Alpha-cluster states of ^{212}Po in a realistic potential model

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Half-lives and α-branching ratios for the members of the ground-state "band" in ^{212}Po are deduced using an α-cluster model. The α-core potential is obtained from a double folding procedure and has integral values close to those obtained in elastic α scattering. The preformation probability is found to be $P = 0.035$. The long half-life of the (18^+) isomer is explained by the prediction of the excitation energy of the 16^+ state to lie above the (18^+) isomer. The (11^-) and (13^-) at $E_x = 2411.3$ and 2771.8 keV are tentatively assigned to the negative parity α-cluster "band."

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Recently we [1] have shown that the use of an optical potential deduced from experimentally known charge distributions by means of the double-folding procedure [2] results in a very good description of elastic and inelastic scattering of α particles on ^{208}Pb for incident energies ranging from near the Coulomb barrier up to 139 MeV. Abele and Staudt [2] have shown that for the system $^{16}\mathrm{O}\otimes\alpha$ these potentials enable not only excellent fits to elastic scattering data but also allow one to describe the excitation energies of α-cluster states in ^{20}Ne and their γ-decay properties. The success of this work motivated the present investigation on the decay properties of the members of the ground state (g.s.) "band" in ^{212}Po described as quasibound states in the system $^{208}\mathrm{Pb}\otimes\alpha$. In a very recent work Buck *et al.* [3] investigated the same system using a phenomenological α-cluster potential with parameters obtained in a global fit to α-decay properties of heavy nuclei. Even though we agree with the conclusion made in this work that the α-cluster configuration contributes a large amount to the structure of the yrast states in ^{212}Po, there are important differences in our approach to this problem.

The levels of the g.s. "band" and two members of a possible negative parity "band" in ^{212}Po where α clustering might be important are shown in Fig. 1.

In the framework of the double-folding model the real part of the optical potential is described by

$$V_N(r) = \lambda_F U_F(r) = \lambda_F \int_0^\infty \int_0^\infty \rho_T(\mathbf{r}_T)\rho_P(\mathbf{r}_P) v_{\mathrm{eff}}(E, \rho_T + \rho_P, s = |\mathbf{r} + \mathbf{r}_P - \mathbf{r}_T|) d\mathbf{r}_P d\mathbf{r}_T, \quad (1)$$

where $\mathbf{r}$ is the separation of the centers of mass of the target (core) nucleus and the α particle (cluster), and $\rho_T(\mathbf{r}_T)$ and $\rho_P(\mathbf{r}_P)$ are the nucleon densities of the cluster and core, respectively. Details of the numerical computation of the potential $U_F(r)$ are given in Refs. [1] and [2]. The volume integral of the folding potential $U_F(r)$ calculated for an α-particle energy of 10 MeV is

$$J_R = \frac{4\pi}{A_T A_P} \int U_F(r) r^2 dr = 264.63 \text{ MeV fm}^3. \quad (2)$$

A_T and A_P are the mass numbers of the core and the cluster, respectively. To this α-core nuclear potential one has to add the Coulomb potential $V_C(r)$, which we take as a uniformly charged sphere of radius $R_c = 7.11$ fm, and a centrifugal term $V_L(r)$ as it arises from the separation of the angular coordinates in this single-particle problem. The total α-core potential has thus only one "free" parameter, the normalization factor λ_F. But from extensive studies on elastic and inelastic scattering of α particles on various nuclei using this potential [1,2] it is known that the volume integral of the nuclear α-core potential has to be in the range 260 MeV fm^3 $< J_R <$ 400 MeV fm^3, thus restricting λ_F to the range $1.0 < \lambda_F < 1.5$.

In the cluster model members of a common "band" are characterized by a common value G, which in the case of an α cluster is given by

$$G = 2N + L = \sum_{i=1}^{4} (2n_i + l_i) = \sum_{i=1}^{4} g_i. \quad (3)$$

In this equation, the so-called Wildermuth condition [4], N is the number of nodes and L the relative angular momentum in the α-core wave function and $2n_i + l_i = g_i$ the corresponding quantum numbers of the nucleons building up the cluster. To satisfy the Pauli principle these individual nucleons have to occupy the $6\hbar\omega$ neutron and $5\hbar\omega$ proton shells, respectively; thus a value of $G \geq 22$ is to be expected.

In the calculation of the α-decay width Γ_α we use the semiclassical approximation outlined in Ref. [5]:

$$\Gamma_\alpha = PF \frac{\hbar^2}{4\mu} \exp\left[-2 \int_{r_1}^{r_2} k(r) dr\right]. \quad (4)$$

The preformation factor P is adjusted to give the measured half-life of the ground state in ^{212}Po. The normalization factor F is according to [5] approximately given by

$$F \int_{r_0}^{r_1} \frac{dr}{2k(r)} = 1 \quad , \quad k(r) = \sqrt{\frac{2\mu}{\hbar^2}|Q - V(r)|}, \quad (5)$$

with the wave number $k(r)$.

The radii r_0, r_1, r_2 are the classical turning points given by the condition $V(r_i) = Q$, with r_0 the innermost turning point. For low L values r_0 ranges from close to zero up to ~ 3.8 fm for $L = 18$; r_1 is found to be ~ 8.5 fm and $r_2 \sim 26$ fm using our potential.

The reduced transition probabilities for γ transitions between cluster states are given by

$$B(EL, L_i \to L_f) = \frac{\beta_L^2 e^2}{2L_i + 1} \left[\int_0^{r_2} u_{N_f L_f}(r) r^L u_{N_i L_i}(r) dr \right]^2 \langle L_f || Y_L^0 || L_i \rangle^2, \tag{6}$$

where the wave functions $u_{NL}(r)$ have been calculated using the subroutine BIND from the distorted-wave Born approximation (DWBA) code DWUCK5 [6] and have been normalized to give

$$\int_0^{r_2} u_{NL}^2(r) dr = 1. \tag{7}$$

The factor β_L^2 accounts for the effect that the moments of the charge radius do not equal the moments of the separation of core and cluster and is given according to Ref. [7] by the expression

$$\beta_L = \frac{A_T^L Z_P + (-1)^L A_P^L Z_T}{(A_T + A_P)^L}. \tag{8}$$

To deduce half-life $T_{1/2}$ values and α-branching ratios b_α one has to calculate the electromagnetic decay width

$$\Gamma_{\rm el} = \frac{(1 + \alpha_{IC}) 2(L+1)}{\epsilon_0 L [(2L+1)!!]^2} \left(\frac{E_\gamma}{\hbar c} \right)^{2L+1} B(EL, L_i \to L_f) , \tag{9}$$

with $1/\epsilon_0 = 18.097$ MeV fm/e^2 and $\hbar c = 197.33$ MeV fm. Especially for low transition energies and high multipolarities internal conversion can be the dominating process; therefore the conversion coefficient α_{IC} has to be taken into account in calculating the total width of the levels. The half-life $T_{1/2}$ and the α-branching ratio b_α are then given by

$$T_{1/2} = \frac{\hbar \ln 2}{\Gamma_\alpha + \Gamma_{\rm el}} \quad , \quad b_\alpha = \frac{\Gamma_\alpha}{\Gamma_\alpha + \Gamma_{\rm el}} . \tag{10}$$

In Table I the results of the present calculation are presented. The normalization factors λ_F have been adjusted to reproduce the experimental excitation energies for the different states in ^{212}Po belonging to the g.s. "band" using a value of $G = 22$ for the global quantum number, since a constant λ_F would result in an inverted spectrum. From the study of elastic and inelastic α scattering [1] one would expect a value of $J_R \sim 350$ MeV fm^3 for the volume integral of the potential corresponding to

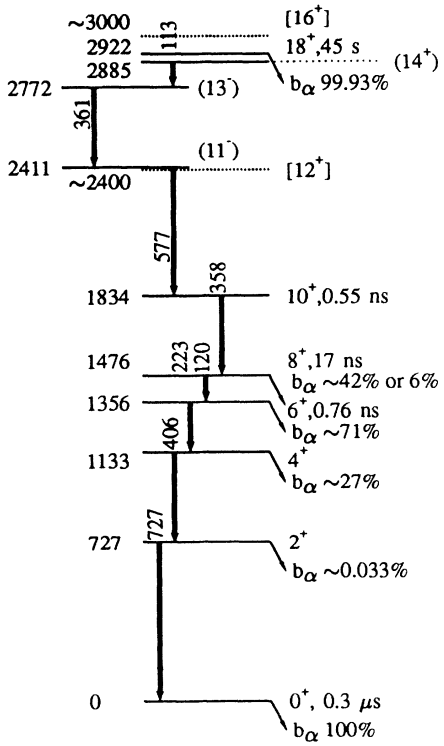

FIG. 1. Partial level scheme for ^{212}Po [12]. Level and transition energies are given in keV, α-branching ratios b_α in %. Known half-lives are quoted next to the spin of the levels. Dashed lines denote levels predicted by the potential model.

TABLE I. Spin J^π and excitation energies E_x [12], and theoretical ($E2$)-conversion coefficients [16] for states belonging to the g.s. "band" of ^{212}Po. The normalization factors λ_F have been determined to reproduce the experimental excitation energies. The α-decay widths Γ_α have been calculated with a preformation probability $P = 0.035$; the "γ-decay" widths $\Gamma_{\rm el}$ include the internal conversion process. The excitation energies (denoted with a superscript b) of the 12^+ and 16^+ members are predicted in interpolating the corresponding values for λ_F.

J^π	E_x (MeV)	$\alpha_{IC}(E2)$[a]	λ_F	Γ_α (eV)	$\Gamma_{\rm el}$ (eV)
0^+	0.000	-	1.203	1.54×10^{-9}	-
2^+	0.727	0.014	1.193	3.96×10^{-8}	9.25×10^{-5}
4^+	1.133	0.055	1.187	5.32×10^{-8}	7.25×10^{-6}
6^+	1.356	0.340	1.179	3.08×10^{-8}	4.75×10^{-7}
8^+	1.476	3.4	1.173	4.03×10^{-9}	6.53×10^{-8}
10^+	1.834	0.076	1.163	7.46×10^{-10}	3.52×10^{-6}
(12^+)	2.4[b]		1.148[b]	1.75×10^{-10}	
(14^+)	2.885		1.132	1.43×10^{-11}	
(16^+)	3.0[b]		1.119[b]	1.48×10^{-13}	
18^+	2.922	$E4{:}2.2\times10^6$	1.105	3.27×10^{-16}	2.54×10^{-21}
(11^-)	2.411		1.228	1.89×10^{-9}	
(13^-)	2.772		1.215	1.35×10^{-10}	

[a]Total conversion coefficient from Ref. [16].
[b]Interpolated from neighboring values.

$\lambda_F \sim 1.35$. This larger volume integral can be obtained if the calculations are performed using a value $G = 24$ for the global quantum number, implying that higher-lying shells do contribute to the structure of the cluster states. It turns out, however, that the predictions for the decay widths of these states do not change very much (less than about 10%) if the number of nodes in the wave function of the relative motion is increased by 1. A rather smooth decrease in the value of the volume integral of the potential is observed as the L value increases. This decrease reflects either a deficiency in the potential or in the model. The relatively successful description of excitation energies and spectroscopic factors using shell model calculations [8] which predict the yrast states in ^{212}Po to arise from $\pi h_{9/2}^2 \otimes \nu g_{9/2}^2$ configurations with a leading $[\pi h_{9/2}^2]_0 \otimes [\nu g_{9/2}^2]_J$ component hints at rather a limitation of the potential model. It has to be pointed out that the change in λ_F enters linearly in the volume integral J_R. This corresponds to a change in J_R of about 3.3% to reproduce the levels from spin 0 to 10, whereas the volume integral of the potential used by Buck *et al.* [3] corresponds to a change by almost 10% for the same levels. Moreover, Buck *et al.* [3] used a global quantum number $G = 20$. This clearly contradicts the fact that ^{208}Pb is a doubly magic nucleus.

In contrast to the recent calculations of Buck *et al.* [3] we need a preformation probability of $P = 0.035$ ($P = 0.027$ for $\lambda_F = 1.355$ which gives the correct Q value for $G = 24$) to reproduce the α-decay width of the g.s. in ^{212}Po. This result is in line with predictions for this factor from the microscopic calculations of Varga *et al.* [9] which give a spectroscopic factor of $S = 0.025$. The theoretical investigations of Varga *et al.* are performed in a combined shell and cluster model. It should be pointed out that the radial wave function of the cluster model has 12 nodes in their calculations. All α-decay widths in Table I are calculated with the same value $P = 0.035$. The electromagnetic transition probabilities are calculated under the assumption of pure cluster configurations ($\mathcal{S} = 1$). The ratio $\mathcal{S}/P >> 1$ can be explained by the fact that only a small fraction of the α-cluster configuration ($\mathcal{S}$) present in ^{212}Po can also exist as a free α particle and a ^{208}Pb core (P) due to the Pauli principle. A more detailed discussion on spectroscopic factors in α decay can be found in Refs. [9] and [10]. In the calculations of Ref. [9] a very high amount of alpha clustering is found ($\mathcal{S} = 0.302$).

In Table I also the 14^+ member of the "band" proposed by Sugawara *et al.* [11] has been used to fix the corresponding value for λ_F. With this one can predict the excitation energy of the missing 12^+ and 16^+ levels of the "band" in interpolating between the λ_F values of the $I = L + 2$ and $I = L - 2$ members of the "band." It is very interesting to note that with this interpolation the excitation energy of the 16^+ member is predicted at about 3 MeV that is *above* the excitation energy of the 18^+ ($T_{1/2} = 45$ s) isomer. The long half-life of the isomer is thus explained by the fact that with $E4$ multipolarity and a transition energy of about 35 keV the width for electromagnetic decay is extremely small even if one accounts for the internal conversion. On the other hand the predicted half-life for the α decay is by about a factor of 20 shorter than the experimental result if the same preformation probability of $P = 0.035$ is assumed. It is clear that for states with spin $I > 8$ the above mentioned $[\pi h_{9/2}]_0^2 \otimes [\nu g_{9/2}^2]_J$ configuration cannot contribute; thus it is not surprising to have different spectroscopic factors for the high spin states. For this explanation to be valid, one has to assume a steeper decrease for λ_F with increasing spin as expected by a linear extrapolation, since in this case the 20^+ and 22^+ members would be energetically below the 18^+ state, thus enabling $E2$ transitions.

A more detailed test of the wave functions obtained in the potential model can be performed in studying the predictions of half-life, $B(EL) \downarrow$ values and α-branching ratios. This comparison of the available experimental information [12] is presented in Table II. For the first excited 2^+ state at $E_x = 727$ keV an α branching of 0.042 is predicted, in very good agreement with the experimental data. Disagreement with the evaluation of the data in Ref. [12] is found for the b_α values for the 4^+ to 8^+ members of the g.s. "band." It has to be pointed out, however, that the values given in Ref. [12] are not directly measured.

Very good agreement is found for the half-life of the 6^+ state as well as for the corresponding $B(E2)$ value. If the experimentally determined $B(E2)$ value and the measured half-life are correct, the b_α value of 71% cannot

TABLE II. Comparison of experimental half-lives $T_{1/2}$(expt), $B(E2)$ values and α branchings with the theoretical predictions in the potential model for states belonging to the g.s. "band" of ^{212}Po.

J^π	E_x (MeV)	$T_{1/2}$(expt)	$T_{1/2}$(theor)	$B(E2)\downarrow$(expt) (W.u.)	$B(E2)\downarrow$(theor) (W.u.)	b_α(expt) (%)	b_α(theor) (%)
0^+	0.000	0.3 μs	0.3 μs[a]	-	-	100	100
2^+	0.727	-	4.9 ps	-	7.41	0.033	0.042
4^+	1.133	-	62 ps	-	10.34	~27	0.72
6^+	1.356	0.76(22) ns	0.90 ns	13.5(37)	10.62	~71	6.08
8^+	1.476	17 ns	7.0 ns	3.95(5) 8.4(20)[b]	10.04	~42 6(1)[b]	5.7
10^+	1.834	0.55(14) ns	0.13 ns	2.2(6)	9.16	no α	0.021
18^+	2.922	45 s	1.6 s	$E4$: $2.3^{-1.4}_{+4.6}$ [c]	$E4$: 5.3	99.93	99.94

[a] Adjusted to experimental half-life using $P = 0.035$.

[b] From the direct measurement in Ref. [13].

[c] $B(E4)$ value deduced from conversion coefficient [15].

be correct. From the experimental data in Table II one can deduce an α branching of about 30%.

Good agreement is found for the b_α value of 6 ± 1 % for the deexcitation of the 17 ns 8^+ isomer at 1476 keV directly measured by Lieder *et al.* [13], but the $B(E2, 8^+ \rightarrow 6^+)$ value is overestimated by a factor of 2.5. Here one should note, however, an experimental fact which was not taken into account in the in-beam work of Refs. [11] and [14]. Lieder *et al.* [13] measured γ rays in coincidence with the 10.18 MeV α-decay line from the 8^+ isomer and observed a 120 keV line in clear coincidence. This implies that this γ ray has to be placed *above* the 1476 keV isomer. In this investigation [13] the $B(E2, 8^+ \rightarrow 6^+)$ value has been deduced to be 8.4(20) Weisskopf units (W.u.), which agrees again very well with the predictions of our model.

In accordance with the experimental finding a very small α branching is predicted for the decay of the 10^+ state at 1834 keV, but the $B(E2)$ value is overestimated in the potential model.

As stated above the potential model underestimates the half-life of the 18^+, but it is very interesting to note that the predicted $E4$-transition probability $B(E4, 18^+ \rightarrow 14^+) = 5.3$ W.u. agrees within the errors with the experimental results obtained by Kudo *et al.* [15] of $B(E4, 18^+ \rightarrow 14^+) = 2.3^{-1.4}_{+4.6}$ W.u.

Two levels with spin (11^-) and (13^-) are reported [11] at an excitation energy of 2411.3 keV and 2771.8 keV, respectively. The (13^-) level is defined by a populating $E1$ transition from the (14^+) level and a deexcitation via an $E2$ transition to the (11^-) level, which itself decays to the 10^+ state of the g.s. "band." This pattern might be explained in the following way: If these negative parity states are interpreted as members of a "band" belonging to the global quantum number $G = 23$ (the corresponding values for λ_F are 1.2281 for the 11^- level and 1.2154 for the 13^- state, respectively), the potential model predicts a $B(E1, 14^+ \rightarrow 13^-)$ value of 45 mW.u. corresponding to a partial half-life of the 14^+ state of about $T_{1/2} \sim 2$ ps. The competing $B(E2, 14^+ \rightarrow 12^+)$ value is predicted by this model to be about 6.7 W.u. corresponding to partial half-life of about $T_{1/2} \sim 0.15$ ns, assuming that the predicted excitation energy of about 2.4 MeV for the 12^+ member of the g.s. "band" is correct. The same arguments hold for the deexcitation of the 11^- state; again the $E1$ branch is much faster than the competing $E2$ branch to the as yet unidentified 9^- member of that negative parity "band," which, however, has to be at a higher excitation energy than the 10^+, since otherwise an $E1$ transition from the 10^+ state should be observed experimentally. Even though this interpretation is very tempting it needs more experimental information, e.g., lifetime measurements, to be confirmed.

In conclusion it was shown that a potential model based on a double-folding potential obtained from nuclear mass densities of the cluster and core is able to reproduce the experimental information for the members of the g.s. "band" in ^{212}Po very well. This potential (which is extensively tested in elastic and inelastic scattering) allows a good description of the data if the global quantum number is taken to be $G = 22$, thus satisfying the Wildermuth condition. In line with theoretical predictions a preformation factor of $P \sim 0.03$ is required to reproduce the half-life of the ground state in ^{212}Po. The α-decay width for the 18^+ isomer is predicted to be too wide, indicating a possible reduction in the amount of α clustering at high spin. The model accounts well for the observed $E2$ strength and even for the $E4$ strength of the 18^+ isomer. The fact that the pure cluster model reproduces the electromagnetic transition probabilities quite well proves that a very high amount of alpha clustering has to be present in the structure of the members of the g.s. "band." From an interpolation of the L dependence of the normalization factors λ_F the 16^+ state is predicted to be at an excitation energy of about 3 MeV, thus explaining the missing electromagnetic decay branch of the 18^+ isomer. In view of some obvious inconsistencies in the experimental data further investigations on the structure of the yrast and yrare states in ^{212}Po are highly desirable. This would also allow one to possibly confirm the interpretation of the high spin negative parity states as members of the odd-parity $G = 23$ "band."

[1] R. Neu and F. Hoyler, Phys. Rev. C **46**, 208 (1992).
[2] H. Abele and G. Staudt, Phys. Rev. C **47**, 742 (1993).
[3] B. Buck, A.C. Merchant, and S.M. Perez, Phys. Rev. Lett. **72**, 1326 (1994).
[4] K. Wildermuth and Y.C. Tang, *A Unified Theory of the Nucleus* (Academic Press, New York, 1977).
[5] S.A. Gurvitz and G. Kälbermann, Phys. Rev. Lett. **59**, 262 (1987).
[6] P.D. Kunz, computer code DWUCK5, University of Colorado, 1978 (unpublished).
[7] B. Buck and A.A. Pilt, Nucl. Phys. **A280**, 133 (1977).
[8] D. Strottmann, Phys. Rev. C **20**, 1150 (1979).
[9] K. Varga, R.G. Lovas, and R.J. Liotta, Phys. Rev. Lett. **59**, 37 (1992).
[10] R. Blendowske, T. Fliessbach, and H. Walliser, Z. Phys. A **339**, 121 (1991).
[11] M. Sugawara, Y. Gono, Y. Ishikawa, and F.M. Fukuda, Nucl. Phys. **A443**, 461 (1985).
[12] A. Artna-Cohen, Nucl. Data Sheets **66**, 171 (1992).
[13] R.M. Lieder, J.P. Didelez, H. Beuscher, D.R. Haenni, M. Müller-Veggian, A. Neskakis, and C. Mayer-Böricke, Phys. Rev. Lett. **41**, 742 (1978).
[14] A.R. Poletti, G.D. Dracoulis, A.P. Byrne, and A.E. Stuchberry, Nucl. Phys. **A473**, 595 (1987).
[15] H. Kudo, T. Nomura, K. Sueki, M. Magara, and N. Yoshida, Nucl. Phys. **A494**, 203 (1989).
[16] R.S. Hager and E.C. Seltzer, Nucl. Data A **4**, 1 (1968).