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# Neutron-proton charge exchange

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We have studied the role of one- and two-pion exchange in the neutron-proton charge exchange cross section and spin-transfer variables. The cross section shows a scaling behavior over a wide energy range, consistent with a one-meson-exchange amplitude, and the spin-transfer observables are nearly invariant on a finite (but smaller) energy range. It is suggested that these data can be understood in terms of the exchange of pions between the quark constituents of the nucleon.

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## I. INTRODUCTION

The investigation of the nucleon-nucleon interaction has traditionally been approached by comparing the cross section, scattering lengths, deuteron binding energy, etc., with a theory expressed in terms of a potential. There are normally a certain number of parameters representing the fundamental coupling constants and vertex ranges of a number of exchanged particles with either a definite or distributed mass. Values of all of the parameters are needed before any results of the theory can be calculated. One might imagine that a laboratory can be found, in the form of a subset of the data, permitting a test of part of the theory without the necessity of fitting all of the parameters.

While it is not obvious *a priori* that such a procedure is possible, the region of neutron-proton charge exchange, at moderate to high energies and small momentum transfer, might well satisfy the conditions for such a separation. The small momentum transfer would seem to imply large distances and high partial waves. In fact, surprisingly, this is not quite the case as we shall see. It has been recognized for some time that the shape of the differential cross section, as a function of momentum transfer, is invariant over a very large energy range [1, 2]. We shall see below that its magnitude scales with the center-of-mass energy, the factor to be expected from an amplitude arising from particle exchange. The fact that an additional energy dependence does *not* enter can be taken as an indication that the iteration of the one-meson-exchange amplitude (such as that performed by the solution of the Schrödinger equation) is not very important since Green's function depends explicitly on the energy. This invariance in shape has sometimes been considered as accidental with no fundamental physical significance. We find this difficult to believe, given the very large energy range over which the scaling occurs.

In terms of a potential, a simple way to explain this behavior is to suppose that the Born approximation is, at least qualitatively, valid and that there is a local (energy independent) potential responsible for the reaction. A natural way to obtain a local potential is by particle exchange and we will regard the problem from this point of view. Note that the validity of the Born approximation requires a potential which is not too strong although the volume integral can be large provided that the form is dispersed in space. Hard cores are to be avoided.

The data [3–5] on the spin-transfer observables are also very interesting. The measured values of the polarization and asymmetry observables are very small in this region, as in fact they must be, since there is always an explicit factor of $\sin\theta$ in their mathematical expression which vanishes for fixed momentum transfer as the energy increases. In a more general statement, the polarization and asymmetry observables are defined relative to the scattering plane which becomes poorly defined for moderate momentum transfer and high energy while the spin transfer variables do not depend on a definition of the scattering plane.

The transfer of spin can be measured in three directions and the values of the spin-transfer observables are *not* small. Recent measurements of these three quantities show that they remain very nearly invariant as a function of momentum transfer, at least for the range of energies measured (laboratory kinetic energy from 485 to 788 MeV). Gersten [6] has pointed out a similar scaling phenomenon in the helicity amplitudes as calculated from phase shifts. Thus there are four observables which show an invariant behavior, making the hypothesis of a coincidence even more difficult to support.

The nucleon-nucleon interaction is treated in this paper taking into account the finite extension of the nucleons. We will consider only that part of the interaction which would arise from the exchange of (possibly interacting) pions between quarks. We consider the pion to have a finite but small radius and take, as an approximation, the limit of this radius going to zero. Thus our degrees of freedom are the quarks and pions with the quarks being limited to the role of a source of pions.

The treatment of the nucleons as objects with finite extent leads to potentials which are finite, even when the nucleons are completely overlapping. This has the feature that the Born approximation will tend to be valid. However, partly because we do not seek a complete theory at this point, we do not attempt a derivation from a fundamental Lagrangian formalism as has been done recently [7, 8]. Since the overall form of pion-nucleon coupling is given by invariance principles there is no freedom in the long-range part of the interaction. That is to say, pion emission by the quarks becomes pion emission by the nucleon since the nucleon has the same spin as the quarks. We discuss and illustrate the method with valence quarks only. For short range, when the nucleons overlap, we consider a modified form of pion exchange nucleon-nucleon interaction with a radial form which depends on the distribution of quarks in the nucleon.

Our principal result is that for one-pion exchange (OPE). While one might have thought that this aspect of the nucleon-nucleon interaction had been completely explored, recent results have been somewhat surprising. With a "form factor" an OPE potential alone is capable of explaining the "external" properties of the deuteron [9–11]. Since the tensor force dominates the deuteron binding, the spin-spin part of the OPE interaction is relatively unimportant in the studies just mentioned. In the present work we shall see that the spin-spin component plays a very important role. It is in this part of the interaction that there may be some indication of pion exchange between quarks, at least the assumption provides a natural basis for the suppression of the $\delta$ function which would be present in the exchange of a "field-theory" pion.

This paper is organized as follows. In Sec. II we illustrate the scaling of the neutron-proton charge exchange cross section over a wide energy range. Section III is devoted to a new insight into one-pion-exchange amplitude. The contribution of the exchange of two pions is considered in Sec. IV and conclusions are given in Sec. V. Appendix A gives the Born approximation to the amplitudes in terms of different components of meson-exchange potentials and Appendix B, formulas to calculate the $r$-space two-pion exchange potentials.

## II. DISCUSSION OF THE CROSS-SECTION DATA

We first examine the data set for the cross section. For neutron-proton scattering, ignoring the production channels, there is only elastic scattering hence there is no experimental distinction between the scattering of a neutron from a proton and the exchange of the two particles. For moderately high energies, however, there are two strong peaks in the angular distribution, one for small angles for the emerging neutron (which defines the elastic scattering region) and one for small angles for the emerging proton (which defines the charge exchange region). The cross section becomes very small at intermediate angles. While at low energies it is difficult to separate the two contributions, above 400 MeV kinetic energy the distinction is clear. In practical cases there will always be some contamination between the two regions.

An excellent data set was obtained at 800 MeV [12, 13], an energy which is high enough that the contamination from the elastic region is small. In data taken over a range of kinetic energies from 200 MeV to 800 MeV (Fig. 1, top) the same charge-exchange peak is seen to the extent that it can be separated from the elastic scattering at the lowest energies. Note that, even though the energies of these data cross the pion production threshold, there is no visible change in the form of the angular distribution as might be expected to occur if the peaking were due to absorption in the nucleon-nucleon channel. At the top of Fig. 1 these data are multiplied by the factor $s(E)/s(800)$, where $s$ is the square of the total center-of-mass energy of the two nucleons at a kinetic energy E. We will discuss the basis for this choice of scaling factor in Sec. III A. The solid points represent the 800 MeV data.

There are also data from 600 to 2000 MeV/$c$ by Shepard *et al.* [14] which show the same effect of invariance with respect to the shape of the peak in the charge-exchange region. The data of Miller *et al.* [15] span the region of 4–11.75 GeV/$c$, those of Stone *et al.* [16] from 5 to 12 GeV/$c$, those of Engler *et al.* [17] from 8 to 22.5 GeV/$c$, those of Kreisler *et al.* [18] from 9.8 to 23 GeV/$c$, and Manning *et al.* [19] give an angular distribution at 8 GeV/$c$. All of these data show the same effect, a strong peak in the charge exchange region with the same shape as a function of momentum transfer.

We have plotted, as representative, the data from Böhmer *et al.* [20] (10.5–22.5 GeV/$c$), Babaev *et al.* [21] (23.5–62.5 GeV/$c$), and Barton *et al.* (75–105 GeV/$c$) [22]. The last mentioned data set extends to even higher energies but, as we shall see shortly, the scaling with the square of the center-of-mass energy appears to end in this region.

Figure 1 shows the data of Böhmer scaled as explained above. Note that the data at different energies around momentum transfers of 350–400 MeV/$c$, where the errors are the smallest, are consistent with each other to within the stated errors ($\pm 7\%$). The scaling factor varies from 4.29 to 8.76 over the energy range of these data. The scaled Böhmer data are about 20–30 % lower than those of Jain *et al.* (shown as the solid dots).

The data of Babaev *et al.* are also shown in Fig. 1. The scaling factor varies from 9.13 to 23.69 over the energy range of the data. Note that the data at 23.5 GeV/$c$ (open stars) scale well with the data of Jain *et al.* while the data of Böhmer *et al.* at 22.5 GeV/$c$ are 20–30 % lower, so there are inconsistencies in normalization of the order of 20% between the two data sets. In fact typical normalization errors are of the order of 30%. Nonetheless the scaling is well established between 1.4 GeV/$c$ and 60 GeV/$c$ since the scaling factor varies by a factor of 23 over this range and the data are consistent with the scaling to the same level as they are with each other ($\approx 20\%$).

After seeing the scaling over such a wide energy range (and thinking of scaling as a high-energy phenomenon) one might believe that it will continue. However, as can be seen in Fig. 1 where the data from Barton *et al.* [22] are shown, it appears to begin to break down above 60 GeV/$c$. The lowest energy data (open triangles over a beam momentum range from 60 to 90 GeV/$c$) from this

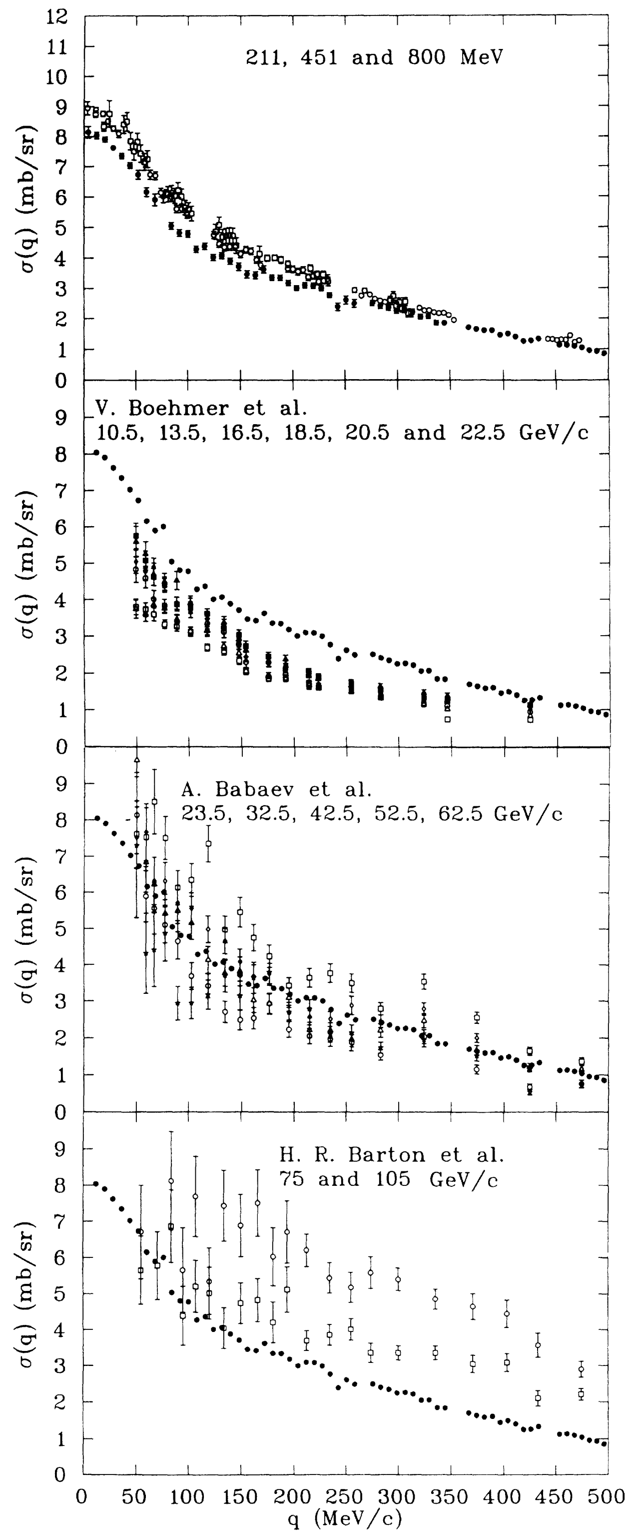


FIG. 1. Scaling of the differential cross section. The data of Jain *et al.* [12] at 800 MeV (1.4 GeV/$c$) is shown as solid circles in each of the figures. Data at 211 MeV is shown as the open boxes and 451 MeV by the open circles. For the data of Böhmer *et al.* [20] the beam momenta are open boxes; 10.5 GeV/$c$, open triangles; 13.5 GeV/$c$, open circles; 16.5 GeV/$c$, open diamonds; 18.5 GeV/$c$, solid boxes; 20.5 GeV/$c$ and solid triangles; 22.5 GeV/$c$. For the data of Babaev *et al.* [21] the beam momenta are solid stars; 23.5 GeV/$c$, solid circles; 32.5 GeV/$c$, open circles; 42.5 GeV/$c$, solid triangles; 52.5 GeV/$c$ and open triangles; 62.5 GeV/$c$. For the data of Barton *et al.* [22] the beam momenta are open triangles; 60–90 GeV/$c$ and solid triangles; 90–120 GeV/$c$.

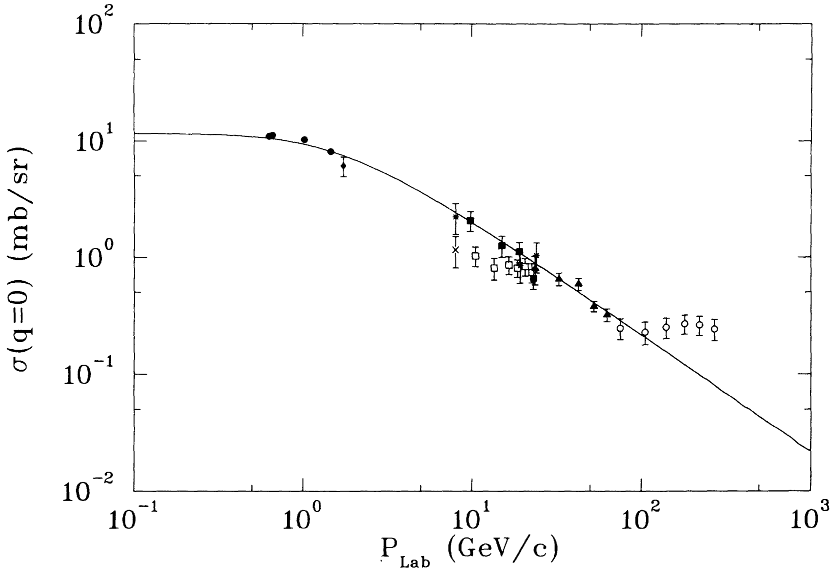


FIG. 2. Scaling of the cross section at $q = 0$. Note that approximate scaling with $s$ holds over a very wide energy range.

set show a departure from the scaling and for the next set (90–120 GeV/$c$ solid triangles) the difference is even greater. This tendency continues with the higher energy data (not shown) departing more and more from the scaling rule. Note that the (unscaled) experimental cross section is continuing to drop in this region (the scaling factor is about 40) so if there is a contribution to the cross section which does not enter with this same scaling factor it will become more important as the energy is increased. Of course experimental background is also a larger problem than at the lower energies.

Another way to display the scaling is to plot the charge-exchange differential cross section at zero momentum transfer as a function of laboratory momentum. Figure 2 shows such a plot with the line indicating the value of the expected cross section, using the above scaling factor, based on the 800 MeV data, i.e., the line is just the function $[s(800)/s(E)]\sigma(800)$. The scaling is seen to hold approximately from very low energies to a laboratory momentum of the order of 100 GeV/$c$.

## III. ONE-PION EXCHANGE

The study of the neutron-proton charge exchange, viewed from the perspective of pion exchange, has a long history. Chew [23] pointed out that by extrapolating to the pion pole in the $np$ charge exchange reaction the residue of the pole could be found and would supply the pion-nucleon coupling constant. A modified version of this method, using the full form of the pion amplitude, was applied by Ashmore *et al.* [24] very successfully even though the observed charge exchange cross section did not resemble the one-pion-exchange cross section since the measured values showed a strong peak at 180 degrees instead of the zero expected from ordinary OPE. Ashmore *et al.* simply parametrized the form of this peak by a polynomial in $\cos\theta$. Bongardt *et al.* [25] studied $np$ charge exchange with the goal of determining the pion-nucleon form factor and parametrized the peak as a Gaussian function. Cass and McKellar [26] and Dominguez and Verwest [27] treated the problem from the point of view of Regge, also to extract the range of this form factor.

We note immediately that the ordinary one-pion exchange, which one might think would be important at small momentum transfer because of its small mass and hence long range, leads us to expect a cross section which is totally wrong for the differential cross section since it predicts zero for the 180° elastic cross section (instead of a maximum). Several workers have associated the peak with the removal of the delta function of the one-pion-exchange potential [28, 6]. It was sometimes assumed that the absorption of the inner partial waves, because of pion production, was the reason for the suppression of the $\delta$ function [28]. However, as we already noted, the character of the peak does not change as the threshold for pion production is crossed or, indeed, as the total cross section for $np$ scattering goes from totally elastic to mostly inelastic.

### A. One-pion-exchange amplitude

As a first orientation to the ordinary one-pion-exchange amplitude we consider the spin-averaged case which has the form

$$\frac{q^2}{q^2+\mu^2}, \tag{1}$$

where $\mathbf{q} = \mathbf{k} - \mathbf{k}'$ and $\mathbf{k}$ and $\mathbf{k}'$ are the $NN$ initial and final momenta in the center of mass. Assuming a neutron beam, if $\mathbf{k}'$ corresponds to the outgoing neutron the momentum transfer is appropriate to elastic scattering while if it corresponds to the final proton then $q$ is the momentum transfer of the charged pion. If one makes the partial-wave expansion of Eq. (1), the amplitudes are seen to have the following interesting property. For all values of $\ell > 0$ the amplitudes are positive while the amplitude for $\ell = 0$ is negative. Since the point at $q = 0$ (where the amplitude is zero) corresponds to the sum of all partial waves, we see that the value of the $\ell = 0$ contribution is equal to the sum of the others. This violates the usual expectation that the high partial waves dominate at high energy and small momentum transfer.

Since the amplitude in Eq. (1) is finite as $q \to \infty$, it leads to a potential with a $\delta$ function in $r$ space

$$V_0(r) \propto \mu^2 \frac{e^{-\mu|\mathbf{r}_1 - \mathbf{r}_2|}}{|\mathbf{r}_1 - \mathbf{r}_2|} + \delta(\mathbf{r}_1 - \mathbf{r}_2), \tag{2}$$

where $\mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2$.

From a strictly pragmatic point of view, this $\delta$ function can be "removed" by modifying the amplitude in two ways which, we point out, are not equivalent. One way is to subtract the asymptotic constant for the limit of large $q$. A second way is to multiply the amplitude by a function of $q$ which tends to zero for large $q$. These two prescriptions are, in general, not equivalent at $q = 0$.

From the point of view of pion exchange between quarks, we now argue, it should be eliminated in *both* ways. Consider the exchange of a pion between two quarks. The two quarks can only communicate at large distances by emitting colorless mesons, the lightest being the pion. If the quarks are separated by a distance greater than the size of the confinement region of the pion then the (spin-averaged) potential will be given by Eq. (2) above. If the two quarks are close together, on a scale of the size of the pion, then the potential given by Eq. (2) is no longer correct and will (usually) be reduced. We can "correct" the potential between pointlike quarks by multiplying by a function which "cuts off" the potential at short distances, e.g.,

$$V(r) = V_0(r)(1 - e^{-(r/R)^2}), \tag{3}$$

where $R$ is the scale of the pion diameter. Transforming back to momentum space and taking the limit as $R \to 0$ we obtain

$$\frac{q^2}{q^2 + \mu^2} - 1 = -\frac{\mu^2}{q^2 + \mu^2}. \tag{4}$$

We have simply set the quark-quark interaction to zero inside the range of $R$ whereas we should have replaced it with some other interaction. If the radius of the pion is very small the error in neglecting this interior potential will also be small.

The true spatial extent of the pion is not well known. The electromagnetic form factor has a range corresponding to a radius of the order of 0.6 fm [29] but it is not clear how this is related to the size of the pion since the photon exchange at the energies used is dominated by vector meson exchange. It has been found [30] that the wave function of the pion may have an extent considerably smaller than the region defined by the radius of the bag.

One way to estimate the size of the pion is to use the exchange of soft photons, i.e., the Coulomb energy difference between the neutral and charged pions. Since this isospin breaking is isotensor in character, the mass difference must arise entirely from electromagnetic effects. Using the naive quark model and the pure Coulomb energy we see that the mass difference is given by (see, e.g., Close [31])

$$m_{\pi^+} - m_{\pi^0} = \frac{1.44}{R}\frac{1}{2},$$

where the factor $\frac{1}{2}$ arises from the product of fractional quark charges. This estimate implies that the distance between $q$ and $\bar{q}$ must be of the order of $R \approx 0.16$ fm, giving a radius of $\approx 0.08$ fm. If this estimate is correct the pion is only 1/10 the size of the nucleon.

Equation (4) is the form of the amplitude to be expected between any two quarks. We now wish to take into account the distribution of the quarks over the two nucleons. Placing the center of mass of the two nucleons at the origin, the centers of the two nucleons will be at $\frac{\mathbf{r}}{2}$ and $-\frac{\mathbf{r}}{2}$ and the coordinates of the six quarks are (see Fig. 3)

$$\mathbf{r}_1 = \frac{\mathbf{r}}{2} + \mathbf{u}_1, \quad \mathbf{r}_2 = \frac{\mathbf{r}}{2} + \mathbf{u}_2, \quad \mathbf{r}_3 = \frac{\mathbf{r}}{2} + \mathbf{u}_3,$$

$$\mathbf{r}_4 = -\frac{\mathbf{r}}{2} + \mathbf{u}_4, \quad \mathbf{r}_5 = -\frac{\mathbf{r}}{2} + \mathbf{u}_5, \quad \mathbf{r}_6 = -\frac{\mathbf{r}}{2} + \mathbf{u}_6,$$

where the vectors $\mathbf{u}_i$ are coordinates of the quarks relative to the center of mass of the nucleons. Taking the expectation value over the wave function of the quarks we have

$$-\frac{\mu^2}{q^2 + \mu^2} \int d\mathbf{u}_1 d\mathbf{u}_4 \rho(\mathbf{u}_1)\rho(\mathbf{u}_4) e^{i\mathbf{q}\cdot\mathbf{r}_1}\, e^{-i\mathbf{q}\cdot\mathbf{r}_4}$$

$$= -\frac{\mu^2}{q^2 + \mu^2} F_n(q) F_p(q), \tag{5}$$

where $F_n(q)$ and $F_p(q)$ represent the Fourier transform of the quark density of the neutron and proton.

Thus we see that the finite size of the *pion* leads to a subtraction of the $\delta$ function while the finite size of the *nucleon* results in a multiplication by a form factor (which would smear the $\delta$ function over the size of the nucleon).

We assume, for simplicity, a density of the form

$$\rho(r) = \frac{\Lambda^2}{4\pi} \frac{e^{-\Lambda r}}{r} \tag{6}$$

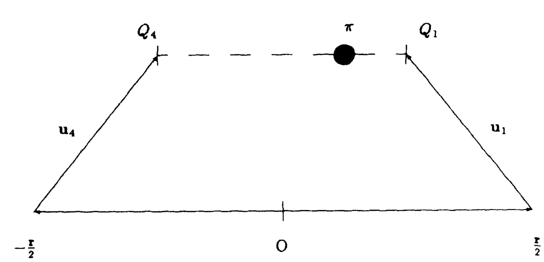


FIG. 3. Position of quarks and the pion in $r$ space.

so that each integral contributes a factor $\frac{\Lambda^2}{\Lambda^2+q^2}$. This density has an rms radius given by $\langle r^2\rangle = \frac{6}{\Lambda^2}$. Thus for a quark density with an rms radius of 0.86 fm (the charge radius of the proton) we expect a value of $\Lambda$ of order of 560 MeV/$c$. Since the quarks in the pion cloud do not constitute a source of pions (to the extent that they are contained as $q\bar{q}$ pairs in true pions), one might expect a smaller radius and hence a larger value of $\Lambda$. For a radius of 0.6 fm we have 806 MeV/$c$ and 0.5 fm corresponds to 967 MeV/$c$. Recently Coon and Scadron [32] have studied the value of $\Lambda$ in relation to the Goldberger-Treiman discrepancy and conclude that $\Lambda$ most likely lies near 800 MeV/$c$.

Note that we have assumed that the quark density in each nucleon is unchanged from its noninteracting value. The two nucleons overlap for only a very short time and presumably do not have the time to rearrange their distributions.

There is an important point with respect to spin. The spin structure of the pion amplitude for charge exchange leads to only two nonzero helicity amplitudes $\phi_2$ and $\phi_3$ (see Appendix A). The first of these represents an amplitude with no helicity flip while the second has helicity flip. Writing the general expansion of these amplitudes in terms of Legendre functions, the first is expressed in ordinary Legendre polynomials, $P_L(\cos\theta)$, while the second is expressed in terms of associated Legendre functions which vanish at 0°. This means that no matter what potential is used $\phi_3$ must vanish at 0° while $\phi_2$ may have a finite value at zero $q$, such as that given by Eq. (4). Hence a first inclination is to convert $\phi_2$ to the form of Eq. (4) and leave $\phi_3$ as it was. In this case one would have the incoherent sum of squares of two amplitudes, one of which vanishes at zero momentum transfer and one of which has a peak at this same point. As we shall see this is not quite the correct procedure but it is close to the truth.

We now return to a treatment of the one-pion-exchange amplitude with a full consideration of the spin degree of freedom. Whether one starts with pseudoscalar or pseudovector coupling, when the amplitude is evaluated in terms of nonrelativistic invariants the expression for the OPE amplitude is the same on shell. It is only when the amplitude is extended to off-shell nucleons that it is necessary to make a choice. Thus when discussing the OPE amplitude (the Born approximation) or iterating this potential in the Schrödinger equation the choice of coupling is irrelevant. When (later) we calculate a potential from the exchange of two pions it is the pseudoscalar form which is used.

Considering the spin dependent amplitude, we see that we must remove the $\delta$ function from the $s$-wave portion of the amplitude only. Since the $s$ wave is given by the angle average

$$\frac{1}{4\pi}\int d\Omega \frac{\sigma_1\cdot\mathbf{q}\sigma_2\cdot\mathbf{q}}{q^2+\mu^2} = \frac{1}{3}\frac{q^2\sigma_1\cdot\sigma_2}{q^2+\mu^2}, \tag{7}$$

we see that the spin-dependent normalization of the subtraction is $\frac{1}{3}\sigma_1\cdot\sigma_2$. Using the tensor form

$$S_{12} \equiv 3\sigma_1\cdot\hat{\mathbf{q}}\sigma_2\cdot\hat{\mathbf{q}} - \sigma_1\cdot\sigma_2 \tag{8}$$

we can separate the amplitude into $s$-wave (spin-spin) and tensor parts

$$\frac{\sigma_1\cdot\mathbf{q}\sigma_2\cdot\mathbf{q}}{q^2+\mu^2} \to \frac{\sigma_1\cdot\mathbf{q}\sigma_2\cdot\mathbf{q}}{q^2+\mu^2} - \frac{1}{3}\sigma_1\cdot\sigma_2 = \frac{1}{3}\frac{q^2 S_{12}}{q^2+\mu^2} - \frac{1}{3}\frac{\sigma_1\cdot\sigma_2\mu^2}{q^2+\mu^2}. \tag{9}$$

We see that it is only the spin-spin part of the amplitude (and potential) which is modified. Note that the residue at the nucleon pole has not been altered by this procedure since it has resulted in the replacement of $q^2$ in the numerator of the spin-spin term by $-\mu^2$, its value at the pole.

We shall use isospin invariance to include the neutral pion exchange. To separate the contributions of the exchange of charged and neutral pions in the expressions it is convenient to write the two momentum transfers involved as

$$p^2 = 2k^2(1-\cos\theta) \tag{10}$$

for the elastic channel ($\pi^0$ exchange) and

$$q^2 = 2k^2(1+\cos\theta) \tag{11}$$

for the charge-exchange channel ($\pi^\pm$ exchange) where $\theta$ is the angle for elastic scattering.

For the amplitudes of Bystricky *et al.* [33] with the $\delta$ function removed we find

$$a = \frac{N}{3}[g(p) + 2g(q)], \tag{12}$$

$$b = \frac{N}{3}[-g(p) - 6f(q)g(q) + 4g(q)], \tag{13}$$

$$c = \frac{N}{3}[-3f(p)g(p) + 2g(p) - 2g(q)], \tag{14}$$

$$d = N[-f(p)g(p) + 2f(q)g(q)], \tag{15}$$

$$e = 0, \tag{16}$$

where we have used the notation

$$g(x) \equiv \left(\frac{\Lambda^2}{x^2+\Lambda^2}\right)^2, \qquad f(x) \equiv \frac{x^2}{x^2+\mu^2}, \tag{17}$$

and the normalization is given by

$$N = \left(\frac{M_n}{m_\pi}\right)^2 \frac{2}{\sqrt{s}}\frac{f_\pi^2}{4\pi}\left(\frac{\Lambda^2-\mu^2}{\Lambda^2}\right)^2. \tag{18}$$

The factor of $\frac{1}{\sqrt{s}}$ is the source of the energy dependence which provided the scaling assumed above. These equations reduce to those of Bystricky *et al.* [34], with the $\delta$ function included and no form factor, if we make the replacements $f(q)g(q) \to f(q)$, $f(p)g(p) \to f(p)$, and

$g(p) = g(q) \to 0$ when $g(p)$ or $g(q)$ is alone. The amplitudes in Eqs. (12)–(16) are also given in Appendix A in terms of OPE potentials in momentum space.

In this notation, when the amplitudes are real, we can write

$$\sigma(\theta) = \frac{a^2 + b^2 + c^2 + d^2}{2}, \tag{19}$$

$$K_{SS}(\theta) = \frac{ac - bd}{\sigma}, \tag{20}$$

$$K_{LL}(\theta) = \frac{ac + bd}{\sigma}, \tag{21}$$

and

$$K_{NN}(\theta) = \frac{a^2 - b^2 + c^2 - d^2}{2\sigma}. \tag{22}$$

The experimental definition of the spin variables are given several places (for example, see Ref. [3]). If the energy is very high we may neglect the contribution from the neutral pion exchange in the charge-exchange region so that the expressions for the amplitudes simplify. In this limit

$$\sigma(\theta) = \tfrac{4}{3} N^2 g^2(q) \left[1 + 3f^2(q) - 2f(q)\right], \tag{23}$$

$$K_{SS}(\theta) = \frac{-\frac{1}{3} + 3f^2(q) - 2f(q)}{1 + 3f^2(q) - 2f(q)}, \tag{24}$$

$$K_{LL}(\theta) = \frac{-\frac{1}{3} - 3f^2(q) + 2f(q)}{1 + 3f^2(q) - 2f(q)}, \tag{25}$$

and

$$K_{NN}(\theta) = K_{LL}(\theta). \tag{26}$$

Thus we see, in this simple limit, that all three spin observables start at $q = 0$ with a value of $-\frac{1}{3}$ and, for large $q$, $K_{SS} \to \frac{1}{3}$, $K_{LL} = K_{NN} \to -\frac{2}{3}$. $K_{SS}$ obtains its negative limit of $-1$ at $q^2 = \frac{\mu^2}{2}$ where $f(q) = \frac{1}{3}$, while $K_{LL}$ has a maximum (with a value of zero) at this same point. The cross section is no longer zero at $q = 0$ but has the value $\frac{4}{3}N^2$.

### B. One-pion-exchange potential

While we have supposed that the Born approximation is qualitatively reasonable, the potential corresponding the amplitudes considered above is needed for the iteration of the OPE, by insertion in the Schrödinger equation, for example. The spin-spin component has the form

$$V_{ss}(r) = \frac{f_\pi^2}{4\pi}\left[\frac{e^{-\mu r} - e^{-\Lambda r}}{r} - \frac{\Lambda^2 - \mu^2}{2\Lambda} e^{-\Lambda r}\right]. \tag{27}$$

We have taken the coupling constant to be normalized at the pion pole (and not at $q = 0$). Note that the potential with the $\delta$ function (the original one-pion-exchange distributed over the quark density in the nucleon, or simply multiplied by a form factor for whatever reason) is given by

$$V_{ss}^{\delta}(r) = \frac{f_\pi^2}{4\pi}\left[\frac{e^{-\mu r} - e^{-\Lambda r}}{r} - \frac{\Lambda(\Lambda^2 - \mu^2)}{2\mu^2} e^{-\Lambda r}\right] \tag{28}$$

so that the large difference seen in the cross section and spin-transfer observables is not obvious in the form of the potential. The difference is a little clearer when one observes that the coefficient of the last term has the proper normalization for a $\delta$ function (proportional to $\Lambda^3$ in the limit of large $\Lambda$) in the second case and not in the first.

The tensor potential remains unchanged by the subtraction of the $\delta$ function:

$$\begin{aligned} V_T(r) = \frac{f^2\mu}{4\pi}\Bigg[ & e^{-\mu r}\left(\frac{1}{\mu r} + \frac{3}{\mu^2 r^2} + \frac{3}{\mu^3 r^3}\right) \\ & -\Lambda^3 e^{-\Lambda r}\left(\frac{1}{\Lambda r} + \frac{3}{\Lambda^2 r^2} + \frac{3}{\Lambda^3 r^3}\right) \\ & -\frac{\Lambda(\Lambda^2 - \mu^2)}{2\mu^3} e^{-\Lambda r}\left(1 + \frac{1}{\Lambda r}\right)\Bigg]. \end{aligned} \tag{29}$$

With the potential given by Eqs. (27) and (29), the binding energy and external observables of the deuteron (including the form factor "$A$" [35]) are correctly given with $\Lambda = 748$ MeV/$c$ and $\frac{f_\pi^2}{4\pi} = 0.079$ or $\frac{g_\pi^2}{4\pi} = \frac{4M^2}{\mu^2}\frac{f_\pi^2}{4\pi} = 14.4$. A similar potential was also used in Ref. [36] in the study of relativistic two-body equations.

### C. Comparison with observables

Even more spectacular than the cross section scaling noted above is the invariance in the new data in the spin variables over the energy range from 485 to 788 MeV. All spin observables are very small in the region of momentum transfer from zero to 500 MeV/$c$ except for the spin-transfer variables $K_{LL}$, $K_{NN}$, and $K_{SS}$. As can be seen in Fig. 4 these quantities change very little when plotted as a function of momentum transfer for a charged particle, $K_{NN}$ more than the others. The values of the spin observables for pure OPE (with the $\delta$ function included) are $K_{NN} = -1$, $K_{SS} = +1$ and $K_{LL} = -1$ (except very close to zero momentum transfer in the charge-exchange channel). These values are seen to have no relationship to the data.

Figure 4 also shows a calculation of the four observables for the full one-pion-exchange amplitude (i.e., with the neutral pion contribution included) with the $\delta$ function removed. It may be seen that there is a considerable improvement over the result with the $\delta$ function included.

The spin observables are calculated with a value of $\Lambda$ of 600 MeV/$c$ and at an energy of 800 MeV. The separation of the two curves in Fig. 4 between $K_{LL}$ and $K_{NN}$ is due to the presence of the neutral pion exchange and is energy dependent. This splitting suggests that measurements of these two quantities at higher energies could be useful to determine whether, as the importance of the neutral pion

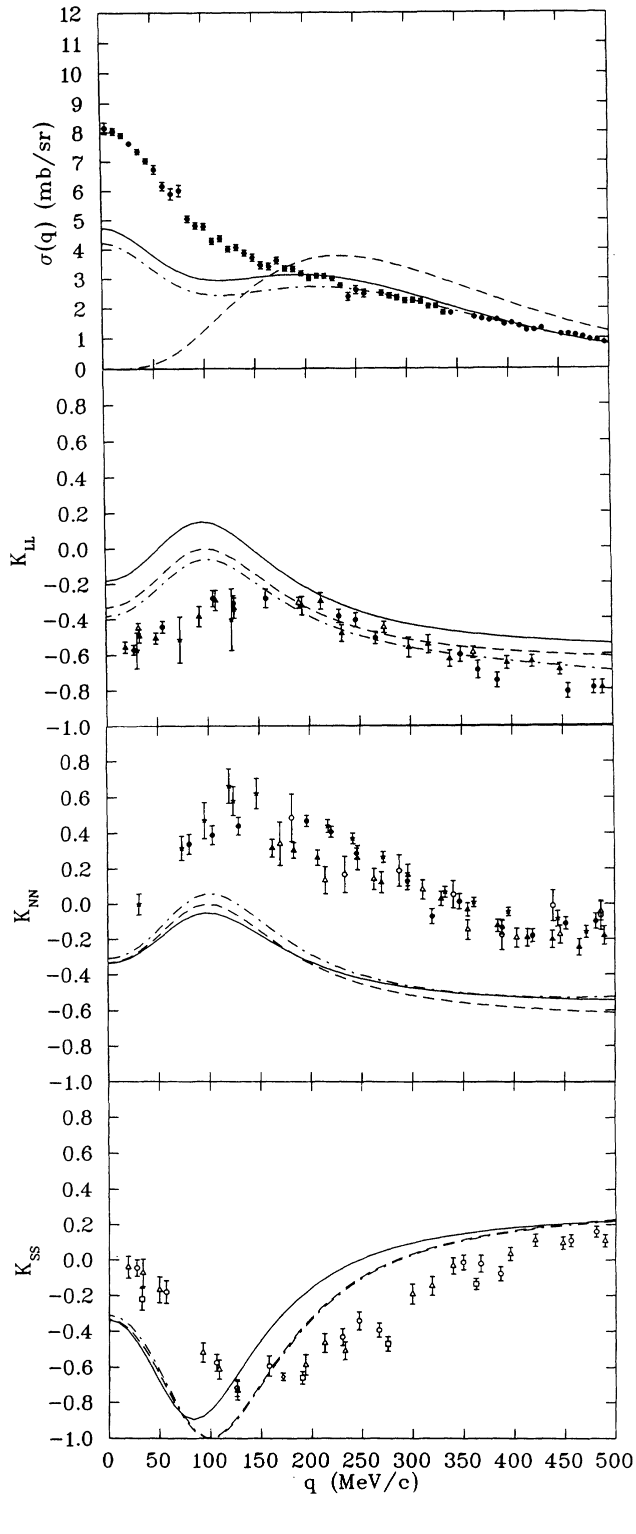


FIG. 4. Comparison of the OPE amplitudes with the four observables. In the figure for the cross section, calculations with the one-pion-exchange amplitude are shown with the $\delta$ function included (dashed curve) removed (dashed-dotted curve). The neutral pion is included in both cases. For the spin observables the curves show the calculation with (dash-dotted) and without (dashed) the exchange of the neutral pion with the $\delta$ function removed. In all four figures the solid curve shows the result of using the potentials corresponding to the dash-dotted curve in the Schrödinger equation. The energies of the spin-transfer variables are $K_{LL}$; 485 MeV open triangles, 635 MeV solid triangles, 788 MeV solid circles, 790 MeV open stars. $K_{NN}$; 425 MeV open triangles, 485 MeV solid triangles, 495 MeV solid circles, 788 MeV open stars, 790 MeV solid stars. $K_{SS}$; 485 MeV open squares, 506 MeV open diamonds, 635 open triangles, 788 MeV open circles.

exchange diminishes, the two observables become closer.

We see that reasonable (at least qualitative) agreement with the spin observables is realized in spite of the fact that there are (essentially) no parameters involved, the value of $\Lambda$ being of minor importance. Except for the exchange of the neutral pion, the spin variables are independent of $\Lambda$ as can be seen from Eqs. (24)–(26).

Also in Fig. 4 the values of $K_{SS}$ are compared with the same calculation. It is seen that the data have a minimum in the region expected from pion exchange but the value of the function at the minimum is not $-1$. This can be taken as an indication of the degree of importance of the iteration of the potential to higher orders or of the exchange of other mesons since, for the one-pion exchange alone there is always a minimum value of $-1$. Note that the minima and maxima of the data do indeed fall near $q = \mu/\sqrt{2}$ providing a definite signal of pion exchange.

Iteration of the OPE potential [Eqs. (27) and (29)], in the Schödinger equation, increases $d\sigma/d\Omega$ of the Born calculation leaving it still below experiment at low $q$ (Fig. 4). The result for the spin-transfer parameter, $K_{NN}$, is not very different from the Born OPE. Its slightly lower maximum does not get closer to experiment. $K_{LL}$ is shifted toward positive values away from experiment. $K_{SS}$ is close to the Born result, however, with the value at its minimum increased from $-1.0$ to $-0.8$ in better agreement with data.

Even though the OPE Born term with the $\delta$ function subtracted and its Schödinger iteration are much closer to data than the OPE with the $\delta$ function included, some additional shorter-range contributions are necessary to have a more realistic model. The next medium-range meson which can carry charge is the $\rho$ with a mass of 770 MeV. One should take into account its relatively large width of 150 MeV. An approach consistent with the present view of pion exchange, developed by the Paris Group [37,38], is to consider the $\rho$ exchange as that of two correlated pions in the $\pi\pi$ $P$ wave. In the next section we study the correction to OPE, arising from two-pion exchange, calculated via dispersion relations following Ref. [37] modified in a schematic manner by considering pion exchange between quarks.

## IV. EXCHANGE OF TWO PIONS

We treat the exchange of heavier mesons from the point of view of the exchange of interacting pions, as was done for the $\sigma$ and $\rho$ mesons in the Paris potential [37]. Note that we wish to obtain the dependence on the radial variable, $r$, of the potential for short range. We will always assume that the asymptotic part of the interaction is as given by the Paris Group [37]. In order to make this reduction it will be necessary to write the same dispersion relation as was used in this case. We put the nucleons in the intermediate state on shell and hence the fourth components of the momenta of the exchanged pions do not enter.

For the exchange of a single pion between quarks there are only two quarks involved in the process, one in each nucleon. For the exchange of two pions three cases are possible: (1) Both pions are emitted by a single quark in one nucleon and both absorbed on one quark in the other nucleon. (2) Both pions are emitted by (absorbed on) one quark in one nucleon and absorbed on (emitted by) two different quarks in the other nucleon. (3) The pions are emitted by two different quarks in one nucleon and absorbed on two different quarks in the other nucleon. In the first case the full momentum transfer must be found on a single quark and the expression is simple. For the other cases the situation is more complicated.

Consider, as an example, the case in which both pions are emitted from quark 1 and are absorbed by quarks 4 and 5. Assuming an uncorrelated quark density in the nucleon we have for the nontrivial part of the expectation value on the quark wave functions:

$$\int d\mathbf{u}_1 d\mathbf{u}_4 d\mathbf{u}_5 \rho(\mathbf{u}_1)\rho(\mathbf{u}_4)\rho(\mathbf{u}_5) e^{i\mathbf{s}\cdot\mathbf{r}_1} e^{-i\mathbf{s}\cdot\mathbf{r}_4} e^{i\mathbf{s}'\cdot\mathbf{r}_1} e^{-i\mathbf{s}'\cdot\mathbf{r}_5}, \tag{30}$$

where $\mathbf{s}$ and $\mathbf{s}'$ are the momenta of the two pions being exchanged. After the change of variables

$$\mathbf{q} = \mathbf{s} + \mathbf{s}' \quad \text{and} \quad \mathbf{t} = \mathbf{s} - \mathbf{s}',$$

we have

$$e^{i\mathbf{q}\cdot\mathbf{r}} \left(\frac{\Lambda^2}{\Lambda^2 + q^2}\right) \left(\frac{\Lambda^2}{\Lambda^2 + (\frac{\mathbf{q}+\mathbf{t}}{2})^2}\right) \left(\frac{\Lambda^2}{\Lambda^2 + (\frac{\mathbf{q}-\mathbf{t}}{2})^2}\right). \tag{31}$$

Thus for all three cases averaging over the quark densities leads to

$$\frac{1}{9}\left(\frac{\Lambda^2}{\Lambda^2 + q^2}\right)^2 + \frac{4}{9}\left(\frac{\Lambda^2}{\Lambda^2 + q^2}\right)\left(\frac{\Lambda^2}{\Lambda^2 + (\frac{\mathbf{q}+\mathbf{t}}{2})^2}\right)\left(\frac{\Lambda^2}{\Lambda^2 + (\frac{\mathbf{q}-\mathbf{t}}{2})^2}\right) + \frac{4}{9}\left(\frac{\Lambda^2}{\Lambda^2 + (\frac{\mathbf{q}+\mathbf{t}}{2})^2}\right)^2\left(\frac{\Lambda^2}{\Lambda^2 + (\frac{\mathbf{q}-\mathbf{t}}{2})^2}\right)^2, \tag{32}$$

where for the weights we have simply taken the number of possible cases without reference to the spin-isospin factors which surely enter. Thus we consider this derivation as a rough evaluation of two-pion exchange between the quarks in the nucleons.

The first term simply multiplies the result obtained for the exchange of a particle between two nucleons as was the case for the pion. The other two terms show a difference, however, due to the structure of the particle being exchanged. Even if the width of the $\rho$ is taken as zero the structure of the particle, considered as a $\pi$–$\pi$ resonance, manifests itself.

In order to include these terms in the modified dispersion relation we must calculate [e.g., for the second term in Eq. (32)] the discontinuity of the integral

$$\int dt \frac{1}{(\frac{q+t}{2})^2+\mu^2}\,\frac{1}{(\frac{q-t}{2})^2+\mu^2}\,\frac{1}{(\frac{q+t}{2})^2+\Lambda^2}\,\frac{1}{(\frac{q-t}{2})^2+\Lambda^2}\,F(t), \tag{33}$$

where $F(t)$ depends on the particular invariant amplitude being evaluated. The first two factors are from the pion propagators and the second two are the last two factors in Eq. (31). The function $F(t)$ is a generic notation for several spectral functions used in the dispersion relations in the Paris potential. In the present case it represents one of the invariant amplitudes of the exchange of two pions, either uncorrelated or interacting in the $s$ or $p$ wave. We assume that there is no dependence of $F(t)$ on $\mathbf{q}$ so the expression that we will obtain is, strictly speaking, only applicable to the spin-independent term. The effect on the spin observables of the exchange of two pions between quarks may be very important but is beyond the scope of the present work. We note, however, that the dependence on spin of the long-range part of the two-pion potential is the same as that given by the Paris potential. Using Cutkosky's rules [39] we obtain

$$\frac{\Lambda^4}{q(\Lambda^2-\mu^2)}\int_0^\infty tdt\frac{1}{2\Lambda^2+2\mu^2+q^2+t^2}\left[\frac{1}{4\mu^2+q^2+t^2}-\frac{1}{4\Lambda^2+q^2+t^2}\right]F(t) \tag{34}$$

or, making the change of variable

$$t'=t^2+4\mu^2,$$

the discontinuity of the second term of Eq. (32) becomes

$$T_2=\frac{\Lambda^2}{\Lambda^2+q^2}\frac{\Lambda^4}{q(\Lambda^2-\mu^2)^2}\int_{4\mu^2}^\infty dt'\left[\frac{1}{t'+q^2}+\frac{1}{4(\Lambda^2-\mu^2)+t'+q^2}-\frac{2}{2(\Lambda^2-\mu^2)+t'+q^2}\right]F(t). \tag{35}$$

The discontinuity of the third term in Eq. (32) can be found in a similar manner:

$$T_3=\frac{\Lambda^8}{(\Lambda^2-\mu^2)^4}\int_{4\mu^2}^\infty dt'\left\{\frac{1}{t'+q^2}+\frac{1}{4(\Lambda^2-\mu^2)+t'+q^2}-\frac{2}{2(\Lambda^2-\mu^2)+t'+q^2}\right.$$
$$\left.+\frac{4}{[4(\Lambda^2-\mu^2)+t'+q^2]^2}-\frac{4}{[2(\Lambda^2-\mu^2)+t'+q^2]^2}+\frac{8}{[4(\Lambda^2-\mu^2)+t'+q^2]^3}\right\}F(t). \tag{36}$$

In order to have the same two-pion-exchange discontinuity across the cut [the same residue for $\rho(t')$ at the pole $t'=-q^2$], and hence the same asymptotic behavior for large $r$, as the dispersion relation used in the Paris potential [37] we identify

$$\rho_{\mathrm{Paris}}(t')=\frac{F(t)}{9}\left[\frac{\Lambda^2}{\Lambda^2+q^2}+2\frac{\Lambda^4}{(\Lambda^2-\mu^2)^2}\right]^2. \tag{37}$$

All formulas necessary to calculate the potentials corresponding to the three different cases are given in Appendix B. For the two-pion exchange considered here we shall, in a first step, include the contribution from the uncorrelated box diagram with two-nucleon intermediate states (QTH) and the correlated two-pion exchange in the $S$ and $P$ waves used by the Paris Group [40]. We leave the rescattering contribution, e.g., the two-pion exchange with a nucleon and a $\Delta$ (1232) and two $\Delta$ intermediate states for a later study. These diagrams will also generate a two-pion range imaginary potential. The latter can be used to take into account, in an optical model approach, the nucleon-nucleon inelasticities present above the pion production threshold. As expected, the QTH and the $S$ wave add little to OPE at small $q$ but they increase $d\sigma/d\Omega$ at forward angles. The $P$ wave gives a $d\sigma/d\Omega$ which is too large, not only at small $q$ but also in the forward region at small $p$. Since a short-range repulsive contribution is needed, we introduce the three-pion-exchange contribution due to the $\omega$ exchange.

We should, as for the two-pion exchange, study the different possibilities for the exchange of three pions between the three quarks of each nucleon. This provides the nucleon size regularization, in terms of $\Lambda$, of the short-range part of the $\omega$ exchange. Going from one-pion to two-pion exchange the mass of the regularization increases from $\Lambda$ to a value $\geq b = \{\Lambda^2+[(\Lambda^2-\mu^2)/\Lambda]^2/2\}^{1/2}\simeq 3\Lambda/2$ (see Appendix B). We expect a similar increase from two- to three-pion exchange and we shall choose a regularization mass of $7\Lambda/3$. With an $\omega$ coupling similar to that used in [38], viz., 11.75, the forward peak comes out with a reasonable value $\sim$ 13 mb but $d\sigma/d\Omega$ at $q=0$ is around 17 mb, still too high. If we decrease the helicity amplitudes $f_-^1(t)$ of the $P$ wave by 15%, $d\sigma/d\Omega$, at $q=0$, drops to 12 mb. A reduction of 30% of $f_-^1(t)$ amplitude gives 8.12 mb. The spin-transfer observables are not very sensitive to these modifications. The results of calculations with this potential are shown in Fig. 5. The agreement with experiment becomes better for $K_{NN}$ at low $q$. The spin-transfer observables $K_{LL}$ and $K_{SS}$ are also in better agreement with experiment at low $q$ ($\lesssim$ 250 MeV/$c$). Similar results are obtained if one uses the $P$-wave helicity amplitudes of Ref. [41] where an uncertainty of 15% at the $\rho$ peak is quoted. Since the $np$ charge exchange differential cross section at low $q$ is very sensitive to the inclusion of the $\rho$ it can constrain the strength of the $\rho NN$ coupling.

It is important to note that the Born approximation for the full potential which gives a quantitative agreement with the data is good to the order of 20% (Fig. 5). Thus one should expect corrections only of this order as the energy is increased. In the present calculation there is a small energy dependence of the two-pion potential itself which could also give some deviation from the pure scaling with $s$.

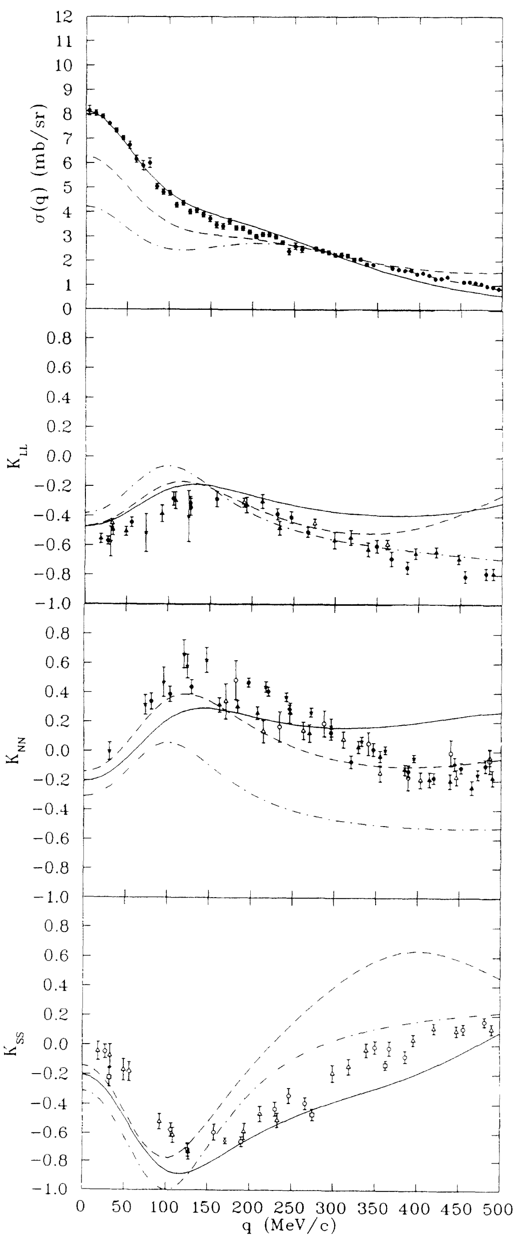


FIG. 5. Comparison of the data with calculations including the two-pion exchange. In each case the dash-dotted curve shows the Born approximation for the full OPE (same as Fig. 4), the dashed curve shows the Born approximation for the full multiple pion exchange potential discussed in the text and the solid curve shows the result of the solution of the Schrödinger equation with the full potential.

## V. CONCLUSIONS

We have investigated the neutron-proton charge exchange reaction and find a number of simplicities which can be understood in a semiquantitative manner. First, the differential cross section scales over a very large energy range with a factor which arises naturally from first order particle exchange. Second, the spin transfer variables are very nearly invariant (viewed as a function of momentum transfer) over the (much narrower but significant) energy range where they have been measured.

A natural understanding of these data is possible in terms of the exchange of pions between point constituents making up the nucleons. For the one-pion-exchange mechanism considered we do not need a detailed model of the source of the interaction because the interaction is normalized at large distances to a field theory which treats the nucleon as a point particle. Thus we can use the pion-nucleon coupling constant obtained from "standard" analyses.

We note that the *subtraction* of the $\delta$ function is necessary, as opposed to its *distribution* over the nucleon size by the multiplication by a form factor. The assumption of the underlying quark structure, with the subtraction arising from a correction due to the finite size of the *pion* provides a natural reason for the removal in this way. It is possible that other arguments can be found for its suppression in this form. In fact Refs. [9, 10] used a similar potential in their calculations of the deuteron properties. In that case the result is insensitive to the exact form of the spin-spin part of the interaction since the potential that binds the deuteron is dominated by the tensor component which is not affected by this subtraction. In the present case the removal of the $\delta$ function in this manner appears essential to obtain a nonzero cross section at 180° and values of the spin-transfer observables which correspond to the data rather than the trivial values of $\pm 1$.

Note that the one-pion-exchange interaction obtained in this manner is "almost" equally valid for all relative distances of the two nucleons, including complete overlap. This is in contrast to the usual approach in which the lower partial waves in the nucleon-nucleon interaction are treated phenomenologically by *replacing* the OPE in these waves. In the present approach it would appear to be more appropriate to *add* some needed supplemental contributions (phenomenological or fundamental) to the OPE already present. The "almost" could be removed in the sentence above if the size of the pion could be reduced to zero. Since there is some contribution to the potential between quarks when they are closer than the diameter of the pion and, since the probability of two quarks being close is greater if the two nucleons are close, the approximation is not independent of internucleon distances.

A common technique for transposing a field theory to finite sized objects is to multiply each vertex by a form factor. The present work seems to indicate that this may not be the proper procedure in the case of the *exchange* of finite sized particles.

For two-pion exchange only a part of the interaction can be expressed in a form resembling the exchange of a $\pi$-$\pi$ resonance (dispersed in mass). Other parts of the interaction contain explicit dependences on the momentum variables which occur because more than one quark in each nucleon may participate in this more complex process. We have again used the principle that at large internucleon distances the neutron and proton can be treated as point particles to obtain the normalization of the potential.

While it would be difficult to consider that the data and its analysis provides the definitive signal for the quark degrees of freedom long searched for in low-energy hadronic physics, the assumption of an underlying quark structure certainly does provide a simple and logical basis for the understanding of the cross section and spin-transfer variables.

## ACKNOWLEDGMENTS

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## APPENDIX A: RELATIONS AMONG AMPLITUDES

In the Born approximation we can write the amplitudes $a$, $b$, $c$, $d$, and $e$ in terms of nonantisymmetrized central, spin-spin, tensor, spin-orbit, and quadratic spin-orbit potentials arising from any meson exchange as follows.

We define the momentum-space potentials by

$$V = V_C + V_{SS}\sigma_1\cdot\sigma_2 + V_T S_{12} + V_{LS}\tilde{\Omega}_{LS} + V_{SO2}\tilde{\Omega}_{SO2}, \tag{A1}$$

where the last two invariants are defined by

$$\tilde{\Omega}_{LS} = \frac{i}{2}(\sigma_1 + \sigma_2)\cdot\mathbf{k}\times\mathbf{k}' \tag{A2}$$

and

$$\tilde{\Omega}_{SO2} = \sigma_1\cdot\mathbf{k}\times\mathbf{k}'\ \sigma_2\cdot\mathbf{k}\times\mathbf{k}'. \tag{A3}$$

One can show that the neutron-proton amplitudes are

$$\begin{aligned} a = {} & \tfrac{1}{2}[W_C + W_{SS} - W_T - V_C - V_{SS} + V_T \\ & + (W_{SO2} - V_{SO2})k^4\sin^2\theta] \\ & - U_C - U_{SS} + U_T - V_{SO2}k^4\sin^2\theta, \end{aligned} \tag{A4}$$

$$\begin{aligned} b = {} & \tfrac{1}{2}[W_C - W_{SS} + W_T - V_C + V_{SS} - V_T \\ & - (W_{SO2} - V_{SO2})k^4\sin^2\theta] - 2U_{SS} - U_T, \end{aligned} \tag{A5}$$

$$\begin{aligned} c = {} & \tfrac{1}{2}[2W_{SS} + W_T - 2V_{SS} - V_T] - U_C + U_{SS} \\ & - U_T + U_{SO2}k^4\sin^2\theta, \end{aligned} \tag{A6}$$

$$d = \tfrac{3}{2}[W_T - V_T + 2U_T], \tag{A7}$$

$$ie = -\frac{k^2}{4}(W_{LS} - V_{LS} + 2U_{LS})\sin\theta. \tag{A8}$$

In Eqs. (A4)–(A8) $W$ represents the contribution of the exchange of an isospin zero particle, $V$ the exchange of a particle with isospin $T = 1$, $T_z = 0$ and $U$ the exchange of a $T = 1$, $T_z \neq 0$ particle. Isospin conservation requires $U(\theta) = V(\pi - \theta)$.

For one-pion exchange Eqs. (A4)–(A8) reduce to

$$a = -\tfrac{1}{2}(V_{SS} - V_T) - U_{SS} + U_T, \tag{A9}$$

$$b = \tfrac{1}{2}(V_{SS} - V_T) - 2U_{SS} - U_T, \tag{A10}$$

$$c = \tfrac{1}{2}(-2V_{SS} - V_T) + U_{SS} - U_T, \tag{A11}$$

$$d = -\tfrac{3}{2}V_T + 3U_T, \tag{A12}$$

$$e = 0. \tag{A13}$$

Here $V$ represents the contribution of the $\pi^0$ and $U$ that of the charged pions and

$$V_{SS}(p) = \frac{2N}{3}[f(p)g(p) - g(p)], \tag{A14}$$

$$V_T(p) = \frac{2N}{3}f(p)g(p), \tag{A15}$$

where $U_{SS}$ and $U_T$ have the same definition with argument $q$. It can be verified that, with these substitutions, Eqs. (A9)–(A12) reduce to Eqs. (12)–(16).

We have for the neutron-proton OPE helicity amplitudes:

$$\phi_1 = \frac{N}{6}\{[g(p) + 2g(q)]\cos\theta - 3g(p) + 6g(q)\}, \tag{A16}$$

$$\begin{aligned} \phi_2 = {} & \frac{N}{6}\{[g(p) + 2g(q)]\cos\theta + 3g(p) - 6g(q) \\ & - 6f(p)g(p) + 12f(q)g(q)\}, \end{aligned} \tag{A17}$$

$$\phi_3 = \frac{N}{6}\{[g(p) + 2g(q)](1 + \cos\theta) - 12f(q)g(q)\}, \tag{A18}$$

$$\phi_4 = \frac{N}{6}\{[g(p) + 2g(q)](1 - \cos\theta) - 6f(p)g(p)\}, \tag{A19}$$

$$\phi_5 = -\frac{N}{6}[g(p) + 2g(q)]\sin\theta. \tag{A20}$$

For charged pion exchange only and with the $\delta$ function included, these amplitudes reduce to

$$\phi_1 = \phi_4 = \phi_5 = 0, \tag{A21}$$

$$\phi_2 = -\phi_3 = 2Nf(q). \tag{A22}$$

TABLE I. Basic functions for the different components of the potential in configuration space.

| Potential | Central and spin-spin | Spin-orbit | Tensor and (spin-orbit)$^2$ |
|---|---|---|---|
| $f^{(0)}(b)$ | $\frac{e^{-br}}{r}$ | $-\frac{be^{-br}}{r^2}\left(1+\frac{1}{br}\right)$ | $\frac{b^2e^{-br}}{r}\left(1+\frac{3}{br}+\frac{3}{b^2r^2}\right)$ |
| $f^{(1)}(b)$ | $\frac{e^{-br}}{2b}$ | $-\frac{e^{-br}}{2r}$ | $\frac{e^{-br}}{2r}(1+br)$ |
| $f^{(2)}(b)$ | $\frac{e^{-br}}{8b^3}(1+br)$ | $-\frac{e^{-br}}{8b}$ | $\frac{re^{-br}}{8}$ |

## APPENDIX B: POTENTIALS IN CONFIGURATION SPACE

In this section we give all formulas necessary to build up, in configuration space, the regularized two-pion-exchange potential considered here and calculated via dispersion relations following the derivation of the Paris Group [37].

From Eqs. (32), (35), (36), and (37), a given component of the two-pion exchange in momentum space can be written as

$$V(q^2) = \int_{4\mu^2}^{\infty} dt' \eta_{\mathrm{Paris}}(t')[V_1(t',q^2) + V_2(t',q^2) + V_3(t',q^2)], \tag{B1}$$

where $V_1(t',q^2)$, $V_2(t',q^2)$, and $V_3(t',q^2)$ correspond to the first, second, and third terms arising from Eq. (32). With the auxiliary definitions, $\lambda^2 = \Lambda^2 - \mu^2$ and $b^2 = \Lambda^2 + \frac{\lambda^4}{2\Lambda^2}$ we have

$$V_1(t',q^2) = \frac{\lambda^8}{4\Lambda^4}\frac{1}{t'+q^2}\frac{1}{(b^2+q^2)^2} \ , \tag{B2}$$

$$V_2(t',q^2) = \frac{\lambda^4}{\Lambda^2}\left[\frac{1}{b^2+q^2} + \frac{\Lambda^2-b^2}{(b^2+q^2)^2}\right]\left[\frac{1}{t'+q^2} + \frac{1}{4\lambda^2+t'+q^2} - \frac{2}{2\lambda^2+t'+q^2}\right] \tag{B3}$$

and

$$V_3(t',q^2) = \left[1 + 2\frac{\Lambda^2-b^2}{b^2+q^2} + \frac{(\Lambda^2-b^2)^2}{(b^2+q^2)^2}\right]\left[\frac{1}{t'+q^2} + \frac{1}{4\lambda^2+t'+q^2} - \frac{2}{2\lambda^2+t'+q^2} + \frac{4\lambda^2}{(4\lambda^2+t'+q^2)^2} - \frac{4\lambda^2}{(2\lambda^2+t'+q^2)^2} + \frac{8\lambda^4}{(4\lambda^2+t'+q^2)^3}\right] \ . \tag{B4}$$

As can be seen from Eqs. (B2)–(B4), in order to obtain the potential in configuration space, one has to take the Fourier transform of the products of poles and multipoles. Systematic formulas can be derived for doing these integrals. Let us define

$$[b_m c_n] = \frac{1}{2\pi^2}\int d\mathbf{q} e^{i\mathbf{q}\cdot\mathbf{r}} \frac{1}{(b^2+q^2)^m}\frac{1}{(c^2+q^2)^n} \ . \tag{B5}$$

One can show that for $m, n \geq 1$,

$$[b_m c_n] = \frac{1}{b^2-c^2}\{[b_{m-1}c_n] - [b_m c_{n-1}]\} \ . \tag{B6}$$

If

$$[b_1 c_0] = f^{(0)}(b) \ , \tag{B7}$$

then

$$[b_2 c_0] = f^{(1)}(b) = -\frac{1}{2b}\frac{d}{db}f^{(0)}(b) \tag{B8}$$

and

$$[b_3 c_0] = f^{(2)}(b) = -\frac{1}{4b}\frac{d}{db}f^{(1)}(b) \ . \tag{B9}$$

The basic functions, entering into play, see, e.g., Eq. (4.20) of [37], are given in Table I for the different components of the potentials.


[1] B. Diu, Nuovo Cimento **20A**, 115 (1974); B. Diu and E. Leader, *ibid.* **28A**, 137 (1975); B. Diu, *ibid.* **37A**, 151 (1977); A. Bouquet and B. Diu, *ibid.* **35A**, 157 (1976).
[2] R. J. N. Phillips, Phys. Lett. **4**, 19 (1963).
[3] K. H. McNaughton *et al.*, Phys. Rev. C **46**, 47 (1992); M. W. McNaughton *et al.*, *ibid.* **44**, 2267 (1991).
[4] D. Axen *et al.*, Phys. Rev. C **21**, 998 (1980).
[5] R. D. Ransome *et al.*, Phys. Rev. Lett. **48**, 781 (1982).
[6] A. Gersten, in *High-Energy Spin Physics: 8th International Symposium,* Proceedings of 8th International Symposium, University of Minnesota, Sept. 1988, AIP Conf. Proc. No. 187, edited by K. J. Heller (AIP, New York,

1989), pp. 691–701; W. M. Alberico and A. Gersten, CERN Report No. TH.3730-CERN (unpublished); A. Gersten, J. Phys. **46**, C2-471 (1985).
[7] K. Bräur *et al.*, Nucl. Phys. **A507**, 599 (1990).
[8] G. O. Liu, M. Swift, A. W. Thomas, and K. Holinde, Nucl. Phys. **A556**, 331 (1993).
[9] T. E. O. Ericson and M. Rosa-Clot, Nucl. Phys. **A405**, 497 (1983).
[10] J. L. Friar, B. F. Gibson, and G. L. Payne, Phys. Rev. C **30**, 1084 (1984).
[11] J. L. Ballot and M. R. Robilotta, Phys. Rev. C **45**, 986 (1992); **45**, 990 (1992); J. L. Ballot, A. M. Eiró, and M. R. Robilotta, *ibid.* **40**, 1459 (1989).
[12] M. Jain *et al.*, Phys. Rev. C **30**, 566 (1984).
[13] B. E. Bonner *et al.*, Phys. Rev. Lett. **41**, 1200 (1978).
[14] P. F. Shepard *et al.*, Phys. Rev. D **10**, 2735 (1974).
[15] E. L. Miller *et al.*, Phys. Rev. Lett. **26**, 984 (1971).
[16] J. L. Stone *et al.*, Phys. Rev. Lett. **38**, 1315 (1977).
[17] J. Engler *et al.*, Phys. Lett. **34B**, 528 (1971).
[18] H. R. Kreisler *et al.*, Nucl. Phys. **B84**, 3 (1975).
[19] G. Manning *et al.*, Nuovo Cimento **XLI A**, 167 (1966).
[20] V. Böhmer *et al.*, Nucl. Phys. **B110**, 205 (1975).
[21] A. Babaev *et al.*, Nucl. Phys. **B110**, 189 (1976).
[22] H. R. Barton *et al.*, Phys. Rev. Lett. **37**, 1656 (1976).
[23] G. F. Chew, Phys. Rev. **112**, 1380 (1958).
[24] A. Ashmore, W. H. Range, R. T. Taylor, B. M. Townes, L. Castillejo, and R. F. Peierls, Nucl. Phys. **36**, 258 (1962).
[25] K. Bongardt, H. Pilkuhn, and H. G. Schlaile, Phys. Lett. **52B**, 271 (1974).
[26] A. Cass and B. H. J. McKellar, Phys. Rev. D **18**, 3269 (1978).
[27] C. A. Dominguez and B. J. Verwest, Phys. Lett. **89B**, 333 (1980).
[28] G. C. Fox, in *Phenomenology in Particle Physics*, edited by C. B. Chiu (California Institute of Technology, Pasadena, CA, 1971), p. 703.
[29] M. Gourdin, Phys. Rep. **11**, 29 (1974).
[30] R. W. Haymaker and T. Goldman, Phys. Rev. D **24**, 743 (1981).
[31] F. E. Close, *The Theory of Quarks and Partons* (Academic, London, 1979), see in particular pp. 401–405.
[32] S. A. Coon and M. D. Scadron, Phys. Rev. C **42**, 2256 (1990); **23**, 1150 (1981).
[33] J. Bystricky, F. Lehar, and P. Winternitz, J. Phys. **48**, 199 (1987).
[34] J. Bystricky, C. Lechanoine-Leluc, and F. Lehar, J. Phys. **39**, 1 (1978).
[35] We have calculated this function with W. B. Kaufmann but we do not show it here.
[36] L. Mathelitsch and H. Garcilazo, Phys. Rev. C **33**, 2075 (1986).
[37] W. N. Cottingham, M. Lacombe, B. Loiseau, J. M. Richard, and R. Vinh Mau, Phys. Rev. D **8**, 800 (1973).
[38] M. Lacombe, B. Loiseau, J. M. Richard, R. Vinh Mau, J. Côté, P. Pirès, and R. de Tourreil, Phys. Rev. C **21**, 861 (1980).
[39] R. E. Cutkosky, J. Math. Phys. **1**, 429 (1960).
[40] B. Bonnier and P. Gauron, Nuovo Cimento **57A**, 261 (1980); P. Gauron, Paris Report No. IPNO/TH 78-07, 1978 (unpublished).
[41] G. Hoehler and E. Pietarinen, Nucl. Phys. **B95**, 210 (1975).