Coherence of nucleonic motion in superdeformed nuclei: Towards an understanding of identical bands

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The moments of inertia of superdeformed nuclei in the Hg-Pb region are investigated. Deformation and pairing effects are treated self-consistently by means of the cranked Strutinsky Lipkin-Nogami approach. Special attention is drawn to the role of the quadrupole pairing force, which is crucial for the quantitative understanding of the moments of inertia, their dependence on N and Z, and hence the identical band phenomenon.

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A new area of nuclear spectroscopy opened up with the discovery of discrete high spin states at superdeformed (SD) shape [1]. With the event of new detector arrays, detailed spectroscopy in the second well is carried out and new features emerge [2,3]. One of the most exciting results relates to the discovery of identical bands (IB), the first example of which was found in mass $A = 150$ region [4]. Shortly afterwards, similar IB were discovered in ^{192}Hg and ^{194}Hg [5] and it was realized that most of the SD bands in the mass $A = 190$ region can be grouped into families of identical bands, with respect to certain central nuclei [2,5]. Similar "families" of IB emerge in the SD $A = 150$ region [3].

Following the experimental discovery, Nazarewicz and co-workers [6] realized that the observation of identical γ-ray sequences in ^{152}Dy and ^{151}Tb, as well as in ^{150}Gd and ^{151}Tb, could be a fingerprint of the pseudospin symmetry at extreme deformation and angular momenta. However, this interesting consequence of the IB phenomenon does not explain the essence of the extremely weak polarization effects due to the odd particle/hole. Other groups tried to gain understanding of the IB phenomenon based on a pure single-particle approach [7]. In the limit of the harmonic oscillator potential the dependence of the moment of inertia on nuclear mass and deformation (quadrupole) parameters is obtained by analytical formulas [8], and, consequently, numerical calculations can be carried out with sufficient accuracy. It has been demonstrated that for certain orbitals the polarization effect, $\delta J^{(2)}$ ($J^{(2)} \equiv dI/d\omega$), is very small, and hence occupying these specific orbitals might result in identical bands [7].

All these considerations and similar algebraic approaches start from simplified nuclear models. Since the occurrence of IB is intimately linked to the understanding of the moments of inertia as a function of N and Z, we believe that more realistic assumptions are needed. The polarizing effects due the presence of additional particles have been investigated previously by means of the self-consistent Hartree-Fock method [9] and in terms of relativistic mean field calculations [10]. Our present study investigates the polarization in the presence of pair correlations, with special attention to the quadrupole component of the pairing force.

In contrast to the Dy region, the SD bands in the Hg region are observed to relatively low rotational frequencies where pair correlations play an important role. The well-known effect of nuclear superfluidity does reduce the moments of inertia with respect to its rigid body value. In addition, the moments of inertia exhibit a smooth increase over a large frequency range which is understood in terms of successive alignment of $\nu j_{15/2}$ and $\pi i_{13/2}$ in the presence of pair correlations. The direct consequence of this interpretation is the expected downturn of $J^{(2)}$ at high spins being a result of exhausted quasiparticle (qp) alignment. Recently, such a downturn was indeed observed experimentally [11].

The presence of pronounced pair correlations in the SD bands in the Hg region evidently excludes the applications of the simple, single-particle or similar algebraic approaches in the explanation of the IB phenomenon. The limitations of the latter method are demonstrated in Fig. 1 where we present the results of cranked-Woods-Saxon calculations with particle number projection (PNP) and without (Δ=0) pair correlations, performed at constant deformation for the SD bands of

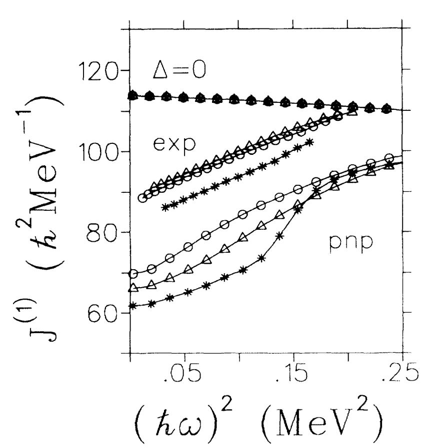

FIG. 1. The kinematical $J^{(1)}$ moments of inertia of ^{190}Hg ($*$), ^{192}Hg ($\bigcirc$), and ^{194} Hg($\triangle$), experiment and theory. The pairing interaction is treated by means of the particle number projection (PNP) before variation approach.

^{190,192,194}Hg. In the single-particle limit the calculations show that by adding two neutrons to, or removing two neutrons from, the ^{192}Hg core the resulting $J^{(1)} \equiv I_x/\omega$ moment of inertia is essentially unchanged. However, the calculated $J^{(1)}$ moments of inertia do not agree at all with the experimental findings. This deficiency is remedied by including monopole pair correlations, which qualitatively account for the main experimental trends in terms of a smoothly increasing moment of inertia [2]. Still, the quantitative agreement between theory and experiment remains unsatisfactory. Moreover, the differences between the moments of inertia for different nuclei now increase considerably as compared to the calculations without pairing. In the paired calculations, the alignment process is sensitively dependent on the Fermi surface [12], yielding large differences as a function of neutron number. The importance of this effect was realized especially after the discovery of SD in ^{190}Hg [13], which triggered the present investigation to include higher order correlations into the particle-particle (p-p) channel.

Already, early investigations by Migdal [14] showed that the gauge invariant pairing interaction is of importance for the moments of inertia (see also [15,16]). Realistic calculations with a quadrupole pairing force [quadrupole-quadropole (QQ) pairing] in the rare earth region further corroborated this effect [17,18]. It was suggested in Ref. [19], and recently estimated by Hamamoto and Nazarewicz [20] in the framework of a simple model, that QQ pairing might be of importance for understanding the alignment process in the SD Hg region. In the present study we therefore consider the Hamiltonian which contains the deformed mean-field potential of Woods-Saxon type and a pairing interaction defined as

$$\bar{v}^{(\lambda\mu)}_{\alpha\beta\gamma\delta} = -G_{\lambda\mu} g^{(\lambda\mu)}_{\alpha\bar{\beta}} g^{*(\lambda\mu)}_{\gamma\bar{\delta}} \tag{1}$$

$$g^{(\lambda\mu)}_{\alpha\bar{\beta}} = \begin{cases} \delta_{\alpha\bar{\beta}}, & \lambda = 0, \quad \mu = 0, \\ \langle\alpha|\tilde{Q}_\mu|\bar{\beta}\rangle, & \lambda = 2, \quad \mu = 0,1,2. \end{cases} \tag{2}$$

The monopole pairing strength G_{00} is determined by the average gap method of Ref. [21] while $G_{2\mu}$ corresponds to the self-consistent coupling constants of [22]. The strength of the quadrupole pairing force stems from the requirement of restoring the local Galilean invariance of the system under λ-pole collective shape oscillations [22]. Consequently, our approach does not involve any adjustable strength parameters. It has been demonstrated by Kishimoto and Sakamoto that effective interactions in the particle-hole (p-h) channel can be realized in general by means of double-stretched operators [23]. Hence, the natural extension to the QQ-pairing force also invokes double-stretched operators ($Q''_\mu = r''^2 Y''_{2\mu}$):

$$\begin{aligned} \tilde{Q}_0 &= Q''_{20}, \\ \tilde{Q}_\mu &= \tfrac{1}{\sqrt{2}}(Q''_\mu + Q''_{-\mu}), \qquad \mu = 1,2. \end{aligned} \tag{3}$$

In order to avoid the commonly encountered problem of a sharp pairing phase transition, we use the Lipkin-Nogami approach. This method, being the approximation of particle number projection before variation, aims to minimize an operator up to second order, $\mathcal{H} = \hat{H}^\omega - \lambda_2(\hat{N}^2 - \langle\hat{N}\rangle^2)$, with λ_2 calculated using certain subsidiary conditions [24]. The resulting Lipkin-Nogami equations (LNC) in the intrinsic frame of reference (cranking model) can be cast into the standard form of cranked Hartree-Fock-Bogolyubov equations (for details we refer the reader to [25–27]). In the case of monopole and quadrupole pairing we derive the following self-consistency condition for evaluating λ_2:

$$\lambda_2 = \frac{1}{4}\sum_{\lambda,\mu} G_{\lambda\mu} \frac{\sum_{\alpha>0}(\kappa^*\rho g^{(\lambda\mu)})_{\alpha\tilde{\alpha}} \sum_{\alpha>0}[\kappa^*(1-\rho)g^{(\lambda\mu)}]^*_{\alpha\tilde{\alpha}} - \sum_{\alpha,\beta>0}\left[(\chi g^{(\lambda\mu)})_{\alpha\beta}(\chi g^{(\lambda\mu)})^*_{\tilde{\beta}\tilde{\alpha}}\right]}{\left(\sum_{\alpha>0}\chi_{\alpha\alpha}\right)^2 - \sum_{\alpha>0}\chi^2_{\alpha\alpha}} \tag{4}$$

where ρ and κ ($\chi = \kappa\kappa^*$) denote the density matrix and pairing tensor, respectively. We use a good signature representation, $\alpha(\tilde{\alpha}) = r = -i(+i)$ where $\hat{T}|r = \mp i\rangle = \overline{|r = \mp i\rangle} = \mp|r = \pm i\rangle$. The contribution to the pairing field is

$$\Delta_{\alpha\bar{\beta}} = -\sum_{\lambda\mu} g^{(\lambda\mu)}_{\alpha\bar{\beta}} \Delta_{\lambda\mu} \tag{5}$$

where

$$\Delta_{\lambda\mu} = G_{\lambda\mu} \sum_{\alpha\beta>0} g^{*(\lambda\mu)}_{\alpha\bar{\beta}} \kappa_{\alpha\bar{\beta}}$$

The use of double-stretched quadrupole operators has several advantages. The coupling constants of the pairing strength are essentially equal by analogy to the spherical case, $G_{20} \approx G_{21} \approx G_{22}$, and almost independent of deformation. Secondly, the modification of qp levels calculated at frequency zero is of the order of 100 keV as compared to the calculations including monopole pairing only, irrespective of deformation. This effect is demonstrated in Table I, where we show the relative shifts of the qp energies of high-j orbitals calculated for SD ^{194}Hg using three different versions of QQ pairing (involving nonstretched, stretched, and double-stretched generators respectively). At least three facts should be noticed here: (i) In the cases of nonstretched and stretched QQ-pairing forces (these are the versions used so far in the literature[1]

[1]In fact in [17] no radial form factor at all was taken into account.

TABLE I. The relative shift of the quasiparticle energies of the high-j orbitals $j_{15/2}$ (neutrons), $i_{13/2}$ (protons) in the presence of different kinds of quadrupole pairing interactions (nonstretched, stretched, and double-stretched). The shift stems essentially from the Y_{20} component of the force. The calculations were done for the nucleus ^{194}Hg at a representative deformation of the SD band ($\beta_2 = 0.480$, $\beta_4 = 0.072$; $\gamma = 0°$).

Orbital	Nonstretched	Stretched	Double-stretched
$\nu j_{15/2}$	+1286 keV	+538 keV	-43 keV
$\pi i_{13/2}$	+1324 keV	+634 keV	+5 keV

[17,18]) these shifts are large and contain spurious, pronounced deformation dependence. (ii) The shifts presented in the table are calculated using the selfconsistent coupling constants [22] which are different for different versions of QQ pairing and remove, in part, the spurious shape dependency of the interaction. For example, the coupling constant for the $K = 0$ component of QQ pairing (G_{20}) responsible for the shifts discussed here is roughly 3(1.5) times smaller for the nonstretched (stretched) force as compared to the double-stretched. Also, the selfconsistent values of the quadrupole pairing gaps are substantial for both nonstretched and stretched versions (in contrast to the double-stretched interaction, see, e.g., Fig. 2). It requires an additional correction of the monopole pairing strength to preserve a roughly constant value of the odd-even mass difference. In the case of deformation self-consistent calculations it must be done

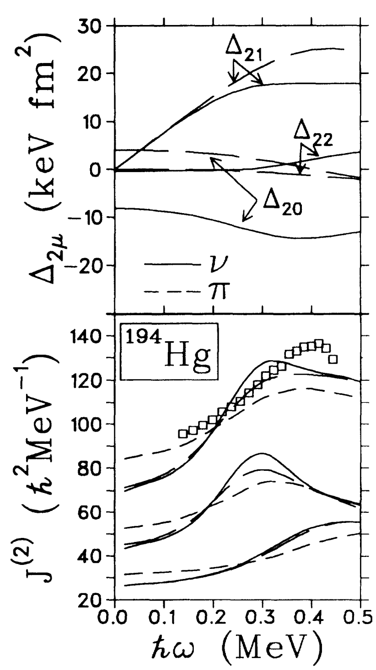

FIG. 2. Comparison of calculated and experimental dynamical (□) $J^{(2)}$ moments of inertia of ^{194}Hg. The pairing interaction is treated within the LNC approach. Calculations including monopole paring only are depicted by solid lines, including the Q_{20} component by long-dashed lines, and the one including Q_{20} and Q_{21} by short-dashed lines.

at each deformation separately. Moreover, due to the state dependence of the interaction it can be done only on the average [19]. (iii) In the case of prolate deformed nucleus, all orbitals with positive quadrupole moment (especially those with low Ω and high j) were shifted *up* [17,18]. On the other hand, within the double-stretched pairing force, the average quadrupole moment is close to zero, and Δ_{20} can be either positive or negative. For example, Δ_{20} of neutrons in SD ^{194}Hg is negative implying that the $\nu j_{15/2}$ orbital is pushed *down* in energy (cf. Table I and Fig. 2).

The above-mentioned facts clearly show that the three versions of QQ pairing are different and cannot be compared directly to each other. Moreover, only the double-stretched version seems to have truly residual character, allowing us to apply the formalism directly to lattice calculations in deformation space. The form of the QQ-pairing force touches the important questions of how to construct residual interactions on top of the *deformed* mean field. Our work seems to suggest that among the hitherto used QQ-pairing forces only the present application (double-stretched) is consistent with the deformed mean field.

Figure 2 shows the self-consistent quadrupole pairing gaps (upper panel) and the corresponding $J^{(2)}$ moments calculated for ^{194}Hg at constant deformation. The calculated quadrupole Δ's are of the order of $\sim 10 - 20$ keV/fm^2 and hence QQ pairing contributes only little to the total energy, $\delta E_{\lambda\mu} = -|\Delta_{\lambda\mu}|^2/G_{\lambda\mu}$, since the self-consistent value of coupling constant is $G^{\nu(\pi)}_{\lambda\mu} \sim$ 1.2 (1.8) keV/fm^4 for neutrons (protons), respectively. Nevertheless, our QQ-pairing force strongly influences $J^{(2)}$, implying that it is of essentially dynamical character.

The action of the Q_{20} component (— —) results in a slight modification of $J^{(2)}$, the effect from protons being negligible, see Fig. 2. These results are very different from previous calculations at normal deformation, where the Q_{20} component played a dominant role [17,18]. In our approach, the key role is played by the Q_{21} term (- - -) acting extremely coherently over the entire frequency range. As a response to cranking, the $\Delta K = 1$ scattering redistributes quasiparticles in a manner which favors the angular momentum gain. The result is an immediate increase of $J^{(2)}$ as expected from early calculations [14–16] and it is qualitatively in agreement with [17,18]. With increasing frequencies, the Coriolis mixing leads to a further enhancement of the $\Delta K = 1$ pair scattering up to the frequency of completed qp alignment or crossing. The crossing frequency becomes slightly shifted and, more importantly, the alignment gain of the high-j orbitals is substantially reduced. Consequently, the $J^{(2)}$ moment of inertia flattens out in agreement with experimental data. Finally, Δ_{22} starts to increase first after the qp alignment, and, therefore, the Q_{22} component does not play an important role in our present investigation (a weak Δ_{22} effect was also found [17]).

Let us allow the following reflection about the rotational structure of SD bands: At normal deformation, the ground state band is crossed by the S band in a region of $10\hbar$–$14\hbar$. Since pair correlations are considerably

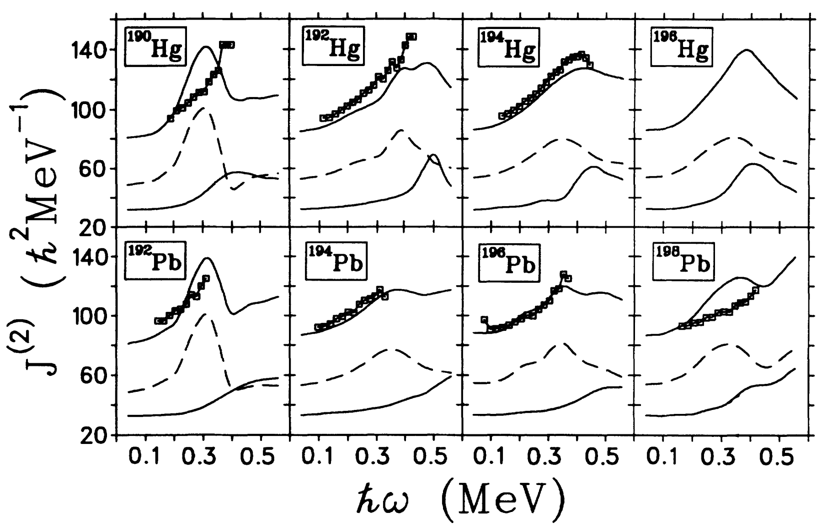

FIG. 3. The systematics of calculated and experimental dynamical moments of inertia $J^{(2)}$ of the SD bands of ^{190}Hg up to ^{198}Pb. The lattice calculations include all components of the QQ-pairing force. Note that the experimental values of ^{198}Pb are not unambiguously assigned to that nucleus.

reduced in the S band, the full interplay of Coriolis force and pairing interaction can be studied essentially in the narrow region below the first band crossing. In contrast, the alignment at SD shape in the Hg-Pb region occurs at spins of $\approx 30\hbar$ (for protons $\approx 40\hbar$), allowing for a more detailed study of the interplay of single-particle motion, rotation, and pairing. Since the alignment occurs so late in SD nuclei, the effects of pairing and especially higher order pair correlations are given a much larger range of influence.

In spite of the remarkable improvement as compared to previous calculations, the high spin behavior of the moments of inertia are not reproduced at constant deformation. Therefore, we included the effects of deformation changes induced by rotation and performed Strutinsky-type calculations on a lattice in deformation space (including quadrupole β_2, γ, and hexadecapole β_4 degrees of freedom). In these calculations we solve the LNC equations self-consistently including the monopole and quadrupole pairing channels. The calculations were performed for the yrast SD bands of even-even Hg and Pb isotopes from $N = 110$–116 and the results are shown in Fig. 3. The discrepancy between all earlier calculations and experiment are, to a large extent, removed. Our calculations demonstrate the simultaneous role of shape changes and pairing correlations. It turns out that an additional coherence arises from the Q_{21} component of the pairing force, resulting in an enhanced stability of the nuclear shape. First after the neutron alignment a deformation shrinking sets in, leading to a prolonged neutron and more rapid proton angular momentum gain.

A few more comments are on place regarding our lattice calculations. The $i_{13/2}$ proton alignment in the Hg isotopes is occurring at a slightly too high frequency as compared to experiment (Fig. 3). A slight lowering of that crossing will further improve the agreement at high spin (see, e.g., ^{192,194}Hg). Similarly, the neutron crossing is too early at $N = 110$. Some of these discrepancies are related to the placement of the single-particle levels in the Woods-Saxon potential used here [28]. A further refinement of the p-h channel in order to better describe the high spin behavior is necessary. One should also point out that, in the presence of simultaneous high-j proton and neutron alignment, a residual proton-neutron (p-n) interaction in the p-h channel can play a role. In contrast to the p-p channel, our studies of the of the QQ p-n interaction show [29] that the effect is limited to a few levels and reveal no coherent result when mixing additional configurations.

A microscopic understanding of the moments of inertia and their dependence on nucleon number is essential for the IB phenomenon. Extending the pairing interaction to higher order and calculating pairing and deformation self-consistently, one is for the first time able to reproduce quantitatively the moments of inertia of the SD bands in the Hg-Pb region over the entire frequency range. In the absence of low-lying band crossings, the SD Hg-Pb nuclei become an excellent tool for detailed studies of the interplay of collective rotation, single-particle motion, and pairing interaction over a multitude of states. The double-stretched QQ-pairing interaction treated within the LNC approach levels out the isotonic differences in the alignment pattern, which were present in previous calculations, without essentially affecting the single-particle structure. Within the mean-field approach it appears difficult to discuss results on a 1 keV level, especially in the presence of interpolation on a lattice. However, our calculations clearly suggest that the additional correlations induced by the QQ-pairing force, essentially by the Q_{21} term only, play a key role in the understanding of the experimentally deduced moments of inertia and hence the IB phenomenon. The nucleonic motion at SD shape appears to be correlated to an extent that overcomes the expected differences due to changes in nucleon number.

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