Observables for polarized neutrons transmitted through polarized targets

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A general and concise formalism is presented for the identification and evaluation of observables, including those that would indicate parity-conservation and/or time-reversal violation by the neutron-nucleus interaction, of experiments on the transmission of polarized neutrons through polarized nuclear targets. Statistical tensors are used for the description of the polarization states of the projectile and target, and the neutron-nucleus total cross section is decomposed into partial cross sections so that each corresponds to different ranks of the projectile and target statistical tensors and to a specific transfer of the orbital angular momentum. Each such partial cross section is associated with a "correlation factor" of particular parity-conservation and time-reversal symmetries, and is measurable by polarizing the projectile and target in states in which the statistical tensors have specific geometries, suggested by the simple geometric properties of the correlation factors.

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Under specific well-defined conditions, the transmission of polarized neutrons through nuclear targets is sensitive to a violation of parity (P) conservation or time-reversal (T) invariance, and, accordingly, neutron-transmission experiments can provide sensitive null tests of these fundamental symmetries in nuclear systems [1–3]. Recently, Conzett [4] has emphasized the null-test nature of such experiments and used the Cartesian formalism for the description of spin-dependent phenomena to identify the P-nonconserving and T-violating observables. However, the formalism of statistical spherical tensors [5] is a much more powerful, if not even indispensable tool for the description of polarization phenomena involving spins greater than 1/2, and tensor-polarized targets of spins $I \geq 1$ are essential for the tests of the T invariance. Gould *et al.* [6] have identified the various P-nonconserving and T-violating terms in the neutron-nucleus total cross section by using the statistical-tensor formalism, and the recent experimental [3] and theoretical work [7] on T violation in neutron-nucleus total cross sections has also employed this formalism.

In this paper, the statistical-tensor methods that have been used in Refs. [6,7] are simplified and consolidated into a general formalism enabling a systematic identification of all the different observables, including those that are P nonconserving and/or T violating, of the neutron-transmission experiments, as well as providing a unified framework for the evaluation of these observables. This is achieved by decomposing the neutron-nucleus total cross section into partial cross sections, which correspond to specific ranks of the projectile and target statistical tensors and to specific transfers of the orbital angular momentum, and which are independent of the magnitude and geometry of the statistical tensors. The geometric aspects of the statistical tensors, peculiar to a given experimental setup, are factored out into separate "correlation factors" of definite and easily identifiable symmetries with respect to parity conservation and time reversal.

It should be noted that the incoherent regime of the neutron transmission through the target is assumed, so that there is no coherent precession of the incident neutron spin that could result in mimicking a T-violating interaction [8,9]. Such coherent-scattering effects can play a significant role only at epithermal neutron energies, and, in principle, it can be checked experimentally that no coherent precession of the spin of the incident neutrons occurs on the transmission through the target.

The optical theorem relates the total cross section σ_t to the forward elastic-scattering amplitude $f_{m'M',mM}(\theta{=}0)$ by

$$\sigma_t = 4\pi\lambda\!\!\!^{-}\,\mathrm{Im}\sum_{mm'MM'}\rho_{mm'}\rho_{MM'}\,f_{m'M',mM}(0), \qquad (1)$$

where $\rho_{mm'}$ and $\rho_{MM'}$ are the density matrices of the projectile and target nuclei, respectively, whose initial polarization states are assumed to be independent. Describing the projectile and target polarization states by statistical tensors [5] $t_{kq}(s)$ and $t_{KQ}(I)$, respectively, rather than by density matrices, and using the partial-wave expansion of the elastic-scattering amplitude [5], the total cross section σ_t for neutrons (spin $s=1/2$) incident on a target nucleus with spin I can be written as

$$\sigma_t = \sum_{kK\lambda} i^{k+K+\lambda}\Big[[t_k(s),t_K(I)]_\lambda,\tfrac{\sqrt{4\pi}}{\hat\lambda}Y_\lambda(\hat{\mathbf{p}})\Big]_0\sigma_{kK\lambda}, \qquad (2)$$

where

$$\sigma_{kK\lambda} = 4\pi\lambda\!\!\!^{-}{}^2\frac{\hat k\hat K}{\hat s\hat I}(2\lambda+1)\,\mathrm{Im}\left[i^{-k-K-\lambda}\sum_{JIjl'j'}(2J+1)\hat l\hat j\hat j'\langle l\lambda 00|l'0\rangle W(JjIK;Ij')\begin{Bmatrix} l & s & j\\ \lambda & k & K\\ l' & s & j'\end{Bmatrix}T^J_{l'j',lj}\right]. \qquad (3)$$

Here, $[\ ,\ ]_k$ denotes a spherical-tensor product of rank k, $\hat{\mathbf{p}}$ is a unit vector along the beam direction, λ is the reduced wavelength, $\hat{k} = (2k+1)^{1/2}$, etc., and $T^J_{l'j',lj} = (1/2i)(S^J_{l'j',lj} - \delta_{ll'}\delta_{jj'})$, where $S^J_{l'j',lj}$ are elements of the elastic-scattering S matrix in the spin-orbit coupling representation; the angular brackets, W, and braces denote the Clebsch-Gordan, Racah, and 9-j coefficients, respectively. The quantities $\sigma_{kK\lambda}$ are partial total cross sections that correspond to different ranks k and K of the statistical tensors and to different values λ of the transfer of the orbital angular momentum. The partial cross sections $\sigma_{kK\lambda}$ are defined so that they are independent of the magnitude and geometry of the statistical tensors, as any such dependence is factored out by the scalar products in (2) [10]. The 9-j coefficient in (3) limits the values of λ to the range from $|k-K|$ to $k+K$, and the phase factors $i^{k+K+\lambda}$ in (2) ensure that the coefficients with which the partial cross sections $\sigma_{kK\lambda}$ enter the sum in (2) are real even when $k+K+\lambda$ is odd.

When the channel-spin coupling representation is used instead of the spin-orbit one, the summation in (3) modifies to

$$\sum_{JlSl'S'} (2J+1)\,(-1)^{S-S'}\hat{l}\hat{S}\hat{S}'\langle l\lambda 00|l'0\rangle W(JlS'\lambda;Sl') \begin{Bmatrix} S & s & I \\ \lambda & k & K \\ S' & s & I \end{Bmatrix} T^J_{l'S',lS}, \qquad (4)$$

with $T^J_{l'S',lS} = (1/2i)(S^J_{l'S',lS} - \delta_{ll'}\delta_{SS'})$, where $S^J_{l'S',lS}$ is now the elastic-scattering S matrix in the channel-spin representation.

The expression of Eq. (2) for the total cross section σ_t does not assume any specific coordinate frame. In experimental practice, however, the devices used to polarize beams and targets produce invariably polarization states with an axial symmetry, and in the coordinate frames with z axes along the directions of axial symmetry, denoted by unit vectors $\hat{\mathbf{s}}_k$ and $\hat{\mathbf{I}}_K$ for the projectile and target, respectively, the statistical tensors are "diagonal": $\tilde{t}_{kq}(s) = \tilde{t}_{k0}(s)\delta_{q0}$ and $\tilde{t}_{KQ}(I) = \tilde{t}_{K0}(I)\delta_{Q0}$ (for simplicity in notation, the subscripts k and K on the unit vectors $\hat{\mathbf{s}}$ and $\hat{\mathbf{I}}$ will be omitted henceforth; for neutrons the statistical-tensor rank k can be at most 1, in any case). Assuming then the existence of such axial-symmetry frames, the general statistical tensors in (2) become proportional to spherical harmonics of the directions $\hat{\mathbf{s}}$ and $\hat{\mathbf{I}}$

$$t_{kq}(s) = \tilde{t}_{k0}(s)\,(\sqrt{4\pi}/\hat{k})\,Y_{kq}(\hat{\mathbf{s}}), \qquad (5)$$

$$t_{KQ}(I) = \tilde{t}_{K0}(I)\,(\sqrt{4\pi}/\hat{K})\,Y_{KQ}(\hat{\mathbf{I}}), \qquad (6)$$

as they are obtained from the tensors $\tilde{t}_{kq}(s)$ and $\tilde{t}_{KQ}(I)$ by appropriate rotations of the axial-symmetry frames, and the total cross section σ_t can be expressed as

$$\sigma_t = \sum_{kK\lambda} \tilde{t}_{k0}(s)\,\tilde{t}_{K0}(I)\,C_{kK\lambda}(\hat{\mathbf{s}}\,\hat{\mathbf{I}}\,\hat{\mathbf{p}})\,\sigma_{kK\lambda}, \qquad (7)$$

where

$$C_{kK\lambda}(\hat{\mathbf{s}}\,\hat{\mathbf{I}}\,\hat{\mathbf{p}}) = i^{k+K+\lambda}\frac{(4\pi)^{3/2}}{\hat{k}\hat{K}\hat{\lambda}}\left[[Y_k(\hat{\mathbf{s}}), Y_K(\hat{\mathbf{I}})]_\lambda, Y_\lambda(\hat{\mathbf{p}})\right]_0. \qquad (8)$$

The scalar quantities $C_{kK\lambda}(\hat{\mathbf{s}}\,\hat{\mathbf{I}}\,\hat{\mathbf{p}})$ contain the geometric features peculiar to a given experimental arrangement, and will be termed the correlation factors. They are defined in (8) with the phase factor $i^{k+K+\lambda}$ so that they are always real [11]. It is now seen from Eq. (7) that the partial cross sections $\sigma_{kK\lambda}$ are each normalized to nominal unity components of the statistical tensors $\tilde{t}_{k0}(s)$ and $\tilde{t}_{K0}(I)$, and to a nominal unity correlation factor $C_{kK\lambda}(\hat{\mathbf{s}}\,\hat{\mathbf{I}}\,\hat{\mathbf{p}})$.

Statistical-tensor expressions of varying generality for the total cross section have been given in the literature on several occasions before [6,7,12–14].

The correlation factor $C_{kK\lambda}(\hat{\mathbf{s}}\,\hat{\mathbf{I}}\,\hat{\mathbf{p}})$ associated with a partial cross section $\sigma_{kK\lambda}$ has simple properties under the symmetry operations of space inversion (i.e., the parity transformation) and time reversal. Under the parity transformation P, the directions of axial symmetry $\hat{\mathbf{s}}$ and $\hat{\mathbf{I}}$ of the spin states do not change, as spins remain unchanged under P, but the beam direction $\hat{\mathbf{p}}$ is reversed, and so, using the property of spherical harmonics $Y_{lm}(-\hat{\mathbf{r}}) = (-1)^l Y_{lm}(\hat{\mathbf{r}})$ in the definition (8), the correlation factor changes as

$$P\,C_{kK\lambda}(\hat{\mathbf{s}}\,\hat{\mathbf{I}}\,\hat{\mathbf{p}}) = (-1)^\lambda C_{kK\lambda}(\hat{\mathbf{s}}\,\hat{\mathbf{I}}\,\hat{\mathbf{p}}). \qquad (9)$$

On the other hand, under time reversal T, the directions of spin axial symmetry $\hat{\mathbf{s}}$ and $\hat{\mathbf{I}}$, as well as the beam direction $\hat{\mathbf{p}}$ are reversed, and [15]

$$T\,C_{kK\lambda}(\hat{\mathbf{s}}\,\hat{\mathbf{I}}\,\hat{\mathbf{p}}) = (-1)^{k+K+\lambda} C_{kK\lambda}(\hat{\mathbf{s}}\,\hat{\mathbf{I}}\,\hat{\mathbf{p}}). \qquad (10)$$

In other words, the correlation factor $C_{kK\lambda}(\hat{\mathbf{s}}\,\hat{\mathbf{I}}\,\hat{\mathbf{p}})$ is "P even" ("P odd") when λ is even (odd); similarly, $C_{kK\lambda}(\hat{\mathbf{s}}\,\hat{\mathbf{I}}\,\hat{\mathbf{p}})$ is "T even" ("T odd") when $k+K+\lambda$ is even (odd). These symmetry properties can be attached also to the partial cross sections $\sigma_{kK\lambda}$, in the sense that their contributions to the total cross section σ_t are proportional to the correlation factors [see Eq. (7)], and so would (would not) change sign under the space-inversed experimental conditions when the corresponding correlation factors are P odd (P even), and similarly for the time-reversed experimental conditions. Thus, a measured nonzero partial cross section $\sigma_{kK\lambda}$ with λ odd is a positive indication of a violation of parity conservation. Similarly a partial cross section with $k+K+\lambda$ odd is an indication of a violation of the time-reversal invariance, provided there is no coherent precession of the neutron spin, which could lead to mimicking of a T-violating $\sigma_{kK\lambda}$

[8,9].

Limiting ourselves to a maximum value $K = 2$ of the target statistical-tensor rank, there are altogether 10 different combinations $(kK\lambda)$ of k, K, and λ, of which (000) involves no spin dependence, and (011) and (101) are formally equivalent (see Table I). It is seen in Table I that out of these first 10 combinations of $(kK\lambda)$, only (122) is associated with a pure T violation, while a nonzero partial cross section $\sigma_{kK\lambda}$ with $(kK\lambda) = (111)$ would indicate both P nonconservation and T violation. The P-nonconserving combinations $(kK\lambda)$, on the other hand, are (011), (101), (121), and (123). Thus neutron-transmission experiments involving only vector polarized beams or targets (i.e., polarizations of statistical-tensor ranks k or $K = 1$) already can be employed as tests of P conservation, while polarized neutron beams and at least tensor rank-2 polarized targets are necessary for tests of the T invariance, leaving aside the "doubly violating" case $(kK\lambda) = (111)$.

Within the developed formalism, analyzing powers, or spin-correlation coefficients in the total cross section are defined naturally in terms of the partial cross sections $\sigma_{kK\lambda}$ as

$$A_{kK\lambda} = \frac{\sigma_{kK\lambda}}{\sigma_{000}}, \tag{11}$$

where σ_{000} is the partial cross section for an unpolarized beam and target. According to the Madison Convention [16], $A_{kK\lambda}$ is an analyzing power when either k or K is zero, otherwise it should be called a spin-correlation coefficient. To simplify the terminology, the name "analyzing coefficient" will be used here generically for both kinds of these observables, unless the values of k, K, and λ are specified.

For an unambiguous identification of the experimental conditions under which a particular analyzing coefficient $A_{kK\lambda}$ is directly measurable, an explicit evaluation turns out to be useful of the corresponding correlation factor $C_{kK\lambda}(\hat{\mathbf{s}}\,\hat{\mathbf{I}}\,\hat{\mathbf{p}})$ in terms of Cartesian constructs (such as the dot and cross vector products) involving the unit vectors $\hat{\mathbf{s}}$, $\hat{\mathbf{I}}$, and $\hat{\mathbf{p}}$. Let us consider, for example, the P-even, T-odd correlation factor $C_{122}(\hat{\mathbf{s}}\,\hat{\mathbf{I}}\,\hat{\mathbf{p}})$. Using the methods of spherical-tensor algebra [17], it can be evaluated to have the following "fivefold" form [7,11]:

$$C_{122}(\hat{\mathbf{s}}\,\hat{\mathbf{I}}\,\hat{\mathbf{p}}) = -\sqrt{\tfrac{3}{10}}\,\hat{\mathbf{s}}\cdot(\hat{\mathbf{I}}\times\hat{\mathbf{p}})(\hat{\mathbf{I}}\cdot\hat{\mathbf{p}}). \tag{12}$$

Explicit Cartesian expressions for the correlation factors were calculated for all the ten combinations $(kK\lambda)$ that are limited by the maximum value $K = 2$ of the target statistical-tensor rank, and the results are collected in Table I [18].

An analyzing coefficient $A_{1K\lambda}$ is then measurable in principle as follows. Polarized neutrons are transmitted through a purely tensor rank-K polarized target and an asymmetry

$$\epsilon = \frac{N^+ - N^-}{N^+ + N^-} \tag{13}$$

is measured. Here N^+ (N^-) is the transmitted number of neutrons in a polarization state $\tilde{t}_{10}(s)$ $[-\tilde{t}_{10}(s)]$ with respect to a suitably oriented axis of neutron polarization $\hat{\mathbf{s}}$; it is assumed that the incident beam has the same intensity in the two polarization states. Provided that the directions $\hat{\mathbf{s}}$, $\hat{\mathbf{I}}$, and $\hat{\mathbf{p}}$ can be chosen so that no partial cross section with $k = 1$ other than $\sigma_{1K\lambda}$ can contribute to the total cross section σ_t, the asymmetry ϵ is related to $\sigma_{1K\lambda}$ by

$$\epsilon = -\tanh[\tilde{t}_{10}(s)\tilde{t}_{K0}(I)C_{1K\lambda}(\hat{\mathbf{s}}\,\hat{\mathbf{I}}\,\hat{\mathbf{p}})\sigma_{1K\lambda}n], \tag{14}$$

where $C_{1K\lambda}(\hat{\mathbf{s}}\,\hat{\mathbf{I}}\,\hat{\mathbf{p}})$ is the correlation factor of the chosen geometry of the directions $\hat{\mathbf{s}}$, $\hat{\mathbf{I}}$, and $\hat{\mathbf{p}}$, and n is the thickness of the target in the number of nuclei per unit area. The analyzing coefficient $A_{1K\lambda}$ itself is then

$$A_{1K\lambda} = -\frac{1}{\tilde{t}_{10}(s)\tilde{t}_{K0}(I)C_{1K\lambda}(\hat{\mathbf{s}}\,\hat{\mathbf{I}}\,\hat{\mathbf{p}})}\,\frac{\operatorname{artanh}\epsilon}{\sigma_{000}n}. \tag{15}$$

Thus to measure the P-even, T-odd spin-correlation coefficient A_{122} [19], to return to the example with $(kK\lambda) =$ (122), the axis of the neutron polarization $\hat{\mathbf{s}}$ is set parallel to $\hat{\mathbf{I}}\times\hat{\mathbf{p}}$, i.e., normal to the plane defined by the axis $\hat{\mathbf{I}}$ of the tensor rank-2 alignment of the target and the beam direction $\hat{\mathbf{p}}$ [20]. In this geometry, the partial cross sections σ_{101}, σ_{121}, and σ_{123} cannot contribute to the total cross section σ_t, which is confirmed by an inspection of the expressions for the correlation factors $C_{101}(\hat{\mathbf{s}}\,\hat{\mathbf{I}}\,\hat{\mathbf{p}})$ and $C_{12\lambda}(\hat{\mathbf{s}}\,\hat{\mathbf{I}}\,\hat{\mathbf{p}})$ with $\lambda = 1$ and 3 in Table I.

However, a single asymmetry measurement would not be sufficient for the determination of the analyzing coefficient $A_{1K\lambda}$ when, for given values of the tensor ranks $k = 1$ and K, there is no geometry of the directions $\hat{\mathbf{s}}$, $\hat{\mathbf{I}}$, and $\hat{\mathbf{p}}$ in which, out of the partial cross sections with $k = 1$, only the partial cross section $\sigma_{1K\lambda}$ can contribute to the total cross section σ_t. Thus to measure the spin-correlation coefficients A_{110} and A_{112}, the P-odd, T-even

TABLE I. The correlation factors $C_{kK\lambda}(\hat{\mathbf{s}}\,\hat{\mathbf{I}}\,\hat{\mathbf{p}})$ and their symmetries under the parity transformation P and time reversal T for various combinations of the statistical-tensor ranks k and K and the orbital angular momentum transfer λ. The positive (negative) sign denotes that the correlation factor is even (odd) under a symmetry transformation.

$(kK\lambda)$	$C_{kK\lambda}(\hat{\mathbf{s}}\,\hat{\mathbf{I}}\,\hat{\mathbf{p}})$	P	T
(000)	1	+	+
(011)	$\sqrt{\frac{1}{3}}\,\hat{\mathbf{I}}\cdot\hat{\mathbf{p}}$	−	+
(022)	$\frac{3}{2}\sqrt{\frac{1}{5}}\,[(\hat{\mathbf{I}}\cdot\hat{\mathbf{p}})^2 - \frac{1}{3}]$	+	+
(101)	$\sqrt{\frac{1}{3}}\,\hat{\mathbf{s}}\cdot\hat{\mathbf{p}}$	−	+
(110)	$\sqrt{\frac{1}{3}}\,\hat{\mathbf{s}}\cdot\hat{\mathbf{I}}$	+	+
(111)	$-\sqrt{\frac{1}{6}}\,\hat{\mathbf{s}}\cdot(\hat{\mathbf{I}}\times\hat{\mathbf{p}})$	−	−
(112)	$\sqrt{\frac{3}{10}}\,[(\hat{\mathbf{s}}\cdot\hat{\mathbf{p}})(\hat{\mathbf{I}}\cdot\hat{\mathbf{p}}) - \frac{1}{3}\hat{\mathbf{s}}\cdot\hat{\mathbf{I}}]$	+	+
(121)	$\sqrt{\frac{3}{10}}\,[(\hat{\mathbf{s}}\cdot\hat{\mathbf{I}})(\hat{\mathbf{I}}\cdot\hat{\mathbf{p}}) - \frac{1}{3}\hat{\mathbf{s}}\cdot\hat{\mathbf{p}}]$	−	+
(122)	$-\sqrt{\frac{3}{10}}\,\hat{\mathbf{s}}\cdot(\hat{\mathbf{I}}\times\hat{\mathbf{p}})(\hat{\mathbf{I}}\cdot\hat{\mathbf{p}})$	+	−
(123)	$\frac{1}{2}\sqrt{\frac{15}{7}}\,[(\hat{\mathbf{s}}\cdot\hat{\mathbf{p}})(\hat{\mathbf{I}}\cdot\hat{\mathbf{p}})^2 - \frac{2}{5}(\hat{\mathbf{s}}\cdot\hat{\mathbf{I}})(\hat{\mathbf{I}}\cdot\hat{\mathbf{p}}) - \frac{1}{5}\hat{\mathbf{s}}\cdot\hat{\mathbf{p}}]$	−	+

analyzing power A_{101} would have to be determined in principle, too, in order to be able to account for possible contributions due to the partial cross section σ_{101} to the measured asymmetries (the P-odd, T-even σ_{101} happens to be, of course, negligibly small under ordinary circumstances).

The measurement of the analyzing powers A_{011} and A_{022} does not involve the use of polarized beams. The P-odd, T-even analyzing power A_{011} is determined by the measurement of the asymmetry (13), where, however, N^+ (N^-) now refers to the transmitted number of neutrons for the target in a polarization state $\tilde{t}_{10}(I)$ $[-\tilde{t}_{10}(I)]$ with respect to a suitably oriented axis $\hat{\mathbf{I}}$ of pure $K = 1$ polarization of the target (a natural choice is, of course, to set $\hat{\mathbf{I}}$ parallel to the beam direction $\hat{\mathbf{p}}$). To determine the analyzing power A_{022}, on the other hand, it is in principle sufficient to use two different settings $\hat{\mathbf{I}}_1$ and $\hat{\mathbf{I}}_2$ of the axis of the tensor rank-2 alignment of the target, and to measure the corresponding asymmetry ϵ. The analyzing power A_{022} [21] is then

$$A_{022} = -\frac{2(\sigma_{000}n)^{-1}\text{artanh}\,\epsilon}{\tilde{t}_{20}(I)\left[C_{022}(\hat{\mathbf{s}}\,\hat{\mathbf{I}}_1\,\hat{\mathbf{p}}) - C_{022}(\hat{\mathbf{s}}\,\hat{\mathbf{I}}_2\,\hat{\mathbf{p}})\right]}, \tag{16}$$

where $C_{022}(\hat{\mathbf{s}}\,\hat{\mathbf{I}}_i\,\hat{\mathbf{p}})$, $i = 1, 2$ are the correlation factors for the two settings of $\hat{\mathbf{I}}$ [22].

In conclusion, it should be stressed that the developed formalism relates rigorously the analyzing coefficients $A_{kK\lambda}$ to the elastic-scattering S matrix. This enables an evaluation of these observables when the elastic-scattering S matrix can be calculated from a neutron-nucleus interaction of appropriate parity and time-reversal symmetries [23]. In fact, a nonzero analyzing coefficient $A_{kK\lambda}$ indicates that there is a term in the neutron-nucleus interaction that has the same symmetries and the same spherical-tensor structure as the corresponding correlation factor $C_{kK\lambda}(\hat{\mathbf{s}}\,\hat{\mathbf{I}}\,\hat{\mathbf{p}})$ [7].

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[15] It should perhaps be emphasized that the correlation factors $C_{kK\lambda}(\hat{\mathbf{s}}\,\hat{\mathbf{I}}\,\hat{\mathbf{p}})$ do not represent any quantum-mechanical operators or states; they are simply ordinary "c numbers," and so the transformations of space inversion and time reversal do not involve here any quantum-mechanical intricacies, such as those of phase conventions, etc.

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[19] The "T-odd spin-correlation coefficient" A_5 of [7] is $A_5 = -(3/40)^{1/2}A_{122}$.

[20] In actual experiments, it may be advantageous to vary the angle θ between $\hat{\mathbf{I}}$ and $\hat{\mathbf{p}}$ and then to fit the asymmetry to the $\sin 2\theta$ behavior expected from the correlation coefficient $C_{122}(\hat{\mathbf{s}}\,\hat{\mathbf{I}}\,\hat{\mathbf{p}})$, see [3].

[21] The "deformation cross section" $\sigma_{\rm def}$ of [7] is $\sigma_{\rm def} = 5^{-1/2}\sigma_{000}A_{022}$.

[22] In actual experiments, it may again be advantageous to vary the angle θ between $\hat{\mathbf{I}}$ and $\hat{\mathbf{p}}$ and then to fit the asymmetry to the $P_2(\cos\theta)$ behavior of the correlation coefficient $C_{022}(\hat{\mathbf{s}}\,\hat{\mathbf{I}}\,\hat{\mathbf{p}})$, as in J. E. Koster, C. R. Gould, D. G. Haase, and N. R. Roberson, Phys. Rev. C **49**, 710 (1994).

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