Isospin dependence of double analog cross sections at $T_\pi = 400$–500 MeV

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At energies in the range 400–500 MeV, pion-induced double charge exchange cross sections leading to the double analog state appear to behave differently for even and odd isospin.

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Pion double charge exchange (DCX) reactions leading to the double isobaric analog state (DIAS) have now been investigated over a wide range of incident pion energies—from 35 to 550 MeV. Most extensive data [1–17] are at 292 MeV, but DIAS cross sections for many nuclei have now been measured [18–20] for $T_\pi = 300$–550 MeV. Published high-energy DIAS cross sections are listed in Table I. These are all for a laboratory angle of 5°, and $T_\pi = 400$–500 MeV.

For energies of 35–292 MeV, the DIAS cross section is observed to vary greatly with incident pion energy. But whenever cross sections for a given nucleus have been measured for more than one energy in the range 400–500 MeV, there is little evidence for variation with energy. This fact is demonstrated in Fig. 1. In the main part of the figure we plot, for the six nuclei with two or more data points, the quantity $\chi^2 = \frac{1}{N-1}\sum_E \left(\frac{\sigma(E)-\bar{\sigma}}{\Delta\sigma(E)}\right)^2$, where the σ's are all from Table I. All the values of χ^2 are significantly less than unity, except for ^{48}Ca. We have also fitted these cross sections to a linear energy dependence: $\sigma(E) = a\frac{T_\pi}{100} + b$, and plotted the quantities $a/\bar{\sigma}$ in the inset of Fig. 1. We conclude there is no statistically significant evidence for energy dependence in these cross sections.

Hence, to increase statistical accuracy, we can energy-average cross sections for any one nucleus. These are included in the last column of Table I. The simplest expression for the N, Z, A dependence of the DIAS cross sections at forward angles is [21]

$$\sigma = g\frac{(N-Z)(N-Z-1)}{2A^\alpha}. \tag{1}$$

Strong absorption, plus Δ dominance, give [21] $\alpha = 10/3$. Realistic calculations suggest α is in the range 2.33–4, with largest values resulting from the use of Woods-Saxon wave functions with realistic binding energies [22]. At $T_\pi = 292$ MeV, an empirical fit to all the data gives [23] $\alpha = 3.24$. However, if the fit is restricted to $T = 1$ only, $\alpha = 2.33$ provides an excellent fit [1,5]. Of course, if zero-degree cross sections go as $A^{-\alpha}$, cross sections at 5° will fall even faster—because (in a diffractive reaction with zero angular momentum transfer) the ratios $\sigma(5°)/\sigma(0°)$ fall with A—roughly as $A^{-0.3}$ here.

Auerbach *et al.* [22] derived simple corrections to the above simple expression, in a seniority model. They also provide a convenient understanding of why $T = 1$ might be different from other isospins. Their model is expected to be applicable within a given shell-model orbital, e.g., $1f_{7/2}$, or perhaps somewhat more globally within a major shell, if generalized seniority is a good approximation. They do not suggest that DIAS amplitudes derived in one mass range might apply to all regions of A. In the present work, our aim is to investigate the isospin dependence more globally. Toward this end, we have fitted the high-

TABLE I. DIAS cross sections at $\theta_L = 5°$, for $T_\pi = 400$–500 MeV.

T	Target	$d\sigma/d\Omega$ (μb/sr) 400 MeV	450 MeV	500 MeV	$\langle d\sigma/d\Omega\rangle$ (μb/sr) (400–500 MeV)
1	^{14}C	3.14±0.39		3.62±0.65	3.27 ± 0.33
	^{18}O	3.06 ± 0.29	2.94 ± 0.33	2.69 ± 0.35	2.92 ± 0.18
	^{26}Mg			1.01±0.32	
	^{42}Ca	0.694±0.146	0.783±0.204	0.755±0.256	0.730 ± 0.108
	^{46}Ti		0.151±0.065		
	^{54}Fe		0.153±0.088		
2	^{44}Ca	1.01±0.184		0.813±0.205	0.922 ± 0.137
	^{52}Cr		0.738±0.143		
	^{56}Fe			0.431±0.094	
3	^{50}Ti		2.22±0.52		
4	^{48}Ca	2.14±0.48	1.46±0.39	2.95±0.77	1.89 ± 0.283
5	^{90}Zr			1.27±0.77	
6	^{80}Se	1.27 ± 0.15		1.55 ± 0.28	1.33 ± 0.128

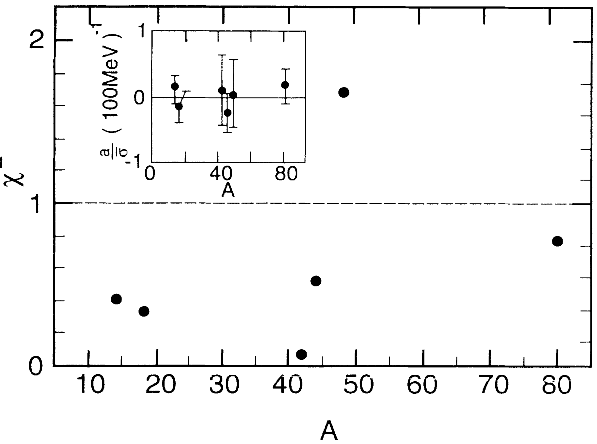

FIG. 1. The quantity $\chi^2 = \frac{1}{N-1}\sum_E \left(\frac{\sigma(E)-\bar{\sigma}}{\Delta\sigma(E)}\right)^2$ is plotted vs A for six nuclei. Values of σ are from Table I. Inset: The quantity $a/\bar{\sigma}$ is plotted vs A, where $\sigma(E) = a\frac{T_\pi}{100} + b$.

energy cross sections with the simple expression (1).

As for 292 MeV, we find that $T = 1$ nuclei require a separate fit, and that $\alpha = 2.37 \pm 0.22$ works best. We also note that cross sections for two $T = 1$ nuclei—viz. ^{18}O and ^{42}Ca—are significantly larger than the general trend. In Fig. 2, we plot the quantities $\sigma' = \sigma\left(\frac{A}{42}\right)^{7/3}$ vs A for $T = 1$ and 2. These are the only values of isospin for which 400–500 MeV DIAS cross sections have been measured for more than one nucleus. We note that $A^{-7/3}$ is reasonable also for $T = 2$.

In the next stage of our analysis we investigate the isospin dependence of the quantities $\sigma(\frac{A}{A_0})^{7/3}$; we average $\sigma(\frac{A}{A_0})^{7/3}$ separately for $T = 1$ and 2. These averages are listed in Table II along with the values for $T = 3$–6. We plot these quantities vs $T(2T-1)$ in Fig. 3. Surprisingly, there is an apparent difference in behavior for even and odd T. For $T = 1$, we give results with and without ^{18}O and ^{42}Ca.

A fit reveals this difference: If we let $y_T = \sigma_0 + \beta T(2T-1)$, odd T cross sections give $\sigma_0 = 0.072 \pm 0.036$ μb/sr, $\beta = 0.173 \pm 0.028$ μb/sr, whereas even T has $\sigma_0 = 0.51 \pm 0.10$ μb/sr, $\beta = 0.082 \pm 0.008$ μb/sr. In other words, even-T σ's increase only half as rapidly with

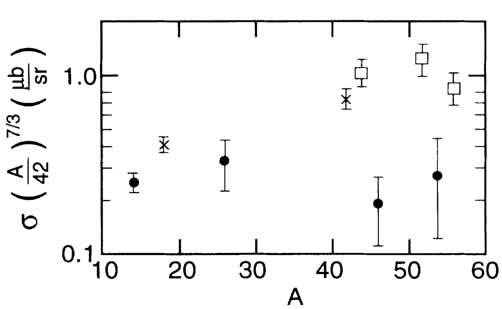

FIG. 2. Plots of $\sigma(A/42)^{7/3}$ vs A for ^{18}O, ^{42}Ca (crosses), other $T = 1$ nuclei (circles) and $T = 2$ (squares).

TABLE II. Values of $y_T \equiv \langle\sigma(\theta)(\frac{A}{42})^{7/3}\rangle$ for DIAS cross sections at high energy.

T	y_T (μb/sr)
1	0.254±0.023[a]
2	0.992±0.104
3	3.26±0.77
4	2.58±0.39
5	7.52±1.65
6	5.98±0.58

[a]This is the value without ^{18}O and ^{42}Ca. If they are included, y_1 is 0.332 ± 0.017.

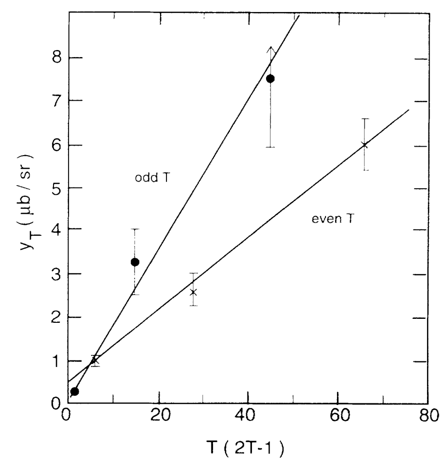

FIG. 3. The quantity y_T from Table II is plotted vs $T(2T-1)$ for odd T (circles) and even T (crosses). Straight lines are the best fits mentioned in the text.

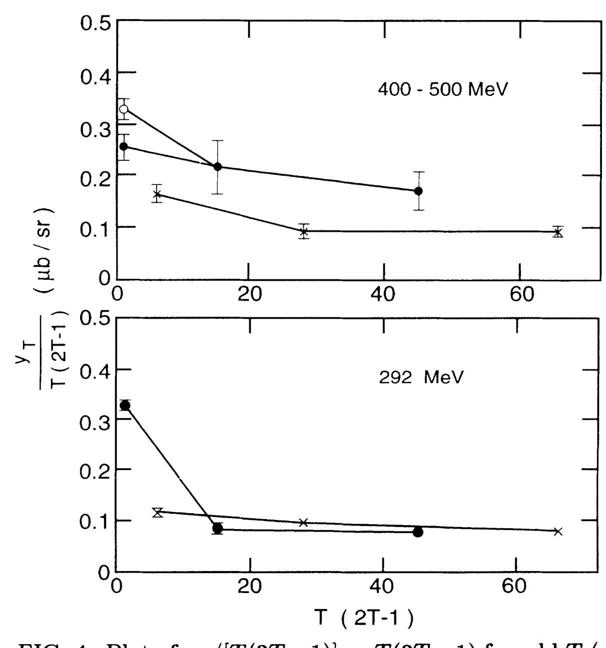

FIG. 4. Plot of $y_T/[T(2T-1)]$ vs $T(2T-1)$ for odd T (circles) and even T (crosses). Top is for 400–500 MeV, bottom for 292 MeV.

$T(2T-1)$ as do odd-T σ's. Both these expressions have total χ^2 less than unity.

In Fig. 4, we plot the quantites $y_T/[T(2T-1)]$ from the present work (top) and for 292 MeV (bottom). At 292 MeV, values for even and odd T behave roughly the same for $T > 2$. The values for both $T = 1$ and 2 are above this trend—markedly for $T = 1$, less so for $T = 2$. In the top half of the figure, the 400–500 MeV values appear to have separated for odd and even T. Furthermore, the $T = 1$ point is more in line with the general odd-T trend than it was at 292 MeV. It thus appears that as T_π increases from 292 MeV to 400–500 MeV, odd-T cross sections increase faster than those for even T, and that this increase has already happened at 292 MeV for $T = 1$. The plots in Fig. 4 slope downward slightly with increasing T. In any two-amplitude description of the type discussed in Ref. [22], the coefficient of the second amplitude has a tendency to decrease (although very slowly for T > 1) with T. The best-fit average ratio of $\sigma'/[T(2T-1)]$ at 400–500 MeV is 0.228 ± 0.018 μb/sr for odd T and 0.103 ± 0.007 μb/sr for even T. The difference is more than five standard deviations.

We have no obvious explanation for the apparent odd-even behavior though a number of possibilities suggest themselves. For example, DCX on even-T targets goes to nuclei that are more "α-particle"-like and hence that may be more deformed and less likely to have good seniority. We know [22] that for ground states, deviations from good seniority tend to decrease DCX cross sections. However, we would not have expected this effect to show up in the DIAS, unless isospin is less well conserved in more α-like nuclei. Isospin mixing (if present) could also be influenced by differences in Q value. A larger separation between the g.s. and the DIAS allows a larger density of $T_<$ 0^+ states in the vicinity of the DIAS and perhaps more mixing. To the extent that the effect suggested here is convincing, perhaps these results will encourage theoretical investigation.

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