34;\omega)$ depends on ω both explicitly via the finite energy exchange $\pm\hbar\omega$ with the medium and implicitly via $W_\omega(12;34)$ which is proportional to the square of the transition matrix element. We expect $W_\omega(12;34)$ to be approximately of the form

$$W_\omega(12;34) \approx W_0(12;34)\mathcal{G}(\omega t_c) \ , \quad (6)$$

where $W_0(12;34)$ is the quantity entering in the standard treatment and the modulation factor $\mathcal{G}$ is proportional to the Fourier transform of the residual interaction responsible for the two-body scattering, with t_c denoting the duration of the particular two-particle encounter.[2] Since $\mathcal{G}(0)=1$ it follows that the stan-

[1] The situation may be rephrased as follows: The term $\delta K(\mathbf{r},\mathbf{p},t)$ in Eq. (1) is a stochastic function governed by a certain distribution function which depends on the phase-space density f, is zero on the average, and has a covariance tensor given by Eq. (2).

[2] For example, if the initial and final nucleon states are described by plane waves, as is appropriate in (locally) uniform matter, we would have $W_\omega(12;34) \sim |\langle\mathbf{p}_1\mathbf{p}_2|V(r_{12})|\mathbf{p}_3\mathbf{p}_4\rangle|^2 \sim |\tilde{V}(\Delta k)|^2$, where $\tilde{V}(k)$ is the Fourier transform of the two-body interaction $V(r)$. Thus, if this quantity is of Yukawa form, $V(r) \sim \exp(-r/a)/r$, then $W \sim [1+(a\Delta k)^2]^{-2} \approx \exp(-\frac{1}{2}\omega^2t_c^2)$. We have here used that $(\Delta\epsilon)^2 = \hbar^2\omega^2$ where the energy change is $\Delta\epsilon = \hbar\Delta\mathbf{k}\cdot\mathbf{v}$, with $\mathbf{v} = \hbar(\mathbf{k}+\mathbf{k}')/m$ being the (average) relative velocity. Furthermore, we have introduced a rough measure of the duration time t_c of the collision by $vt_c = 2a$.

dard theory is recovered by putting $C(\mathbf{p},\mathbf{p}';\omega)$ equal to $C(\mathbf{p},\mathbf{p}',\omega=0)=\hat{C}(\mathbf{p},\mathbf{p}')$.

In our present study we adopt the simple form $\mathcal{G}(\omega t_c) = \exp(-\omega^2 t_c^2/2)$, which expresses the expected suppression of the high-frequency components as a result of the finite duration of a two-particle encounter. The effective interaction range a may be estimated from the residual cross section $\pi a^2 \approx \sigma_{NN} \approx 40$ mb. Furthermore, the relative speed of two colliding nucleons is $v \approx \frac{3}{2} v_F$, since the active nucleons lie in the Fermi surface. We can then obtain a rough estimate of the duration time, $t_c \approx 2a/v \approx 5$–$6$ fm/c. This is a fairly brief length of time as compared with the typical free travel time for a nucleon, $t_{\rm mfp} \approx \lambda/v_F \approx 20$–$30$ fm/c. One would therefore expect that the treatment developed in Ref. [13] will still be applicable, but with suitably modified transport coefficients. We show below that this is indeed the case and that the source terms $\mathring{\mathcal{D}}_k^{\nu\nu'}$ responsible for the agitation of the unstable collective modes in nuclear matter are simply replaced by effective coefficients of the form $\mathcal{D}_k^{\nu\nu'}(t) = \mathring{\mathcal{D}}_k^{\nu\nu'} \chi_k^{\nu\nu'}(t)$. The formal developments are accompanied by numerical illustrations corresponding to typical scenarios.

III. UNSTABLE NUCLEAR MATTER

Following Ref. [13], we start from uniform nuclear matter having a momentum density distribution of Fermi-Dirac form, $f(\mathbf{r},\mathbf{p},t{=}0){=}f^0(\epsilon)$, and then consider the early development of the deviations caused by the stochastic part of the collision integral, $\delta f(\mathbf{r},\mathbf{p},t) = f(\mathbf{r},\mathbf{p},t) - f^0(\epsilon)$. The corresponding linearized Boltzmann-Langevin equation is then

$$\frac{\partial}{\partial t}\delta f + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\delta f - \frac{\partial}{\partial \mathbf{r}}\delta U\cdot\frac{\partial}{\partial \mathbf{p}} f^0 = \bar{I}_0 * \delta f + \delta K_0 \ , \quad (7)$$

where $\bar{I}_0$ is the linearized approximation to the average collision integral $\bar{K}$, and the stochastic term $\delta K_0(\mathbf{r},\mathbf{p},t) \equiv \delta K_0[f^0(\mathbf{r},\mathbf{p},t)]$ represents the rate of fluctuations generated in the specified initial state.

Because of the initial translational invariance, it is convenient to perform a Fourier transform with respect to the position $\mathbf{r}$:

$$\delta f(\mathbf{r},\mathbf{p},t) = \sum_{\mathbf{k}} \frac{1}{\sqrt{\Omega}} f_{\mathbf{k}}(\mathbf{p},t) e^{i\mathbf{k}\cdot\mathbf{r}} \ , \quad (8)$$

where Ω is the large volume of matter considered. The Fourier coefficients $f_k(\mathbf{p},t)$ are then governed by the corresponding transform of the above linearized equation,

$$\frac{\partial}{\partial t} f_{\mathbf{k}} + i\mathbf{k}\cdot\mathbf{v} f_{\mathbf{k}} - i\mathbf{k}\cdot\mathbf{v}\frac{\partial f^0}{\partial \epsilon}\frac{\partial U_k}{\partial \rho}\rho_{\mathbf{k}} = \bar{I}_0 * f_{\mathbf{k}} + K_{\mathbf{k}} \ . \quad (9)$$

Here $\rho_{\mathbf{k}}(t)$ and $\partial U_k/\partial\rho$ are the Fourier transforms of the induced density fluctuations $\delta\rho(\mathbf{r},t)$ and the associated change in the self-consistent potential δU, respectively, and $K_{\mathbf{k}}(\mathbf{p},t)$ is the Fourier transform of the fluctuating part of the collision term $\delta K_0(\mathbf{r},\mathbf{p},t)$. For example, $\rho_{\mathbf{k}}(t) = \Omega^{-1/2}\int d\mathbf{r}\, \exp(-i\mathbf{k}\cdot\mathbf{r})\delta\rho(\mathbf{r},t)$. We note that the spatial locality of the noise term in (1) causes the problem to decouple,

$$\prec K_{\mathbf{k}}(\mathbf{p})^* K_{\mathbf{k}}(\mathbf{p}') \succ = \delta_{\mathbf{k}\mathbf{k}'} C(\mathbf{p},\mathbf{p}';t-t') \ , \quad (10)$$

which simplifies the treatment significantly.

By summing the above equation of motion over the momentum $\mathbf{p}$, it is possible to obtain a dispersion relation determining the frequencies ω_k of the periodic solutions:

$$\frac{\partial U_k}{\partial \rho}\int \frac{d\mathbf{p}}{h^D}\frac{\mathbf{k}\cdot\mathbf{v}}{\mathbf{k}\cdot\mathbf{v}-\omega}\frac{\partial f^0}{\partial \epsilon} = 1 \ . \quad (11)$$

Thus, inside the appropriate spinodal zone, there are two imaginary solutions associated with a given wave vector $\mathbf{k}, \omega_k = \pm i/t_k = \pm i\gamma_k$. The possible collective modes can therefore be represented as

$$f_{\mathbf{k}}(\mathbf{p},t) = A_{\mathbf{k}}^+(t) f_{\mathbf{k}}^+(\mathbf{p}) + A_{\mathbf{k}}^-(t) f_{\mathbf{k}}^-(\mathbf{p}) \ , \quad (12)$$

where the eigenfunctions are of the form

$$f_{\mathbf{k}}^{\pm}(\mathbf{p}) = \frac{\partial U_k}{\partial \rho}\frac{\mathbf{k}\cdot\mathbf{v}}{\mathbf{k}\cdot\mathbf{v}\mp i\gamma_k}\frac{\partial f^0}{\partial \epsilon} \quad (13)$$

and have the normalization $h^{-D}\int d\mathbf{p}\, f_{\mathbf{k}}^{\pm}(\mathbf{p}) = 1$.

A. Random-phase approximation treatment

The agitation of unstable collective modes in nuclear matter was treated by Colonna, Chomaz, and Randrup [13], by performing an orthogonal projection of the dynamics onto the space spanned by the collective modes. In the present study we shall employ a simpler treatment which can be derived by considering the time-dependent response function and ignoring the single-particle poles, in analogy to what is done in random-phase approximation (RPA) treatments [17]. These results can readily be transformed into the form appropriate for the treatment given in Ref. [13]. The two methods represent different approximations and they are identical if the dynamics is entirely collective, in which case they yield the correct evolution of the system, within the general bounds of linear-response theory.

We then define the two auxiliary functions

$$Q_{\mathbf{k}}^{\pm}(\mathbf{p}) = \frac{\mathcal{N}_k}{\gamma_k \mp i\mathbf{k}\cdot\mathbf{v}} \ , \quad (14)$$

which have the following convenient orthonormality property:

$$\int \frac{d\mathbf{p}}{h^D} Q_{\mathbf{k}}^{\nu}(\mathbf{p})^* f_{\mathbf{k}}^{\nu'}(\mathbf{p}) = \delta_{\nu\nu'} \ , \quad (15)$$

provided that the normalization constant $\mathcal{N}_k$ is taken as

$$\mathcal{N}_k = \left[\frac{\partial U_k}{\partial \rho}\frac{2}{t_k}\int \frac{d\mathbf{p}}{h^D}\frac{(\mathbf{k}\cdot\mathbf{v})^2}{[(\mathbf{k}\cdot\mathbf{v})^2+\gamma_k^2]^2}\frac{\partial f^0}{\partial \epsilon}\right]^{-1} \ . \quad (16)$$

Insertion of the collective form (12) into the equation of motion (9) and subsequently projecting onto the functions $Q^{\nu}_{\mathbf{k}}(\mathbf{p})$ leads to the following equations of motion for the collective amplitudes $A^{\nu}_{\mathbf{k}}(t)$:

$$\frac{d}{dt}A^{\nu}_{\mathbf{k}}(t) = \frac{\nu}{t_k}A^{\nu}_{\mathbf{k}}(t) + F^{\nu}_{\mathbf{k}}(t) , \tag{17}$$

if the average part of the collision term is neglected, as is justified by the fact that its effect on the eigenfrequency ω_k is rather small [18]. Moreover, the effective (fluctuating) collective forces are given by

$$F^{\nu}_{\mathbf{k}}(t) = \int \frac{d\mathbf{p}}{h^D} Q^{\nu}_{\mathbf{k}}(\mathbf{p})^* K_{\mathbf{k}}(\mathbf{p},t) . \tag{18}$$

These quantities vanish on the average, $\prec F^{\nu}_{\mathbf{k}}(t) \succ = 0$, and the associated correlation function has the following spectral resolution:

$$\begin{aligned}\prec F^{\nu}_{\mathbf{k}}(t)^* F_{\mathbf{k}'}(t') \succ &= \delta_{\mathbf{k}\mathbf{k}'} \int \frac{d\omega}{2\pi} e^{-i\omega(t-t')} \tilde{C}^{\nu\nu'}_{k}(\omega) \\ &= C^{\nu\nu'}_{k}(t-t') , \end{aligned} \tag{19}$$

where the collective correlation kernel is given by

$$\begin{aligned}\tilde{C}^{\nu\nu'}_{k}(\omega) &= \int \frac{d\mathbf{p}}{h^D} \int \frac{d\mathbf{p}'}{h^D} Q^{\nu}_{\mathbf{k}}(\mathbf{p}) \tilde{C}(\mathbf{p},\mathbf{p}';\omega) Q^{\nu'}_{\mathbf{k}}(\mathbf{p}')^* \\ &= \frac{1}{2} \int \frac{d\mathbf{p}_1}{h^D} \int \frac{d\mathbf{p}_2}{h^D} \int \frac{d\mathbf{p}_3}{h^D} \int \frac{d\mathbf{p}_4}{h^D} \Delta Q^{\nu}_{\mathbf{k}} (\Delta Q^{\nu'}_{\mathbf{k}})^* \\ &\quad \times W(12;34;\omega) f_1 f_2 \bar{f}_3 \bar{f}_4 , \end{aligned} \tag{20}$$

with

$$\Delta Q^{\nu}_{\mathbf{k}} \equiv Q^{\nu}_{\mathbf{k}}(\mathbf{p}_3) + Q^{\nu}_{\mathbf{k}}(\mathbf{p}_4) - Q^{\nu}_{\mathbf{k}}(\mathbf{p}_1) - Q^{\nu}_{\mathbf{k}}(\mathbf{p}_2) \tag{21}$$

representing the change in the observable $Q^{\nu}_{\mathbf{k}}(\mathbf{p})$ due to the collision process $12 \to 34$.

The four momentum integrations in the expression (20) for the correlation kernel are constrained by the energy-conserving δ function contained in the basic transition rate $w(12;34;\omega)$ given in Eq. (5). For temperatures small in comparison with the Fermi energy, $T \ll \epsilon_F$, the integrand is effectively confined to the region near the Fermi surface, due to the appearance of the factor $f_1 f_2 \bar{f}_3 \bar{f}_4$. When, furthermore, the energy exchange is small as well, $\hbar\omega \ll \epsilon_F$, then the energy and angular parts of the integrations approximately decouple. This feature was exploited recently for the derivation of simple analytical approximation to the Boltzmann-Langevin transport coefficients [19], utilizing methods developed in condensed matter physics [20]. Proceeding in a similar manner, we obtain

$$\begin{aligned}\int_0^\infty d\epsilon_1 \int_0^\infty d\epsilon_2 \int_0^\infty d\epsilon_3 \int_0^\infty d\epsilon_4 f_1 f_2 \bar{f}_3 \bar{f}_4 \delta(\Delta\epsilon \mp \hbar\omega) \\ \approx \pm \frac{\hbar\omega}{6} \frac{(\hbar\omega)^2 + (2\pi T)^2}{1 - \exp(\mp\hbar\omega/T)} . \end{aligned} \tag{22}$$

This approximation is good when both the temperature T and the characteristic energy $\hbar\omega$ are small in comparison with the Fermi energy ϵ_F. However, the correlation kernel $\tilde{C}^{\nu\nu'}_{k}(\omega)$ given in(2) involves frequencies well above the Fermi level, and so we need to improve the above approximation somewhat. (It should also be noted that the Fermi energy is reduced by more than a factor of 2 in that part of the spinodal region where the fastest-growing modes occur, $\rho \approx 0.3\rho_0$.) On the basis of numerical evaluations of the constrained integral in (22), we have found that the above approximation can be extended towards high frequencies by dividing by the factor $[1 + \frac{1}{6}(\hbar\omega/\epsilon_F)^2/(1 + 2(T/\epsilon_F)^2)]$. The ω dependence is then correct to better than 10% up to well above $\hbar\omega = 100$ MeV, which is fully adequate for our present purposes.

It is then possible to factor out the ω dependence of the correlation kernel,

$$\tilde{C}^{\nu\nu'}_{k}(\omega) \approx 2\mathring{\mathcal{D}}^{\nu\nu'}_{k} \tilde{\chi}(\omega) , \tag{23}$$

where $2\mathring{\mathcal{D}}^{\nu\nu'}_{k} = \tilde{C}^{\nu\nu'}_{k}(\omega = 0)$ is the collective diffusion co-

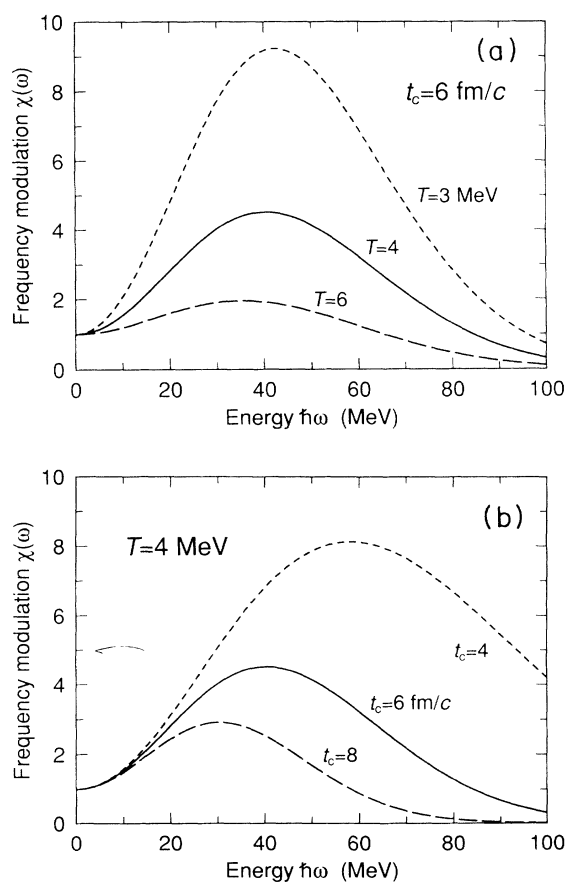

FIG. 1. Frequency modulation. The function $\tilde{\chi}(\omega)$ expressing the frequency modulation of the collective correlation kernels [see Eq. (23)]. (a) explores a range of temperatures $T = 3, 4, 6$ MeV for a fixed value of the memory time $t_c = 6$ fm/c; (b) keeps the temperature fixed at $T = 4$ MeV while varying the memory time $t_c = 4, 6, 8$ fm/c. The density is kept at the value $\rho = 0.3\rho_0$ near which the fastest growth occurs [14,15].

efficient (the "source term") obtained when the collision kernel has no memory, corresponding to the study made in Ref. [13]. The frequency modulation is conveniently given by the function

$$\tilde{\chi}(\omega) \approx \frac{1+\left(\frac{\hbar\omega}{2\pi T}\right)^2}{1+\frac{1}{6}\left(\frac{\hbar\omega}{\epsilon_F}\right)^2 / \left[1+2\left(\frac{T}{\epsilon_F}\right)^2\right]} \frac{\hbar\omega}{2T}$$

$$\times \coth\left(\frac{\hbar\omega}{2T}\right) e^{-(1/2),\omega^2 t_c^2} = \mathcal{F}_\rho\left(\frac{\hbar\omega}{2T}\right) \mathcal{G}(\omega t_c) \ . \quad (24)$$

This key function is displayed in Fig. 1 for a range of temperatures T and for several different assumptions about the duration time t_c. The function generally starts out at $\tilde{\chi}(\omega = 0) = 1$ and initially exhibits an increase reflecting $\mathcal{F}_\rho(\hbar\omega/2T)$, before ultimately subsiding due to the strong decay of $\mathcal{G}(\omega t_c)$. For our standard choice of the memory time, $t_c = 6$ fm/c, the peak is situated near $\hbar\omega \approx 40$ MeV. Naturally, the peak shifts inversely in response to changes in t_c. It should also be noted that the peak broadens at the temperature T is raised, in accordance with our expectation that a classical description (i.e., no frequency dependence, as in the standard BL model) should emerge at high temperatures.

The Fourier transform of the frequency modulation function

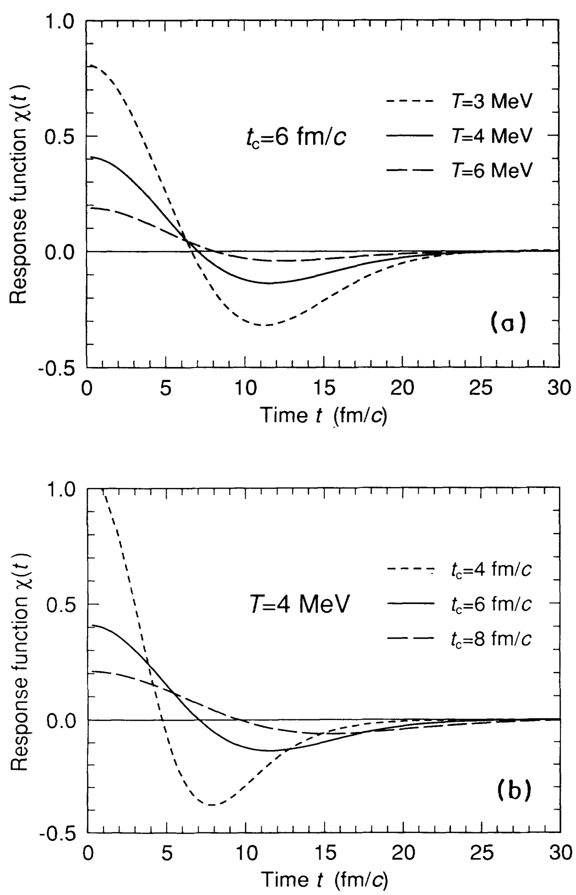

FIG. 2. Response function. The response function $\chi(t)$ given in Eq. (25), for the same scenarios as in Fig. 1.

$$\chi(t) = \int \frac{d\omega}{2\pi} e^{-i\omega t} \tilde{\chi}(\omega) = \frac{C(\mathbf{p},\mathbf{p}';t)}{\hat{C}(\mathbf{p},\mathbf{p}')} \ , \quad (25)$$

is the time-dependent response function for the system. This quantity is shown in Fig. 2 for the same scenarios as considered in Fig. 1. Starting out from $\chi(t = 0) > 0$, the response function exhibits an oscillatory decay. The first sign change occurs for $t \approx t_c$, and typically only the first minimum is noticeable. As expected, the time dependence of the response function grows gentler as either the memory time or the temperature is increased.

B. The collective evolution

The equation of motion (17) for the collective amplitudes can readily be solved formally,

$$A_{\mathbf{k}}^{\nu}(t) = e^{\nu\gamma_k t}\left[A_{\mathbf{k}}^{\nu}(0) + \int_0^t dt' e^{-\nu\gamma_k t'} F_{\mathbf{k}}^{\nu}(t')\right] \ . \quad (26)$$

In the presently considered scenario, the initial system consists of uniform nuclear matter and so the initial amplitudes all vanish, $A_{\mathbf{k}}^{\nu}(0) = 0$. Since the random forces vanish on the average, $\prec F_{\mathbf{k}}^{\nu}(t) \succ = 0$, it then follows that the amplitudes remain zero on the average, $\prec A_{\mathbf{k}}^{\nu}(t) \succ = 0$.

However, each individual history displays a random evolution of $A_{\mathbf{k}}^{\nu}(t)$ and the development of the average magnitude of the amplitudes is, as usual, conveniently described by the associated covariance coefficients

$$\begin{aligned}\sigma_k^{\nu\nu'}(t) &\equiv \prec A_{\mathbf{k}}^{\nu}(t)^* A_{\mathbf{k}}^{\nu'}(t) \succ \\ &= e^{(\nu+\nu')\gamma_k t} \int_0^t dt' \int_0^t dt'' e^{-\nu\gamma_k t' - \nu'\gamma_k t''} \\ &\quad \times \prec F_{\mathbf{k}}^{\nu}(t')^* F_{\mathbf{k}}^{\nu'}(t'') \succ \ , \end{aligned} \quad (27)$$

which depend only on the magnitude k of the wave number $\mathbf{k}$, due to the isotropy of the initial state.

When the memory time vanishes the correlation function in the integrand of (27) is proportional to $\delta(t'-t'')$ and the scenario considered in Ref. [13] emerges. The covariance coefficients $\mathring{\sigma}_k^{\nu\nu'}$ then satisfy the following simple equation of motion:

$$\frac{d}{dt}\mathring{\sigma}_k^{\nu\nu'}(t) = 2\mathring{\mathcal{D}}_k^{\nu\nu'} + \frac{\nu+\nu'}{t_k}\mathring{\sigma}_k^{\nu\nu'}(t) \ , \quad (28)$$

which has the solution

$$\mathring{\sigma}_k^{\nu\nu'}(t) = 2\mathring{\mathcal{D}}_k^{\nu\nu'} e^{(\nu+\nu')\gamma_k t} \int_0^t dt' e^{-(\nu+\nu')\gamma_k t'} \ , \quad (29)$$

recalling that the initial fluctuations vanish, $\mathring{\sigma}_k^{\nu\nu'}(0) = 0$.

With a finite memory time, we need instead to insert the nontrivial resolution (19) into the integrand of (27). It is then useful to introduce the following pair of correlation functions for a given collective mode:

$$\begin{aligned}\chi_k^{\pm}(t) &\equiv \int_0^t dt' e^{\pm\gamma_k(t-t')} \chi(t-t') \\ &= \int_0^t dt' \int \frac{d\omega}{2\pi} e^{(\pm\gamma_k + i\omega)(t-t')} \tilde{\chi}(\omega) \ , \end{aligned} \quad (30)$$

and to adopt a convenient notation for the sums, $\chi_k^{\nu\nu'}(t) \equiv \chi_k^{\nu}(t) + \chi_k^{\nu'}(t)$. The functions $\chi_k^{\nu}(t)$ are illustrated in Fig. 3 for a few typical temperatures, employing the standard value $t_c = 6$ fm/c and considering the density $\rho = 0.3\rho_0$ for which the fastest growth of the unstable modes occurs. The growth times have been chosen to represent the fastest mode for the specified temperature and density, $t_k = 24, 36$ fm/c for $T = 4, 6$ MeV, respectively. These functions start out from zero, display a first maximum at $t \approx t_c$, and then approach a constant limiting value. We note that for large times $\chi_k^{+}(t)$ is generally below $\chi_k^{-}(t)$, because the exponentially increasing weight factor in the integrand for χ_k^{+} enhances the effect of the first negative part of $\chi(t)$. Whereas the limiting value $\chi_k^{+}(\infty)$ may then become negative, $\chi_k^{-}(\infty)$ is always positive.

It is now easy to see that when a memory time is included the collective correlation coefficients (27) satisfy the following modified equation of motion:

$$\frac{d}{dt}\sigma_k^{\nu\nu'}(t) = 2\mathring{\mathcal{D}}_k^{\nu\nu'}\chi_k^{\nu\nu'}(t) + \frac{\nu+\nu'}{t_k}\sigma_k^{\nu\nu'}(t) . \qquad (31)$$

This equation is of the same general form as (28), but it has a time-dependent diffusion coefficient, $\mathcal{D}_k^{\nu\nu'}(t) = \mathring{\mathcal{D}}_k^{\nu\nu'}\chi_k^{\nu\nu'}(t)$. The solution is then given by

$$\sigma_k^{\nu\nu'}(t) = 2\mathring{\mathcal{D}}_k^{\nu\nu'}e^{(\nu+\nu')\gamma_k t}\int_0^t dt' e^{-(\nu+\nu')\gamma_k t'}\chi_k^{\nu\nu'}(t')$$
$$= \mathring{\sigma}_k^{\nu\nu'}(t)\bar{\chi}_k^{\nu\nu'}(t) , \qquad (32)$$

where the renormalization coefficients can be expressed as suitable time averages of the collective correlation functions

$$\bar{\chi}_k^{\nu\nu'}(t) = \frac{\int_0^t dt' e^{-(\nu+\nu')\gamma_k t'}[\chi_k^{\nu}(t') + \chi_k^{\nu'}(t')]}{\int_0^t dt' e^{-(\nu+\nu')\gamma_k t'}} . \qquad (33)$$

This result is readily obtained by multiplying and dividing (32) by the denominator in (33). Since $\chi_k^{\nu\nu'}(t)$ approaches a constant in time, the weighted average $\bar{\chi}_k^{\nu\nu'}(t)$ also approaches a constant value as the upper limit t grows large; we denote this limiting value by simply $\bar{\chi}_k^{\nu\nu'}$.

Obviously the two mixed correction functions are identical, $\bar{\chi}_k^{+-}(t) = \bar{\chi}_k^{-+}(t)$, and it is elementary to demonstrate that the two diagonal correction functions are identical as well, $\bar{\chi}_k^{++}(t) = \bar{\chi}_k^{--}(t)$ (first change the order of

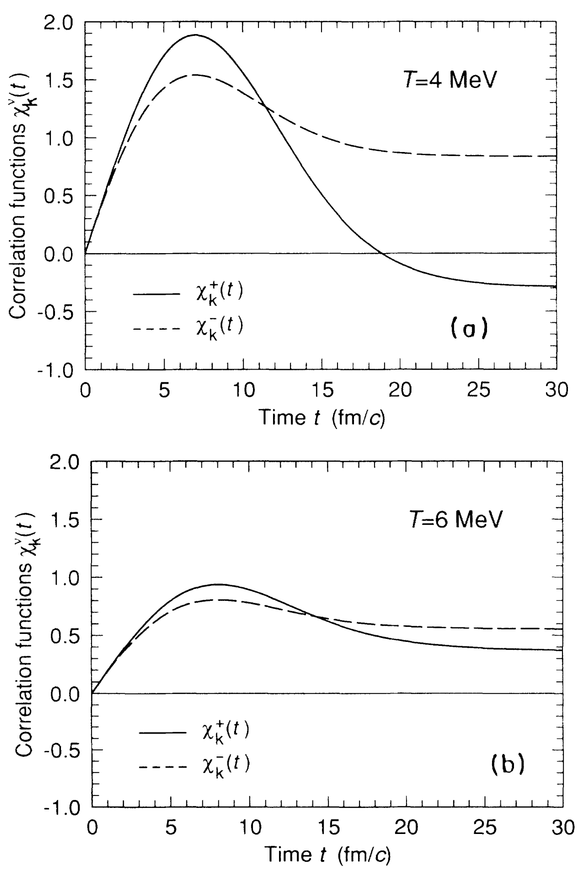

FIG. 3. Collective correlation functions. The correlation functions $\chi_k^{+}(t)$ (solid) and $\chi_k^{-}(t)$ (dashed) associated with unstable collective modes having a wave number of magnitude ki, as given in Eq. (3). (a) $T = 4$ MeV and $t_k = 24$ fm/c; (b) $T = 6$ MeV and $t_k = 36$ fm/c. The amplification times t_k are the fastest ones at the particular temperature [15].

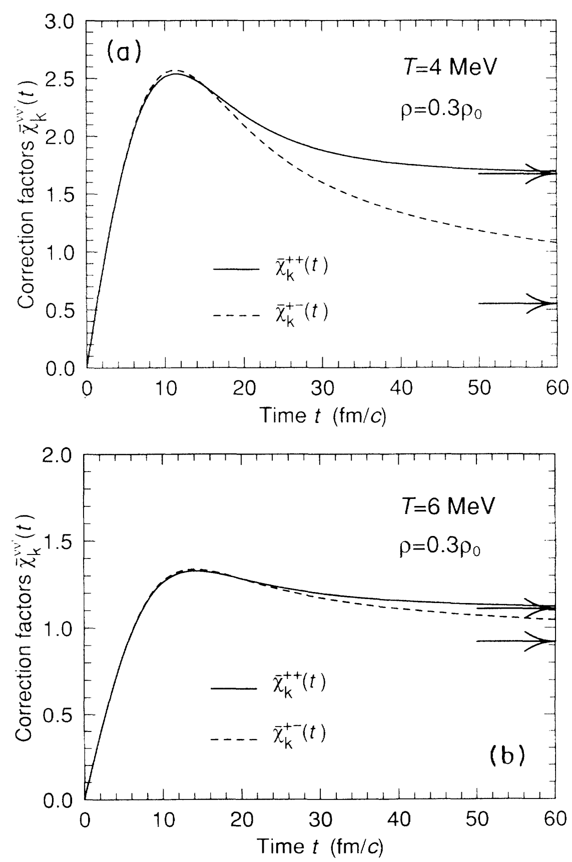

FIG. 4. Correction factors. The correction factors $\bar{\chi}_k^{\nu\nu'}$ determining the time dependence of the covariance coefficients describing the agitation of collective modes in unstable nuclear matter, for the same two cases as in Fig. 3. The arrows indicate the asymptotic values to which the factors tend at large times.

integration and then replace the integration variables t' and t'' by $t-t'$ and $t-t''$, respectively). These functions are shown in Fig. 4, for the same cases as considered in Fig. 3. It is important to note that the approach towards the limiting values occurs fairly slowly, especially for the mixed coefficient $\bar{\chi}_k^{+-}(t)$, for which the weight factor is a constant. It is therefore useful to note that the weighted time averages in (33) may be related to the long-time behavior of the integrands $\chi_k^{\nu}(t)$, since those asymptotic values are attained significantly earlier,

$$\bar{\chi}_k^{++} = \bar{\chi}_k^{--} = 2 \lim_{t\to\infty} [\chi_k^{-}(t)] > 0 \ , \tag{34}$$

$$\bar{\chi}_k^{+-} = \bar{\chi}_k^{-+} = \lim_{t\to\infty} [\chi_k^{+}(t) + \chi_k^{-}(t)] \ . \tag{35}$$

It also follows that the mixed coefficient approaches a smaller value than does the diagonal one, $\bar{\chi}_k^{+-} < \bar{\chi}_k^{++}$. Finally, we note that the diagonal coefficient can be written on a simple alternative form directly in terms of the frequency modulation function $\tilde{\chi}(\omega)$:

$$\bar{\chi}_k^{\nu\nu} = t_k \int \frac{d\omega}{2\pi} \frac{\tilde{\chi}(\omega)}{1+\omega^2 t_k^2} \ . \tag{36}$$

It follows from the above analysis that at sufficiently large times, when the correction factors $\bar{\chi}_k^{\nu\nu'}(t)$ have attained their limiting values $\bar{\chi}_k^{\nu\nu'}$, the behavior of the covariance coefficients $\sigma_k^{\nu\nu'}(t)$ is equal to what would result with a vanishing memory time, except that the time-independent source terms $\mathring{\mathcal{D}}_k^{\nu\nu'}$ are replaced by the renormalized quantities $\mathring{\mathcal{D}}_k^{\nu\nu'} \chi(\infty)_k^{\nu\nu'}$ which are also independent of time. (Of course, as the time keeps increasing, the collective amplitudes will ultimately grow beyond the linear-response domain and the analysis breaks down.) The limiting values $\bar{\chi}_k^{\nu\nu'}$ therefore give a good indication of the magnitude of the memory-time effect on the growth of unstable modes.

Table I displays calculated values of the correction factors $\bar{\chi}_k^{\nu\nu'}$ corresponding to the fastest-growing collective mode on a grid in the density-temperature phase plane. Generally the diagonal renormalization coefficient exceeds unity, whereas the mixed coefficient is less than 1. Consequently, the agitation of the two collective modes is enhanced, while their coupling is suppressed. Near $\rho = 0.3\rho_0$ and $T = 4$ MeV these effects amount to about 50%, while they fall off as the temperature is increased.

TABLE I. Correction factors. The correction factors $\bar{\chi}_k^{++}$ and $\bar{\chi}_k^{+-}$ for the effective diffusion coefficients $\mathcal{D}_k^{\nu\nu'}$ governing the spontaneous agitation of collective modes with wave number k in unstable nuclear matter prepared at a uniform density ρ and having a specified temperature T. For the indicated grid in ρ and T, the correction factors $\bar{\chi}_k^{\nu\nu'}$ for the fastest mode are displayed [15]. The duration time has been taken to be $t_c = 6$ fm/c.

$\bar{\chi}_k^{++}/\bar{\chi}_k^{+-}$	$\rho = 0.2\rho_0$	$\rho = 0.3\rho_0$	$\rho = 0.4\rho_0$	$\rho = 0.5\rho_0$
$T = 6$ MeV	1.06/0.94	1.11/0.92	1.14/0.91	1.10/0.95
$T = 5$ MeV	1.20/0.84	1.29/0.79	1.30/0.84	1.23/0.91
$T = 4$ MeV	1.50/0.64	1.67/0.55	1.66/0.69	1.46/0.85
$T = 3$ MeV	2.29/0.11	2.66/0.00	2.49/0.41	2.03/0.71

IV. DISCUSSION

The present investigation has focused on the effect of a finite memory time in the stochastic term of the nuclear Boltzmann-Langevin equation on the agitation of collective modes in unstable nuclear matter, by applying a recently proposed extension of the BL model [12] in conjunction with the previously developed transport treatment of spinodal matter [13]. In the standard BL model, the stochastic term is local in time, as would be appropriate if the two-body collisions can be considered as instantaneous. Under such idealized circumstances, density undulations are generated only indirectly as the local rearrangements in momentum space are propagated by the mean field. By contrast, the extended BL model maintains the random force for a finite length of time and thereby provides a direct coupling between the two-body collision process and the collective modes.

The finite memory time introduces a certain gentleness into the source term which is reflected in the suppression of high-frequency components, as expressed by the modulation function $\mathcal{G}(\omega t_c)$. However, since $t_c \ll t_k$ this has little effect on the agitation of the collective modes. Indeed, the factor $\mathcal{G}(\omega_k t_c)$ produces a reduction of only about 2% for the fastest mode, so it is apparent that this effect is not essential.

Much more important is the quantum-statistical enhancement expressed by the factor $\mathcal{F}_\rho(\hbar\omega/2T)$. (In the case of stable collective modes, the factor $\mathcal{F}$ guarantees that the appropriate quantum-statistical equilibrium is approached [12], whereas the standard BL treatment leads to a classical (Boltzmann) equilibrium occupation of the collective modes, $P_k \sim \exp(-\hbar\omega_k/T)$.) Since the characteristic energy $E_k = \hbar/t_k$ exceeds the temperature T, the factor $\mathcal{F}$ causes a significant enhancement of the collective source terms $\mathcal{D}_k$ and, consequently, the density undulations will grow correspondingly larger in the course of a given time interval. The effect depends strongly on the temperature T and the growth time t_k, but for the fastest mode and the most typical temperatures, the enhancement factor is 50–100 %, as shown in Table I.

The memory time presents the shortest time scale in the problem, in so far as it is expected to be given approximately by the duration of an individual two-body collision, $t_c \approx a/v_F \approx 6$ fm/c (twice the range divided by the relative speed, which is about the Fermi speed since the collisions only involve states in the Fermi surface). This is considerably shorter than the growth time for even the fastest unstable mode, which is about $t_k \approx 20$ fm/c. It is also fairly short compared to the free propagation time of a nucleon between collisions, $t_{\text{free}} \approx \lambda/v_F \approx 20$ fm/c. One might then expect that the transport process would retain a Markovian character.

However, the finite memory time in the microscopic collision kernel gives rise to a nontrivial modulation of the effective source terms that agitate the unstable modes in the spinodal phase domain, $\mathcal{D}_k^{\nu\nu'}(t) = \mathring{\mathcal{D}}_k^{\nu\nu'}\chi_k^{\nu\nu'}(t)$. As a consequence, the evolution of the covariance coefficients $\sigma_k^{\nu\nu'}(t)$ then deviates from what would be obtained without a memory time, as expressed by the time-dependent correction factors $\bar{\chi}_k^{\nu\nu'}(t)$, of which examples are shown in Fig. 4. These factors can deviate significantly from unity, particularly in the domain where the fastest growth occurs. It therefore appears important to incorporate such memory effects in BL simulations, especially in the presence of instabilities.

The correction factors ultimately attain constant values, and the evolution is then similar to what the standard treatment would give, except for the time-independent renormalization of the source terms. But this limiting simplicity emerges only relatively slowly, particularly for the mixed factor, as is evident from the illustration in Fig. 4.

The present analysis has been confined to the idealized scenario of initially uniform nuclear matter, which can be subjected to near-analytic treatment by previously developed methods. Nevertheless, our conclusions are expected to hold for more complicated dynamical scenarios, such as may be encountered in nuclear collisions. Thus, if qualitatively reliable results are to be obtained from numerical simulations based on the BL model, it appears necessary to refine the treatment to take account of the memory time in an appropriate manner.

ACKNOWLEDGMENTS

S.A. thanks the NSD Theory Group at LBL for its hospitality while the major part of this work was carried out. This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Nuclear Physics Division of the U.S. Department of Energy under Contracts No. DE-AC03-76SF00098 and No. DE-FG05-89ER40530.

[1] S. Ayik and C. Gregoire, Phys. Lett. B **212**, 269 (1988); Nucl. Phys. **A513**, 187 (1990).
[2] W. Cassing and U. Mosel, Prog. Part. Nucl. Phys. **25**, 235 (1990).
[3] P. Bonche, S. E. Koonin, and J. W. Negele, Phys. Rev. C **13**, 226 (1979).
[4] L. W. Nordheim, Proc. R. Soc. London Ser. A **119**, 689 (1928).
[5] E. A. Ühling and G. E. Uhlenbeck, Phys. Rev. **43**, 552 (1933).
[6] P. Schuck, R. W. Hasse, J. Jaenicke, C. Gregoire, B. Remaud, F. Sebille, and E. Suraud, Prog. Part. Nucl. Phys. **22**, 181 (1989).
[7] W. Cassing, V. Metag, U. Mosel, and K. Niita, Phys. Rep. **188**, 363 (1990).
[8] J. Randrup and B. Remaud, Nucl. Phys. **A514**, 339 (1990).
[9] G. F. Burgio, Ph. Chomaz, and J. Randrup, Phys. Rev. Lett. **69**, 885 (1992).
[10] S. Ayik and M. Dworzecka, Phys. Rev. Lett. **54**, 534 (1985); Nucl. Phys. **A440**, 424 (1985).
[11] S. Ayik and D. Boilly, Phys. Lett. B **276**, 263 (1992); **284**, 482(E) (1992).
[12] S. Ayik, Z. Phys. A **350**, 45 (1994).
[13] M. Colonna, Ph. Chomax, and J. Randrup, Nucl. Phys. **A567**, 637 (1994).
[14] M. Colonna and Ph. Chomaz, Phys. Rev. C **49**, 1908 (1994).
[15] J. Randrup, in *Nucleus-Nucleus Collisions V*, Taormina, Italy, 1994 [Nucl. Phys. A (to be published)].
[16] D. Kiderlen and H. Hofmann, Phys. Lett. B (to be published).
[17] E. M. Lifshitz and L. P. Pitaevskiĭ, *Physical Kinetics* (Pergamon, New York, 1981), p. 209; S. Ayik, Ph. Chomaz, M. Colonna, and J. Randrup, Report No. LBL-35987 (in preparation).
[18] J. Randrup, Report No. LBL-35848 (in preparation).
[19] J. Randrup and S. Ayik, Nucl. Phys. **A572**, 489 (1994).
[20] G. Baym and C. Pethick, in *The Physics of Liquid and Solid Helium, Part II*, edited by K. H. Bennemann and J. B. Ketterson (Wiley, New York, 1976), p. 1.