Simple phenomenology for the ground-state bands of even-even nuclei

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It is observed that the even-even nuclei having identical N_pN_n/Δ values exhibit identical excitation energies and/or energy ratios ($E_J/E_2, E_J/E_4$) in their ground-state bands (g.s.b.). N_p (N_n) is the valence proton (neutron) number and Δ is the average pairing gap, $(\Delta_p + \Delta_n)/2$. It is shown that this correlation is a manifestation of a very simple but uniform dependence of the excitation energies of the g.s.b. of even-even nuclei, spanning a wide mass region ($70 \leq A \leq 244$), on the ratio of their pairing and n-p interaction energies. An explicit form of this dependence is phenomenologically deduced and subsequently used to study the development of collectivity, existence of subshell closures and some other nuclear systematics.

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One of the most interesting discoveries in recent nuclear structure investigations is the existence of identical bands (IB) in pairs of even-even as well as in adjacent odd-even nuclei spanning an extended region of spin and deformation. Several explanations have been put forward to understand the origin of this phenomenon in the superdeformed and normally deformed states. A review of the experimental evidences and critical analysis of the theoretical explanations for the superdeformed identical bands can be found in Ref. [1]. So far as the low spin identical bands observed in pairs of even rare-earth [2] and actinide [3] nuclei are concerned, attempts have been made to understand this phenomenon in terms of some simple systematics of the moments of inertia in the rare-earth region [4–6] or from symmetry considerations [7]. However, a satisfactory explanation of the fact that some nuclei exhibit identical excitation energies upto $J^\pi \simeq 10^+$–12^+, and a vast majority of them do not, is still lacking and further investigations are necessary. The purpose of the present article is to show that this phenomenon is a manifestation of a very interesting but simple correlation which exists between the excitation mode of the ground-state bands (g.s.b.) of even-even nuclei and the ratio of their N_pN_n product and the average pairing gap Δ. N_p and N_n are the numbers of valence proton and neutron, respectively, whereas Δ is the average of their proton and neutron pairing gaps (Δ_p, Δ_n). This correlation is not restricted to any specific mass region; rather it extends over a wide region of mass, charge, and equilibrium shape.

Casten *et al.* [2] have pointed out that pairs of nuclei in the rare-earth region, which show identical excitation energies, have nearly equal values of the product of their valence nucleon numbers N_p and N_n. However, an analysis of the existing data shows that the converse is not true. In Table I, the experimental excitation energies of three nuclei which have identical N_pN_n products are shown. For all the even-even nuclei considered in this paper, the experimental data are taken from Refs. [8,9] and recent Nuclear Data Sheets. It can be seen that two of them ($^{158}_{66}$Dy, $^{170}_{72}$Hf) show identical transition energies but they are different from those of the third one ($^{180}_{74}$W). Before we go into the significance of this discrepancy we would like to present another interesting correlation which exists among the excitation spectra of nuclei, not necessarily belonging to the same mass region. In Table II, the experimental level energies of several groups of nuclei are shown. For each group, the excitation energies of its members are quite different, but their energy ratios are very close to each other. Since the energy ratios, e.g., $R_4 = E_4/E_2$, rather than the absolute energies, give better information about the underlying excitation mechanism, it can be said that they exhibit identical excitation properties. It may be noticed that, in some cases, nuclei exhibiting identical energy ratios have nearly identical N_pN_n products; but there are exceptions also. The question is whether the phenomena of identical transition energies and identical energy ratios have anything common between them.

The low energy property in any nucleus is expected to be determined, at least qualitatively, by the competition of two opposing forces, i.e., short-range pairing (p-p, n-n) and long-range deformation producing n-p interactions. A measure of the pairing interaction energy in an even nucleus can be obtained from the neutron (proton) pairing gaps [Δ_p (Δ_n)], which are estimated from the difference in experimental binding energies of neighboring odd and even nuclei. Moreover, it has been shown by Casten *et al.* [10,11] that the product N_pN_n can be approximately related to the integrated strength of the n-p interactions. Therefore, it appears that the ratio N_pN_n/Δ, where $\Delta = (\Delta_p + \Delta_n)/2$, rather than the product N_pN_n, may be a better parameter for studying the identity in excitation modes in pairs of nuclei. In Table III, we have listed the N_pN_n/Δ values for the nuclei whose excitation

TABLE I. Excitation energies (in keV) of three rare-earth nuclei whose N_pN_n products are equal (=160).

Isotope	$J = 2$	4	6	8	10
$^{158}_{66}$Dy$_{92}$	98.9	317.3	638.9	1044	1520
$^{170}_{72}$Hf$_{98}$	100.8	322.0	642.8	1043	1505
$^{180}_{74}$W$_{106}$	103.6	337.6	688.5	1138	1664

TABLE II. Nuclei exhibiting nonidentical excitation energies but identical energy ratios (E_J/E_4) in their ground-state bands. The energies are given in keV. In the last column, N_pN_n products are listed.

Isotope	$J = 2$	4	6	8	10	N_pN_n
$^{168}_{70}Yb_{98}$	87.7	286.5	585.3	970.0	1424	192
	0.31	1.0	2.04	3.39	4.97	
$^{172}_{72}Hf_{100}$	95.2	309.3	628.1	1037	1521	180
	0.31	1.0	2.03	3.35	4.92	
$^{180}_{74}W_{106}$	103.6	337.6	688.5	1138	1664	160
	0.31	1.0	2.04	3.37	4.93	
$^{182}_{74}W_{108}$	100.1	329.4	680.5	1144	1712	144
	0.30	1.0	2.07	3.47	5.20	
$^{232}_{92}U_{140}$	47.6	156.6	322.8	541.2	806.0	140
	0.30	1.0	2.06	3.46	5.15	
$^{124}_{54}Xe_{70}$	354.0	878.7	1548	2330	3170	48
	0.40	1.0	1.76	2.65	3.61	
$^{130}_{56}Ba_{74}$	357.0	901.0	1593	2396	3261	48
	0.40	1.0	1.77	2.66	3.62	
$^{80}_{38}Sr_{42}$	385.7	980.5	1763	2699	3764	80
	0.39	1.0	1.80	2.75	3.84	
$^{106}_{44}Ru_{62}$	270.0	714.8	1296	1974		72
	0.38	1.0	1.81	2.76		

energies are shown in Tables I and II. The pairing gaps Δ_n and Δ_p are calculated from the atomic masses [12] of a sequence of isotopes or isotones using the third difference of energies [13]. The correlation between the excitation modes and the N_pN_n/Δ ratios in nuclei spanning a wide mass region can be easily seen. Nuclei having similar N_pN_n/Δ values, even if they belong to different mass regions, show identical energy ratios, i.e., their ground-state spectra differ only by an overall (energy) scaling factor. If, for some reasons, this scaling factor becomes unity, then not only the energy ratios but also the absolute energies become identical. So this phenomenon of identical bands is not restricted to pairs of nuclei belonging to a particular region, rather it has a more universal character. We may call it the phenomenon of identical excitation modes (IEM) in even-even nuclei. It may further be noted that although some of the nuclei (e.g., ^{170}Hf, ^{180}W) have similar N_pN_n values but they exhibit neither IB nor IEM as their Δ values are different. On the other hand, some of them exhibit IEM (e.g, ^{168}Yb, ^{172}Hf, ^{180}W), because of their identical N_pN_n/Δ ratio, although their N_pN_n and Δ values are different. For comparison, the P factors defined in Ref. [10] as $N_pN_n/(N_p+N_n)$ are also listed for these nuclei in Table III. The correlation between the P factor and the excitation mode does not appear to be so striking, at least with the shell closures used in this paper. It has been shown recently [5,14] that the P factor and the ratio ϵ/Δ, where ϵ is the deformation parameter, are strongly correlated. Although the parameters N_pN_n/Δ and ϵ/Δ appear to be very similar, there is a subtle difference between the two. N_pN_n is related to the n-p interaction strength, whereas the deformation parameter ϵ is likely to be determined by the combined effect of the n-p interaction and the pairing correlation. This may be one of the reasons for the observed difference in the dependence of the excitation mode on the N_pN_n/Δ and P-factor values.

The above analysis shows that the energy ratios of the g.s.b.'s of the even-even nuclei can possibly be expressed as a simple function of their N_pN_n/Δ values. In fact, we have found that the g.s.b. energies of the even-even nuclei encompassing an extended region of mass, charge ($34 \leq Z \leq 94$, $70 \leq A \leq 244$), and equilibrium shapes can be expressed through the following relations:

$$E_2 = \alpha_0(\Delta/N_pN_n) \quad (1)$$

and

$$\Delta E_\gamma(I) = \alpha_I(\Delta/N_pN_n)^{1/2I} , \quad (2a)$$

where

$$\Delta E_\gamma(I) = E_\gamma(I+2 \to I) - E_\gamma(I \to I-2) . \quad (2b)$$

It can be easily verified that the above relations lead to the following expression for the energy ratios of the g.s.b.:

$$E_I/E_2 = I/2 + \sum_J^{2,I-2,2} \alpha'_J \frac{(I-J)}{2}(\kappa)^{(2J-1)/2J} , \quad (3)$$

where

$$\kappa = N_pN_n/\Delta \quad \text{and} \quad \alpha'_J = \alpha_J/\alpha_0 . \quad (4)$$

For example, the energy ratio E_{10}/E_2 can be calculated from the following expression:

$$E_{10}/E_2 = 5 + 4\alpha_2'(\kappa)^{3/4} + 3\alpha_4'(\kappa)^{7/8} + 2\alpha_6'(\kappa)^{11/12} + \alpha_8'(\kappa)^{15/16} . \quad (5)$$

The most interesting feature of Eq. (3) is its universal nature. The energy ratios of the g.s.b. of all the even-even nuclei exhibiting collective excitations ($2.1 \le R_4 \le 3.33$) in the mass region $70 \le A \le 244$ can be obtained with good accuracy using the same parameter values α_J'. The parameter sets α_J' have been determined by fitting the excitation energies of the g.s.b. in several nuclei situated in the $A = 80$, 100, 110, 160, and 230 regions, and are listed in Table IV. As expected, they are not exactly identical. However, the parameters α_2' and α_4' (and to some extent α_6' also) lie within a narrow range over such a wide mass region. From Eqs. (3) and (5) it can be seen that they are the most significant parameters in determining the energy ratios. As a consequence we have found that using any of these parameter sets the experimental energy ratios of all the nuclei under consideration can be reproduced with good accuracy.

We have calculated the energy ratios of the g.s.b. of about 180 nuclei extending from Se ($Z = 34$) to Pu ($Z = 94$) isotopes using the parameter set α_J' determined from the g.s.b. energies of ^{162}Er (Table IV). The N_pN_n value for each nucleus is calculated using standard shell closures at $N, Z = 28, 50, 82, 126, 184$, with the following exception: for $62 \le Z \le 72$ and $84 \le N \le 88$, a subshell closure at $Z = 64$ is assumed. The method of calculating Δ_p, Δ_n has been mentioned earlier.

The calculated values are in good agreement with their corresponding experimental values in more than 90% of the cases. The exceptions are those which lie in the midshell region, showing strong saturation effect, mainly in their 2_1^+ excitation energy. In addition, good agreement is not achieved in a few cases ($\simeq 10$) with the shell closures used in this paper. The calculated and experimental energy ratios of the g.s.b. of some representative cases (showing two extreme limits of the quality of agreement) are shown in Table V. In some cases (e.g., ^{160}Dy, ^{238}U)

TABLE III. The N_pN_n/Δ ratios for the nuclei whose excitation energies are shown in Tables I and II. The average pairing gap $\Delta = (\Delta_p + \Delta_n)/2$ is given in MeV. For comparison the P-factor values defined in Ref. [10] as $N_pN_n/(N_p + N_n)$ are also listed.

Isotope	N_pN_n	Δ	N_pN_n/Δ	P	Comments
$^{160}_{68}$Er$_{92}$	140	1.13	124	5.83	IB
$^{168}_{72}$Hf$_{96}$	140	1.10	127	5.83	
$^{160}_{66}$Dy$_{94}$	192	0.97	197	6.86	IB
$^{168}_{70}$Yb$_{98}$	192	1.01	190	6.86	
$^{156}_{66}$Dy$_{90}$	128	1.21	105	5.33	IB
$^{178}_{76}$Os$_{102}$	120	1.06	113	4.62	
$^{236}_{92}$U$_{144}$	180	0.82	219	6.43	IB
$^{238}_{92}$U$_{146}$	200	0.92	217	6.67	
$^{80}_{38}$Sr$_{42}$	80	1.62	49.3	4.44	IEM
$^{106}_{44}$Ru$_{62}$	72	1.47	49.0	4.0	
$^{124}_{54}$Xe$_{70}$	48	1.37	35.2	3.0	IEM
$^{130}_{56}$Ba$_{74}$	48	1.34	35.8	3.43	
$^{168}_{70}$Yb$_{98}$	192	1.01	190	6.86	
$^{172}_{72}$Hf$_{100}$	180	0.96	188	6.43	IEM
$^{180}_{74}$W$_{106}$	160	0.87	184	5.71	
$^{182}_{74}$W$_{108}$	144	0.83	173	5.54	IEM
$^{232}_{92}$U$_{140}$	140	0.82	171	5.38	
$^{180}_{74}$W$_{106}$	160	0.87	184	5.71	Non-Ident.
$^{170}_{72}$Hf$_{98}$	160	1.03	156	6.15	

TABLE IV. The values of the parameters α'_J extracted from the g.s.b. energies of ^{76}Se, ^{102}Ru, ^{110}Pd, ^{162}Er, and ^{232}U. Numerical values of the average pairing gap Δ expressed in keV, are used in Eq. (3) to extract the parameter values.

Nucl.	α'_2	α'_4	α'_6	α'_8	α'_{10}
^{76}Se	5.55	6.52	3.59	1.13	
^{102}Ru	4.31	5.75	3.12		
^{110}Pd	5.17	4.73	3.51	6.21	
^{162}Er	4.97	5.48	5.01	4.27	3.24
^{232}U	4.87	5.57	5.56	5.10	4.60

we have found that the agreement between the calculated and experimental values for higher spin states becomes much better if the E_J/E_4 ratios are considered. This is due to the fact that the experimental E_2 energy shows a saturation effect near the midshell region whereas in the empirical relation [Eq. (3)] no such effect is incorporated. In fact, the E_J/E_4 ratios can be deduced from the E_J/E_2 ratios listed in Table V by using the relation

TABLE V. Calculated and experimental (given in parentheses) energy ratios (E_J/E_2) in some representative cases. Calculations have been done using α'_J extracted from g.s.b. energies of ^{162}Er, listed in Table IV.

	E_J/E_2			
Nucl.	$J=4$	6	8	10
^{76}Se	2.34(2.38)	3.9(4.0)	5.7(5.9)	7.6(7.7)
^{80}Kr	2.34(2.33)	3.9(3.9)	5.7(5.5)	7.6(7.1)
^{100}Ru	2.31(2.27)	3.8(3.8)	5.5(5.7)	7.4(7.6)
^{110}Pd	2.45(2.47)	4.2(4.2)	6.3(6.2)	8.5(8.4)
^{150}Sm	2.15(2.18)	3.4(3.6)	4.7(5.2)	6.1(6.9)
^{188}Hg	2.35(2.43)	4.0(4.3)	5.8(5.9)	
^{232}Th	3.13(3.30)	6.23(6.8)	10.1(11.3)	14.7(16.9)
^{156}Dy[a]	2.92(2.93)	5.6(5.6)	8.9(8.8)	12.8(12.5)
^{178}Os[a]	2.98(3.01)	5.8(5.8)	9.3(9.0)	13.3(12.7)
^{160}Dy[a]	3.40(3.26)	7.3(6.7)	12.2(11.1)	18.0(16.4)
^{168}Yb[a]	3.40(3.25)	7.1(6.6)	11.9(11.0)	17.6(16.2)
^{160}Er[a]	3.04(3.09)	6.0(6.1)	9.6(9.7)	13.9(14.0)
^{168}Hf[a]	3.06(3.11)	6.0(6.1)	9.7(9.8)	14.0(14.0)
^{236}U[a]	3.50(3.31)	7.6(6.9)	12.9(11.6)	19.2(17.4)
^{238}U[a]	3.50(3.31)	7.6(6.8)	12.9(11.5)	19.1(17.3)
^{124}Xe[b]	2.40(2.48)	4.1(4.4)	6.0(6.6)	8.1(8.9)
^{130}Ba[b]	2.40(2.52)	4.1(4.4)	6.1(6.7)	8.2(9.1)
^{80}Sr[b]	2.52(2.54)	4.4(4.6)	6.7(7.0)	9.1(9.8)
^{106}Ru[b]	2.52(2.64)	4.4(4.8)	6.6(7.3)	
^{182}W[b]	3.33(3.29)	6.8(6.8)	11.4(11.4)	16.7(17.1)
^{232}U[b]	3.32(3.25)	6.8(6.7)	11.3(11.3)	16.6(16.8)

[a]Pairs of nuclei showing identical transition energies. Pairing gaps in ^{238}U have been calculated with a second difference [13] as the binding energy of ^{236}Th is not available.
[b]Pairs of nuclei showing identical energy ratios.

$$E_J/E_4 = (E_J/E_2)/R_4 .$$

The α_0 values for all the nuclei under consideration, extracted from the experimental E_2 energies using Eq. (1), are shown in Fig. 1. The α_0 values for several nuclei are also listed (Table VI) separately. Although there is a significant scattering in α_0 values over the entire mass region, but in a specific mass region, they lie within a narrow range. Therefore, identical transition energies are observed only in pairs of nuclei situated within the same mass region, whereas identical energy ratios are observed even for those nuclei which are situated in different mass regions (e.g., ^{182}W and ^{232}U). Since the valence nucleons of the nuclei situated in the same mass region are filling up similar single particle orbitals, it is expected that the α_0 values for these nuclei will be close to each other.

The universal character of Eq. (3) with a global parameter set α'_J has several important consequences: (i) the nuclei which have identical N_pN_n/Δ values show identical energy ratios; (ii) if two nuclei have identical N_pN_n/Δ values as well as E_2 energies, then they will exhibit identical excitation energies; (iii) since the N_pN_n product and the Δ value for any nucleus can be determined empirically (provided necessary binding energy data are available), the energy ratios of the g.s.b. of the even-even nuclei in this mass region can be easily predicted; (iv) the individual character of the nuclei (i.e., specificity of the single particle orbitals occupied by the valence nucleons, detailed features of the residual interactions and the resulting shape, etc.) is manifested through an overall scaling factor in energy, i.e., α_0 in Eq. (1), as if all necessary information are contained in the excitation energy of the 2^+ state itself. The last feature is consistent with our earlier observation [15] that the E_2 energies of these nuclei, when plotted against the factor, Δ/N_pN_n, exhibit three distinct branches corresponding to different

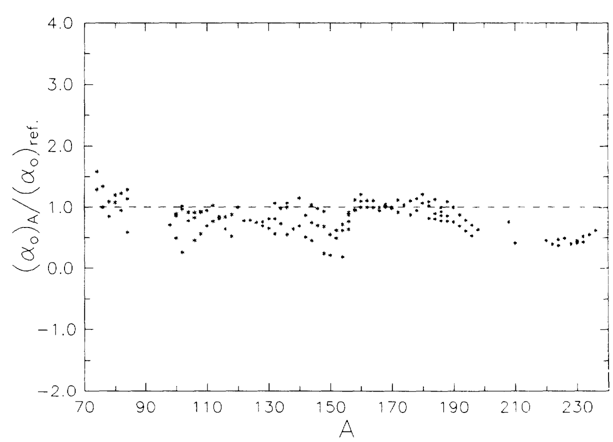

FIG. 1. The ratios $(\alpha_0)_A/(\alpha_0)_{ref}$ of the nuclei in the mass range $A \simeq$ 70–240 are plotted against mass number A. The parameters α_0 are determined from the experimental $E_{2_1^+}$ energies using Eq. (1). $(\alpha_0)_{ref} = \alpha_0(^{162}\mathrm{Er})$ is listed in Table VI.

TABLE VI. The α_0 values for several nuclei, deduced from Eq. (1).

Nucl.	α_0	Nucl.	α_0	Nucl.	α_0
$^{76}_{34}$Se	15.7	$^{160}_{66}$Dy	17.1	$^{188}_{80}$Hg	12.2
$^{100}_{44}$Ru	13.6	$^{162}_{68}$Er	15.8	$^{210}_{86}$Rn	6.5
$^{104}_{46}$Pd	12.4	$^{168}_{70}$Yb	16.7	$^{230}_{90}$Th	6.5
$^{112}_{48}$Cd	12.2	$^{168}_{72}$Hf	15.9	$^{232}_{90}$Th	6.8
$^{134}_{56}$Ba	11.6	$^{176}_{74}$W	17.7	$^{232}_{92}$U	8.1
$^{150}_{62}$Sm[a]	3.3	$^{178}_{76}$Os	15.1	$^{234}_{92}$U	8.8
$^{156}_{66}$Dy	14.5	$^{192}_{78}$Pt	14.0	$^{236}_{92}$U	9.9

[a]Nuclei with $84 \le N \le 88$ show very low α_0 values.

equilibrium shapes of these nuclei.

In even-even nuclei, the R_4 value, which can be expressed as

$$R_4 = 2 + \alpha'_2 \left(\frac{N_p N_n}{\Delta}\right)^{3/4} , \tag{6}$$

is a good indicator of collectivity. In Fig. 2, the R_4 values of the nuclei under consideration are plotted against the factor $(N_pN_n/\Delta)^{0.75}$. The compact correlation observed in this plot shows that the ratio N_pN_n/Δ, rather than the product N_pN_n, is a more appropriate structure parameter in the study of the systematics of ground-band properties. For the $B(E2, 0_1^+ \to 2_1^+)$ values, existence of a global correlation with the N_pN_n products was pointed out earlier [16]. It would be interesting to study similar systematics in terms of the N_pN_n/Δ ratios.

In Table VII, the calculated and experimental energy ratios are shown for some nuclei situated in the midshell region. In order to highlight the saturation effect in the excitation energies of their 2_1^+ states, the E_J/E_6 values are tabulated. One interesting feature of the excitation energies of the midshell nuclei belonging to rare-earth and actinide regions is the identical nature of their experimental energy ratios. This feature is also nicely reproduced in this paper. In an earlier work [17] we have shown that the saturation effect in the $E_{2_1^+}$ energies and $B(E2, 0_1^+ \to 2_1^+)$ values can be taken care of in a phenomenological way, if instead of N_pN_n, an effective value

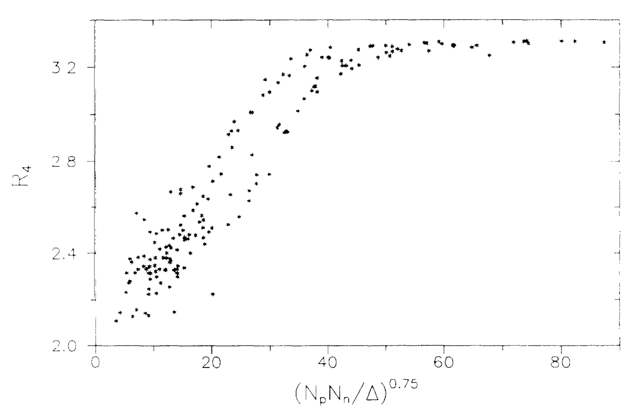

FIG. 2. Experimental R_4 values of the nuclei mentioned in the text are plotted against the factor $(N_pN_n/\Delta)^{0.75}$ [Eq. (6)].

TABLE VII. Calculated and experimental (given in parentheses) energy ratios (E_J/E_6) in some rare-earth and actinide nuclei near midshell showing saturation effect.

J^π	$^{164}_{66}$Dy$_{98}$	$^{176}_{72}$Hf$_{104}$	$^{238}_{94}$Pu$_{144}$	$^{236}_{94}$Pu$_{142}$
2^+	0.11(0.15)	0.12(0.15)	0.11(0.15)	0.12(0.15)
4^+	0.45(0.48)	0.46(0.49)	0.45(0.48)	0.46(0.48)
6^+	1.0(1.0)	1.0(1.0)	1.0(1.0)	1.0(1.0)
8^+	1.74(1.68)	1.72(1.67)	1.74(1.70)	1.73(1.69)
10^+	2.64(2.51)	2.58(2.48)	2.64(2.55)	2.60(2.53)

$(N_pN_n)_{\text{eff}}$ is used as a structure variable. However, these detailed features are not of much importance so far as the main conclusions of our present study are concerned.

Another important aspect of the present study is that the existence of subshell closure within a major shell for some specific values of neuron proton numbers can be predicted from very simple considerations. It is found that the energy ratios of the nuclei with $84 \le N < 90$ and $62 \le Z \le 72$ can only be reproduced if we assume a subshell closure at $Z = 64$. Equation (3), along with a universal set of coefficients α'_J, reproduces the energy ratios of the even-even nuclei spanning a wide mass region with good accuracy. So they can be used with some confidence to probe the existence of subshell closure in specific regions just from the knowledge of a few excited states in one or two nuclei even if other supporting data are not available.

In conclusion, it may be said that the excitation mechanisms of the ground-state band of the even-even nuclei spanning a very wide region of mass, charge, and equilibrium shape show an unexpected simplicity and uniformity in their dependence on the residual short-range pairing and long-range deformation producing neutron-proton interactions. This dependence is so simple and universal in character, that the excitation energies of such complex systems, consisting of so many nucleons interacting with each other, can be calculated from one simple but universal relation and practically using only one adjustable parameter, i.e., the energy scale factor. A number of very interesting features such as the occurrence of identical bands and/or identical energy ratios, existence of subshell closure, extent of saturation in excitation energies in the midshell region, etc., can be studied and in some cases predicted by using these relations. As a result, they may be used to extract much useful information about the nuclear structure of the nuclei far away from the β-stability line where new data are becoming available in recent times. The very fact that the energy ratios of the g.s.b. of a nucleus belonging to the actinide region (e.g., $^{232}_{92}$U) can be predicted from the knowledge of the same in a nucleus in the $A = 80$ region (e.g., $^{76}_{34}$Se) appears to be far more "puzzling" than the observation of identical transition energies in a pair of rare-earth nuclei.

Finally, such a remarkable correlation between the excitation energies and the ratio of the two most important components of the residual two-body interactions over such a wide mass region is interesting not only in terms of nuclear phenomenology but also from the standpoint of microscopic theory of nuclear structure.

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