Two-body pion absorption on ^{3}He at threshold

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It is shown that a drastic reduction of the ratio of the rates of the reactions $^3\mathrm{He}(\pi^-, nn)$ and $^3\mathrm{He}(\pi^-, np)$ for stopped pions is obtained once the effect of the short range two-nucleon components of the axial charge operator for the nuclear system is taken into account. In a calculation using simple wave functions but employing realistic models for the nucleon-nucleon interaction in the construction of these components of the axial charge operator, the predicted ratios can be brought to within 10–20 % of the empirical value.

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I. INTRODUCTION

A natural Ansatz for the description of nuclear pion absorption and production reactions in the near threshold regime, where the reactions mainly involve S-wave pions, is to express the reaction amplitudes in terms of the axial charge density operator of the nuclear system [1]. This operator is formed of a single nucleon term and a two-nucleon "exchange current" term. The latter separates into a long-range pion exchange term, first derived in [2], and a short-range term, the most important part of which can be determined directly from the nucleon-nucleon interaction [3]. Direct evidence for the short-range component was recently found by the quantitatively successful explanation of the reaction $pp \to pp\pi^0$ near threshold [4], to which the main pion exchange term does not contribute [1]. This important role of the short-range part of the axial exchange current operator has been substantiated in subsequent work [5,6].

In this paper we apply the Ansatz of describing nuclear S-wave pion absorption by the axial charge operator to the case of absorption of stopped negative pions on ^{3}He. This reaction, for which the isospin structure of the nuclear states involved is richer than that in the case of the reaction $pp \to pp\pi^0$, is less selective and therefore the rate also has an important long-range pion exchange contribution. Yet if only this rescattering contribution is taken into account in addition to the single nucleon operator an overprediction by more than an order of magnitude of the empirical ratio [7] of the rates for production of nn and np pairs results. The objective of this work is to investigate whether this problem can be resolved by the effect of the short-range contributions to the axial charge operator, which were recently found in Ref. [1] to provide the explanation for the cross section for the reaction $pp \to pp\pi^0$ near threshold.

As a first step, we only consider the ratio between the total absorption rates which can be reasonably estimated by a calculation using simplified wave functions. The initial three-nucleon wave function is assumed to be of an s-wave harmonic oscillator form. The nuclear distortion effects on the absorbed pions and the outgoing nucleons are also neglected. These simplifications lead to a formulation in which the role of short-range mechanisms can be most easily exhibited. We will show that when realistic models for the nucleon-nucleon interaction are employed in the construction of the short-range components of the axial exchange charge operator, a very satisfactory account of the recent data [7] on the predicted ratio of the two-nucleon absorption processes $^3\mathrm{He}(\pi^-, nn)p$ and $^3\mathrm{He}(\pi^-, np)n$ is obtained. The short-range corrections reduce the predicted value by the required large factor so that the final predicted ratio exceeds the empirical value 6.3 ± 1.1 [7] by only 10–20 %, the range of variation arising from the differences between the potential models considered.

In Sec. II we review the derivation of the S-wave pion absorption operator, as it appears when constructed from the axial charge operator. The formalism for calculating the absorption rate for stopped pions by a pair of nucleons in a given eigenchannel is then presented in Sec. III. The numerical results and discussions are given in Sec. IV.

II. THE S-WAVE PION ABSORPTION OPERATOR

We shall take the effective Lagrangian for the interaction between low energy pions and nuclei to have the form

$$\mathcal{L}(x) = -\frac{1}{f_\pi}\vec{A}_\mu(x)\cdot\partial^\mu\vec{\phi}(x)\,, \tag{1}$$

where $\vec{\phi}$ is the pion field, f_π is the pion decay constant, and $\vec{A}_\mu$ the axial current density of the nuclear system. This Lagrangian density is the direct generalization of Weinberg's effective Lagrangian for the pion nucleon system [8]. In the case of a single nucleon the axial current operator has the form

$$\vec{A}_\mu(0) = ig_A\bar{u}(p')\gamma_\mu\frac{\vec{\tau}}{2}u(p)\ , \tag{2}$$

and thus in this case Eq. (1) is equivalent to the usual pseudovector πNN coupling model, since by the Goldberger-Treiman relation the coefficient $g_A/2f_\pi$ can be expressed as

$$\frac{g_A}{2f_\pi} = \frac{f_{\pi NN}}{m_\pi}\ , \tag{3}$$

where $f_{\pi NN}$ is the pseudovector coupling constant.

The interaction Eq. (1) implies the S-wave pion production and absorption at threshold is determined by the axial charge operator, which in the case of a single nucleon is

$$\vec{A}_0(0) = -g_A\sigma\cdot\mathbf{v}\frac{\vec{\tau}}{2}\ , \tag{4}$$

where $\mathbf{v}$ is the nucleon velocity operator.

The most important two-nucleon contribution to the axial charge operator is found in Ref. [3] to be the $\pi\rho$-exchange operator, which has the form

$$\vec{A}_0^{(\pi\rho)}(0) = i\frac{g_A m_\rho^2}{2f_\pi^2}\frac{\sigma^2\cdot\mathbf{k}_2}{(k_1^2+m_\rho^2)(k_2^2+m_\pi^2)}(\vec{\tau}^1\times\vec{\tau}^2) + (1\leftrightarrow 2)\ . \tag{5}$$

Here the symbol $(1\leftrightarrow 2)$ represents interchange of all the coordinates of the nucleon pair. We denote the fractions of the momentum $\mathbf{k}$ of the absorbed pion that are imparted to the two nucleons as $\mathbf{k}_1$ and $\mathbf{k}_2(\mathbf{k}=\mathbf{k}_1+\mathbf{k}_2)$. In the limit $m_\rho\to$ infinite the $\pi\rho$ exchange operator Eq. (5) reduces to the pion exchange operator of Ref. [2]. As noted in Ref. [9], it in this limit also corresponds to the isospin asymmetric term $(\vec{\tau}^1\times\vec{\tau}^2)$ in the usual form for the S-wave pion rescattering operator calculated using the usual pseudovector πNN coupling and the conventional effective Hamiltonian for S-wave πN scattering [10,11]:

$$T = -\frac{8\pi i}{\sqrt{2}\omega_\pi}\frac{f_{\pi NN}}{m_\pi}\frac{\sigma^2\cdot\mathbf{k}_2}{m_\pi^2+k_2^2}\left\{\frac{\lambda_1}{m_\pi}\vec{\tau}^2 - i\frac{\lambda_2}{2m_\pi^2}(\omega_\pi+\omega_2)\vec{\tau}^1\times\vec{\tau}^2\right\} + (1\leftrightarrow 2)\ . \tag{6}$$

Here ω_2 is the energy of the exchanged pion and ω_π that of the initial pion. The coefficients $\lambda_{1,2}$ are the following combinations of the two S-wave pion-nucleon scattering lengths $a_{1,3}$:

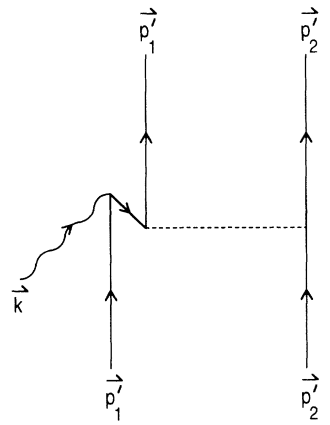

FIG. 1. Axial exchange current operator arising from intermediate $N\bar{N}$ pairs that contribute to the $NN\leftrightarrow NN\pi$ reaction.

$$\lambda_1 = -\tfrac{1}{6}m_\pi(a_1+2a_3),\quad \lambda_2 = \tfrac{1}{6}m_\pi(a_1-a_3)\ . \tag{7}$$

The $\pi\rho$ component Eq. (5) of the axial exchange charge operator is obtained by choosing the following values for the coupling constants $\lambda_{1,2}$ [9]:

$$\lambda_1 = 0,\quad \lambda_2 = \frac{m_\pi^2}{16\pi f_\pi^2}\simeq 0.04\ , \tag{8}$$

which agree well with the values extracted from πN phase shift analyses [12,13].

The short-range components of the axial exchange charge operator that were found to be important for the cross section of the reaction $pp\to pp\pi^0$ near threshold correspond to two-nucleon operators that arise when the axial field excites intermediate negative energy components of the nucleon (nucleon-antinucleon "pairs"), which are then deexcited by the short-range components of the nucleon-nucleon interaction (Fig. 1) [1]. Although these contributions are most readily derived by employment of explicit boson exchange models, the corresponding operators can be constructed directly from any existing nucleon-nucleon interaction model, if that is expressed in terms of Lorentz invariant spin-amplitudes as, e.g., the Fermi invariants. This construction has been presented in detail in Refs. [3,14].

In the decomposition of the nucleon-nucleon interaction in terms of Fermi invariants the short-range components are contained in the amplitudes S, V, T, and A (the P amplitude contains the long-range pion exchange term, which has to be dropped once the $\pi\rho$ exchange contribution above is included explicitly in the pion rescattering contribution). Numerically the most important of these short-range axial exchange charge operators are those that are associated with the scalar (S) and vector (V) invariants:

$$\vec{A}^0(S) = \frac{g_A}{m_N^2}\left([v_S^+(\mathbf{k}_2)\vec{\tau}^1 + v_S^-(\mathbf{k}_2)\vec{\tau}^2]\sigma^1\cdot\mathbf{P}_1 + \frac{i}{2}v_S^-(\mathbf{k}_2)\vec{\tau}^1\times\vec{\tau}^2\sigma^1\cdot\mathbf{k}_2\right) + (1\leftrightarrow 2)\ ,$$
$$\vec{A}^0(V) = \frac{g_A}{m_N^2}\left[[v_V^+(\mathbf{k}_2)\vec{\tau}^1 + v_V^-(\mathbf{k}_2)\vec{\tau}^2]\left(\sigma^1\cdot\mathbf{P}_2 + \frac{i}{2}\sigma^1\times\sigma^2\cdot\mathbf{k}_2\right) + \frac{i}{2}v_V^-(\mathbf{k}_2)\vec{\tau}^1\times\vec{\tau}^2\sigma^1\cdot\mathbf{k}_2\right] + (1\leftrightarrow 2)\ . \tag{9}$$

Here we have defined the nucleon momentum operators $\mathbf{P}_{1,2}$ for the two nucleons as $(\mathbf{p}_{1,2}+\mathbf{p}'_{1,2})/2$, respectively.

The potential components v_S and v_V can be given a direct interpretation as scalar and vector meson exchange potentials. In the case of a single meson exchange model these would have the expressions

$$v_S^{\pm}(\mathbf{q}) = -\frac{(g_S^{\pm})^2}{(m_S^{\pm})^2+q^2}\,, \tag{10}$$

$$v_V^{\pm}(\mathbf{q}) = \frac{(g_V^{\pm})^2}{(m_V^{\pm})^2+q^2}\,. \tag{11}$$

Here $m_S^{\pm}$ and $m_V^{\pm}$ are the masses of the isospin 0 and 1 scalar and vector mesons, respectively, and $g_S^{\pm}$ and $g_V^{\pm}$ are the corresponding scalar- and vector meson-nucleon coupling constants. Below we shall use the notation σ and δ to denote the isospin 0 and 1 scalar mesons, respectively.

Using the expressions above in defining the effective Lagrangian Eq. (1), the transition T matrix for the absorption of a pion with charge α and momentum $\mathbf{k}$ by a nuclear system can now be written as

$$S_{fi} = \delta_{fi} - i2\pi\delta^{(4)}(P_f - P_i - k)\langle\Psi_f|A_\alpha(\mathbf{k})|\Psi_i\rangle\,, \tag{12}$$

where the axial charge density is formed as the sum of one- and two-nucleon contributions

$$A_\alpha(\mathbf{k}) = A_\alpha^{(1)}(\mathbf{k}) + A_\alpha^{(2)}(\mathbf{k})\,. \tag{13}$$

We shall here restrict the expressions to the limit $\mathbf{k}\to 0$. The choice of the kinematic variables for a two-nucleon system are indicated in Fig. 1. For the one-body operator $A_\alpha^{(1)}(\mathbf{k})$ we obtain from Eq. (4) the matrix element in the momentum representation,

$$\langle\mathbf{p}'_1|A_\alpha^{(1)}|\mathbf{p}_1\rangle = C(2\pi)^3(-1)^\alpha\tau_\alpha^1\frac{\sigma^1\cdot\mathbf{P}_1}{m_N}\,. \tag{14}$$

Here the coefficient C has been defined as

$$C = i\frac{\omega_\pi(k)}{(2\pi)^9/2}\frac{1}{\sqrt{2\omega_\pi(k)}}\frac{f_{\pi NN}}{m_\pi}\,. \tag{15}$$

In terms of the kinematical variables in Fig. 1, the contribution of pion S-wave rescattering term Eq. (6) to two-body axial exchange charge operator $A_\alpha^{(2)}(\mathbf{k})$ is given by the matrix element

$$\langle\mathbf{p}'_1\mathbf{p}'_2|A_\alpha^{(\pi)}(\mathbf{k})|\mathbf{p}_1\mathbf{p}_2\rangle = C\frac{8\pi}{\omega_\pi(k)}v_\pi(\mathbf{q})(-1)^\alpha\left\{\frac{\lambda_1}{m_\pi}\tau_\alpha^2 - \frac{[\omega_\pi(q)+\omega_\pi(k)]\lambda_2}{2m_\pi^2}i[\vec{\tau}^1\times\vec{\tau}^2]_\alpha\right\}\sigma^2\cdot\mathbf{q} + (1\leftrightarrow 2)\,. \tag{16}$$

Here v_π represents the pion exchange interaction without coupling constants:

$$v_\pi(\mathbf{q}) = \frac{1}{\mathbf{q}^2+m_\pi^2}F_\pi^2(\mathbf{q})\,, \tag{17}$$

where $F_\pi(\mathbf{q})$ represents the vertex form factor, which here has been taken to be the same at the S- and the P-wave vertices.

From the short-range components of the axial charge operators defined in Eqs. (9) we obtain the following explicit contributions for the scalar and vector exchange mechanisms (σ,ω for isoscalar and δ,ρ for isovector):

$$\langle\mathbf{p}'_1\mathbf{p}'_2|A_\alpha^{(\sigma)}(\mathbf{k})|\mathbf{p}_1\mathbf{p}_2\rangle = C(-1)^\alpha\tau_\alpha^1\frac{-1}{m_N^2}v_\sigma(\mathbf{q})\sigma^1\cdot\mathbf{P}_1 + (1\leftrightarrow 2)\,, \tag{18}$$

$$\langle\mathbf{p}'_1\mathbf{p}'_2|A_\alpha^{(\delta)}(\mathbf{k})|\mathbf{p}_1\mathbf{p}_2\rangle = C(-1)^\alpha\tau_\alpha^2\frac{-1}{m_N^2}v_\delta(\mathbf{q})\sigma^1\cdot\mathbf{P}_1 + (1\leftrightarrow 2)\,, \tag{19}$$

$$\langle\mathbf{p}'_1\mathbf{p}'_2|A_\alpha^{(\omega)}(\mathbf{k})|\mathbf{p}_1\mathbf{p}_2\rangle = C(-1)^\alpha\tau_\alpha^1\frac{-1}{m_N^2}v_\omega(\mathbf{q})\left\{\sigma^1\cdot\mathbf{P}_2 + \frac{i}{2}\sigma^1\times\sigma^2\cdot\mathbf{q}\right\} + (1\leftrightarrow 2)\,, \tag{20}$$

$$\langle\mathbf{p}'_1\mathbf{p}'_2|A_\alpha^{(\rho)}(\mathbf{k})|\mathbf{p}_1\mathbf{p}_2\rangle = C(-1)^\alpha\frac{-1}{m_N^2}v_\rho(\mathbf{q})\left(\tau_\alpha^2\{\sigma^1\cdot\mathbf{P}_2 + \frac{i}{2}\sigma^1\times\sigma^2\cdot\mathbf{q}\} - \frac{i}{2}[\vec{\tau}^1\times\vec{\tau}^2]_\alpha\sigma^1\cdot\mathbf{q}\right) + (1\leftrightarrow 2)\,. \tag{21}$$

Here we have used the notation

$$v_i(\mathbf{q}) = \frac{S_i g_i^2}{\mathbf{q}^2+m_i^2}F_i(\mathbf{q})^2\,, \tag{22}$$

where $S_i=-1$ for $i=\sigma,\delta$, $S_i=+1$ for $i=\omega,\rho$, and the g_i's are the corresponding meson-nucleon-nucleon coupling constants. In the case of the Bonn potential model [15] the form factors and coupling constants are determined by a fit to nucleon-nucleon scattering data (here we use the parameters values in Table A.3 of Ref. [15]). Note that the terms due to the tensor coupling of ρ and ω vector meson were found [3] to be unimportant and hence are neglected here.

For a general nucleon-nucleon potential, such as the Paris potential [16] used in this work, the derivation of axial charge operator can be done [3,14] by the expansion of the potential in terms of Fermi invariants. The resulting expressions are the same as above except that the radial functions $v_\sigma(q), v_\delta(q), v_\omega(q)$, and $v_\rho(q)$ are replaced, respectively, by $v_S^+(q), v_S^-(q), v_V^+(q)$, and $v_V^-(q)$. In this work we consider the Paris potential [16] and use the $v_S^{\pm}(q)$ and $v_V^{\pm}(q)$ calculated in Ref. [14].

The above expressions are used in the numerical calculations of π absorption on ^{3}He at threshold reported below.

III. TRANSITION RATE FOR THE REACTION ^{3}He(π, NN)

The kinematics of the reaction $^3\mathrm{He}(\pi, NN)$ in the ^{3}He rest frame is illustrated in Fig. 2. As we are interested only in the ratio between the absorption rates, which is not expected to depend sensitively on the wave functions, we shall neglect the nuclear distortion effect on the absorbed pions and the outgoing nucleons. For simplicity the ^{3}He wave function is taken to be of a simple harmonic oscillator form with the oscillator parameter value $b = 1.36$ fm. It then has the separable form $\sim \Phi(\mathbf{P})\phi_{S,T}(\mathbf{p})$, where $\mathbf{P}$ and $\mathbf{p}$ are the total and the relative momentum of a pair of nucleons in ^{3}He, respectively. With these simplifications and the kinematic variables shown in Fig. 2, the total transition rate of the reaction $^3\mathrm{He}(\pi, NN)$ is then determined by the quantity

$$P_{m_{\tau_1}m_{\tau_2};\alpha} = \frac{3}{2j+1}\sum_{m_j}\sum_{m_{s_1},m_{s_2}}\int d\mathbf{P}\,d\mathbf{p}\,\delta\left(\omega_\pi(\mathbf{k}) - \epsilon - \frac{3P^2}{4m_N} - \frac{p^2}{m_N}\right)|\Phi(\mathbf{P})|^2$$
$$\times\left|\sum_{S,T}\sum_{M_S,M_T}\langle SsM_Sm_{s_3}|jm_j\rangle\langle T\tau M_Tm_{\tau_3}|tm_t\rangle\langle \mathbf{p}_1m_{s_1}m_{\tau_1},\mathbf{p}_2m_{s_2}m_{\tau_2}|A|\phi_{S,T}^{M_S,M_T},\mathbf{k},\alpha\rangle\right|^2 , \quad (23)$$

where $\tau = s = 1/2, j = 1/2$, and $t = m_t = 1/2$ are the spin and isospin quantum numbers of ^{3}He, and obviously $m_{s_3} = m_j - M_S, m_{\tau_3} = m_t - M_T$. The isospin quantum numbers and the momenta of the ejected two nucleons are (m_{τ_1}, m_{τ_2}) and $\mathbf{p}_1 = \mathbf{P}+\mathbf{p}/2, \mathbf{p}_2 = \mathbf{P}-\mathbf{p}/2$. The momentum and the charge of the pion are respectively denoted $\mathbf{k}$ and α. The outgoing two-nucleon state is properly antisymmetrized.

In the stopped pion limit $\mathbf{k} \to 0$, Eq. (23) can be written in the partial-wave decomposed form

$$P_{m_{\tau_1}m_{\tau_2};\alpha} = \sum_{\gamma,\beta}\langle T'M_T'|\tau\tau m_{\tau_1}m_{\tau_2}\rangle\langle T\tau M_tm_{\tau_3}|tm_t\rangle$$
$$\times\langle T'M_T'|T1M_T\alpha\rangle g(\gamma,\beta) , \quad (24)$$

where

$$g(\gamma,\beta) = \tfrac{3}{2}\int d\mathbf{P}|\Phi(\mathbf{P})|^2|\langle\gamma,p|A|\phi_\beta\rangle|^2 , \quad (25)$$

with $p = [m_N(m_\pi - \epsilon - 3P^2/4m_N)]^{1/2}$. Here we have introduced abbreviated partial-wave notations $\beta : [LS]JT$ for the initial two-nucleon bound state, and $\gamma : [L'S']J'T'$ for the ejected two nucleons. The matrix element of A has contributions from one-body term and two-body terms defined in Sec. II

$$\langle\gamma,p|A|\phi_\beta\rangle = \langle\gamma,p|A^{(1)}|\phi_\beta\rangle + \sum_i\langle\gamma,p|A^{(i)}|\phi_\beta\rangle , \quad (26)$$

where $i = \pi,\sigma,\delta,\rho,\omega$. The contribution from the one-body term is

$$\langle\gamma,p'|A_\alpha^{(1)}|\Phi_\beta\rangle = -B(2\pi)^3\frac{f_{\pi NN}}{m_\pi}\frac{p}{m_N}\phi_{[LS]JT}(p) , \quad (27)$$

where $\phi_{[LS]JT}(p)$ is a general bound state wave function. For an S-wave harmonic oscillator wave function of ^{3}He, we have $L = 0, J = S$ and only $[S,T] = [0,1],[1,0]$, are permitted by the Pauli principle. The radial wave function is $\phi_{[0,S]S,T}(p) = \phi(p) = Ne^{-b^2p^2}$. The angular momentum coupling in Eq. (27) is isolated in the coefficients B. We find that

$$B = i\sqrt{\frac{m_\pi}{2}}\frac{1}{(2\pi)^{9/2}}\hat{L}\hat{L}'\hat{S}'(-1)^{(J-S'+1)}\delta_{J'J}\delta_{MM'}$$
$$\times\begin{pmatrix}1 & L' & L\\ 0 & 0 & 0\end{pmatrix}\begin{Bmatrix}S & L & J\\ L' & S' & 1\end{Bmatrix} . \quad (28)$$

The matrix element of the pion S-wave rescattering term is

$$\langle\gamma,p'|A_\alpha^{(\pi)}|\phi_\beta\rangle = B\frac{8\pi f_{\pi NN}}{m_\pi}\frac{1}{m_\pi^2}[A_{L'L}^{(\pi)}(L') - B_{L'L}^{(\pi)}(L)]$$
$$\times\left\{\lambda_1\langle S'T'||O_1||ST\rangle + \frac{\omega_\pi(k)+m_\pi}{m_\pi}\lambda_2\langle S'T'||O_3||ST\rangle\right\} , \quad (29)$$

where

$$A_{L'L}^{(\pi)}(l) = \int p^2dpF_l^{(\pi)}(p',p)\phi_{[LS]JT}(p)p ,$$
$$B_{L'L}^{(\pi)}(l) = p'\int p^2dpF_l^{(\pi)}(p',p)\phi_{[LS]JT}(p) , \quad (30)$$

with

$$F_l^{(\pi)}(p',p) = (2\pi)\int_{-1}^{+1}P_l(x)dx\,v_\pi(\mathbf{p}'-\mathbf{p}) , \quad (31)$$

where $x = \hat{p}' \cdot \hat{p}$. Defining similar partial-wave projected integrals for the $i = \sigma, \delta, \rho$, and ω meson exchange contributions, we then get the expressions

$$\langle \gamma, p' | A_\alpha^{(\sigma)} | \phi_\beta \rangle = B \frac{g_A}{2f_\pi} \frac{1}{2m_N^2} \{ A_{L'L}^{(\sigma)}(L') + B_{L'L}^{(\sigma)}(L) \} \langle S'T' || O_1 || ST \rangle \,, \tag{32}$$

$$\langle \gamma, p' | A_\alpha^{(\delta)} | \phi_\beta \rangle = B \frac{g_A}{2f_\pi} \frac{1}{2m_N^2} \{ A_{L'L}^{(\delta)}(L') + B_{L'L}^{(\delta)}(L) \} \langle S'T' || O_2 || ST \rangle \,, \tag{33}$$

$$\langle \gamma, p' | A_\alpha^{(\omega)} | \phi_\beta \rangle = B \frac{g_A}{2f_\pi} \frac{1}{2m_N^2} \{ -(A_{L'L}^{(\omega)}(L') + B_{L'L}^{(\omega)}(L)) \langle S'T' || O_1 || ST \rangle + \sqrt{2}(A_{L'L}^{(\omega)}(L') - B_{L'L}^{(\omega)}(L)) \langle S'T' || O_4 || ST \rangle \} \,, \tag{34}$$

$$\begin{aligned} \langle \gamma, p' | A_\alpha^{(\rho)} | \phi_\beta \rangle = B \frac{g_A}{2f_\pi} \frac{(1+\kappa)}{2m_N^2} \{ & -[A_{L'L}^{(\rho)}(L') + B_{L'L}^{(\rho)}(L)] \langle S'T' || O_2 || ST \rangle \\ & + \sqrt{2}[A_{L'L}^{(\rho)}(L') - B_{L'L}^{(\rho)}(L)] (\langle S'T' || O_4 || ST \rangle + \langle S'T' || O_3 || ST \rangle) \} \,. \end{aligned} \tag{35}$$

The reduced matrix elements in the above equations are defined by the spin-isospin operators

$$\vec{O}_1 = \vec{\tau}^1 \sigma^1 - \vec{\tau}^2 \sigma^2 \,,$$

$$\vec{O}_2 = \vec{\tau}^2 \sigma^1 - \vec{\tau}^1 \sigma^2 \,,$$

$$\vec{O}_3 = \frac{i}{\sqrt{2}} \vec{\tau}^1 \times \vec{\tau}^2 (\sigma^1 + \sigma^2),$$

$$\vec{O}_4 = (\vec{\tau}^1 + \vec{\tau}^2) \frac{i}{\sqrt{2}} \sigma^1 \times \sigma^2 \,. \tag{36}$$

It is then easy to show that

$$\begin{aligned} &\langle S'T' || \vec{O}_1 || ST \rangle \\ &\quad = 6 \hat{S} \hat{T} [(-1)^{(S+T)} - (-1)^{(S'+T')}] \\ &\qquad \times \begin{Bmatrix} T' & T & 1 \\ 1/2 & 1/2 & 1/2 \end{Bmatrix} \begin{Bmatrix} S' & S & 1 \\ 1/2 & 1/2 & 1/2 \end{Bmatrix}, \end{aligned} \tag{37}$$

$$\begin{aligned} &\langle S'T' || \vec{O}_2 || ST \rangle \\ &\quad = 6 \hat{S} \hat{T} [(-1)^{(S+T')} - (-1)^{(S'+T)}] \\ &\qquad \times \begin{Bmatrix} T' & T & 1 \\ 1/2 & 1/2 & 1/2 \end{Bmatrix} \begin{Bmatrix} S' & S & 1 \\ 1/2 & 1/2 & 1/2 \end{Bmatrix}, \end{aligned} \tag{38}$$

$$\begin{aligned} \langle S'T' || \vec{O}_3 || ST \rangle &= 6\sqrt{18} \hat{S} \hat{T} [(-1)^S + (-1)^{S'}] \\ &\quad \times \begin{Bmatrix} T' & T & 1 \\ 1/2 & 1/2 & 1 \\ 1/2 & 1/2 & 1 \end{Bmatrix} \begin{Bmatrix} S' & S & 1 \\ 1/2 & 1/2 & 1/2 \end{Bmatrix}, \end{aligned} \tag{39}$$

$$\begin{aligned} \langle S'T' || \vec{O}_4 || ST \rangle &= 6\sqrt{18} \hat{S} \hat{T} [(-1)^T + (-1)^{T'}] \\ &\quad \times \begin{Bmatrix} S' & S & 1 \\ 1/2 & 1/2 & 1 \\ 1/2 & 1/2 & 1 \end{Bmatrix} \begin{Bmatrix} T' & T & 1 \\ 1/2 & 1/2 & 1/2 \end{Bmatrix}. \end{aligned} \tag{40}$$

The employment of uncorrelated wave functions restricts the applicability of the formalism presented above to the calculation of total rates only. The prediction of angular distributions would require that at least the final-state interactions be treated [6,17]. The employment of harmonic oscillator model wave functions, which do not take into account the short-range repulsive interaction between the nucleons also will overestimate the short-range part of the two-body operator. It was found in Ref. [18] that the correlations would reduce the matrix elements of the short-range components of the axial charge operator by only 10–15 % in the case of the potential models considered here. The main reason is that the two-nucleon axial charge operators used are constructed from realistic nucleon-nucleon interaction models, which are regularized at short range. This overestimate is expected to be even smaller in the present pion absorption calculation, since the outgoing two nucleons are in P states near threshold, and thus the radial wave functions vanish at short range by the centrifugal barrier. The formulation presented in this section is therefore sufficient for our exploratory investigation.

IV. RESULTS AND DISCUSSION

It is interesting to first note that all transition matrix elements, Eq. (27) and Eqs. (29)–(35), depend on the

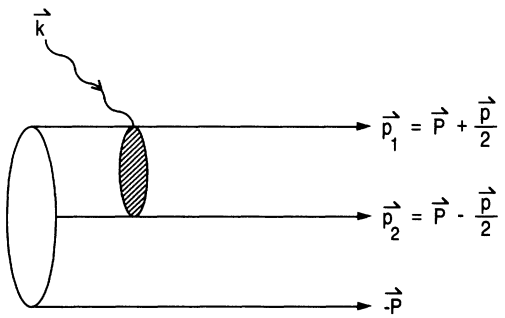

FIG. 2. Kinematics of ^{3}He(π, NN) reaction in the rest frame of ^{3}He.

same geometric factor B defined in Eq. (28). Evaluating this factor of an initial S-wave $(L = 0)NN$ pair, it is easy to see that only the following transitions can occur in the reaction $^3\mathrm{He}(\pi, NN)$:

$$f_{11} = g(^3P_0, ^1S_0) ,$$
$$f_{10} = g(^3P_1, ^3S_1) .$$

Here the rate factor g is that defined in Eq. (25), and we use the standard spectroscopic notation $^{2S+1}L_J$. The subindices on the f's indicate the total isospin T' and T of the final and initial two-nucleon systems. Evaluation of the Clebsch-Gordan coefficients in Eq. (24) leads to the following ratio of the absorption probabilities [$P_{m_{\tau_1} m_{\tau_2};\alpha}$ defined in Eq. (24)] for negative pions:

$$R = \frac{^3\mathrm{He}(\pi^-, nn)}{^3\mathrm{He}(\pi^-, np)} = \frac{P_{-1/2,-1/2;-1}}{P_{-1/2,1/2;-1}}$$
$$= 1 + 6\frac{f_{10}}{f_{11}} . \qquad (41)$$

The pion rescattering term $A^{(\pi)}$, Eq. (29), gives a very small contribution to f_{11} because its main (λ_2) term depends on the isospin operator $\vec{\tau}^1 \times \vec{\tau}^2$ and therefore its matrix element between two $T = 1$ states vanishes. The remaining term gives only a small contribution because the coefficient $\lambda_1 \simeq 0$, as discussed in Sec. III. On the other hand, the pion rescattering term gives a large contribution to the transition between $T = 1$ and $T = 0$ states (i.e., f_{10}) and hence the prediction based on the sum of the single nucleon contribution $A^{(1)}$ and the pion rescattering term $A^{(\pi)}$ is very large, as shown in the first row of Table I.

In Table I we also show the effects caused by the short-range absorption mechanisms as calculated using the Bonn [15] and Paris [16] potential models. By comparing rows 1–3, it is seen that the short-range corrections due to the σ and ω exchange mechanisms (or more generally from the isospin independent scalar and vector exchange mechanisms) drastically reduce the predicted ratio. The contributions from isovector δ and ρ exchanges are less important, but are significant in further reducing the predicted ratio toward the experimental value [7]. Although the contributions from the individual two-nucleon mechanisms differ between the two potential models, the net result obtained by taking all of them into account is quite similar and close to the experimental value. As pointed out in Sec. III above we do not expect the numerical results to be very dependent on the schematic wave function model used here, although the short-range mechanisms may be overestimated here by 10–15 %. This situation is in agreement with that found for the reaction $pp \to pp\pi^0$ near threshold [1].

The results presented in Table I further strengthen the case for the importance of the short-range pion absorption mechanisms [1]. It will be interesting to explore whether this model also is able to resolve several long-standing problems associated with nuclear pion absorption at higher energies. In the case of the two-nucleon system, all existing unitary πNN calculations

TABLE I. The ratios $R = ^3\mathrm{He}(\pi^-, nn)/^3\mathrm{He}(\pi^-, np)$ as predicted using the Bonn [11] and Paris [12] potential models in the construction of the two-nucleon operators are compared with the experiment value [7]. The absorption mechanisms included in each calculation, defined in Eq. (27) and Eqs. (29)–(35), are indicated in each row.

Mechanism	R_{Bonn}	R_{Paris}	$R_{\mathrm{expt.}}$
$A^{(1)} + A^{(\pi)}$	533.42	533.42	
$A^{(1)} + A^{(\pi+\sigma)}$	74.30	208.12	
$A^{(1)} + A^{(\pi+\sigma+\omega)}$	12.40	16.81	
$A^{(1)} + A^{(\pi+\sigma+\omega+\delta)}$	11.77	11.88	
$A^{(1)} + A^{(+\pi+\sigma+\omega+\delta+\rho)}$	9.786	8.0349	6.3 ± 1.1

[19] based on pion rescattering mechanisms (including S and $P\pi N$ partial waves) fail to give satisfactory descriptions of the analyzing powers for the reactions $\pi^+ d \to pp$ and $NN \to \pi NN$. Although the short-range absorption mechanisms that involve intermediate $N\bar{N}$ pairs (Fig. 1) is in general weaker than the pion rescattering term, they can nevertheless have large effects on the polarization observables through interference with the large pion rescattering amplitude. This can be explored straightforwardly within the unitary πNN formalism developed by Lee and Matsuyama [20].

It will be also interesting to investigate the effect of the short-range absorption mechanisms on $^3\mathrm{He}(\pi^-, np)$ reaction in the Δ region. This reaction involves small contribution from the Δ excitation, since the dominant $N\Delta$ S-wave intermediate state is excluded [21,22] for an initial $^1S_0$ pp pair in ^{3}He. The angular distributions of this reaction as well as the energy dependence of the ratio of $^3\mathrm{He}(\pi^+, pp)/^3\mathrm{He}(\pi^-, np)$ have not been well understood [21]. The calculation of Ref. [6] only succeeded in describing the shapes of angular distributions up to the Δ region, but not the energy dependence of the absolute normalization. It will be interesting to see whether this problem can be resolved by using the unitary formulation of Ref. [20] and including the short-range absorption mechanisms described in this work. A rigorous theoretical calculation of two-body absorption on ^{3}He is essential for extracting from the data the contribution from three-body absorption mechanisms which have been empirically established in Ref. [21].

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