Thermal equilibrium of the nuclear system in the ^{40}Ca(35 MeV/nucleon)+^{40}Ca reaction

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A semiclassical simulation is made for the reaction ^{40}Ca(35 MeV/nucleon)+^{40}Ca. The experimental fragment charge dispersion in the reaction is reproduced reasonably. Time evolutions of the fragment charge dispersion and of the average square of the component of particle momentum seem to indicate that the nuclear system formed in the intermediate energy nucleus-nucleus collisions might approach thermal equilibrium in the case of nearly central collisions.

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I. INTRODUCTION

The formation, disassembly, and associated phase transitions of a hot nucleus are the objects of intensive studies both experimentally and theoretically in recent years. The main motivation for this interest is that the hot nucleus does not exist in nature but is formed in energetic nuclear collisions. The properties of the hot nucleus, such as its decay and associated phase transitions (liquid-gas phase transition, for instance), must be different from that of a nucleus in a conventional state. Therefore these studies may shed light on the nuclear interaction and equation of state.

The works in this field can be roughly cataloged into two branches: one is the dynamical approaches [1–8] and the other is the statistical approaches [9–24]. However, the more successful one is the equilibrium statistical models [12–24].

Many investigations on the problem of whether the nuclear system formed in high or intermediate energy nuclear collisions can reach thermal equilibrium before breakup have been reported both experimentally [25–27] and theoretically [2,8,28]. However, there is no unique answer yet.

In this paper we use the program of semiclassical simulation of intermediate energy nucleus-nucleus collisions (SSIENC) [5,6] to study the problem of the nuclear system in the reaction ^{40}Ca(35 MeV/nucleon)+^{40}Ca approaches to thermal equilibrium before breakup. SSIENC is quite similar to the quantum molecular dynamics (QMD) approach [2–4]. The simulated results of time evolutions of the fragment charge dispersion and the average square of the component of particle momentum (particle refers to the nucleon and/or Δ resonance hereafter) seem to prefer a positive answer. In addition, experimental result of fragment charge dispersion is also reproduced reasonably.

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II. INGREDIENTS OF THE MODEL

In SSIENC, the colliding nuclei are considered spheres with a radius of $1.14A^{1/3}$, where A refers to the mass number of projectile (A_p) or target (A_t) nucleus. Each nucleon is depicted as a Gaussian distribution both in $\mathbf{r}$ and $\mathbf{p}$, and the corresponding distribution function is

$$f(\mathbf{r},\mathbf{p},t)=\frac{1}{(\pi\hbar)^3}\exp[-\alpha^2(\mathbf{r}-\mathbf{r}_0)^2] \times\exp\left(-\frac{(\mathbf{p}-\mathbf{p}_0)^2}{(\hbar\alpha)^2}\right) \quad (1)$$

($\alpha=0.7$). The initial centroid of nucleon Gaussian distribution in spatial space ($\mathbf{r}_0$) is uniformly distributed in the sphere of the parent nucleus (at rest) under the nonoverlapping condition.

Nucleon-nucleon interaction is regarded as a sum of the Skyrme-type interaction, the long-range Yukawa-type interaction, and the Coulomb interaction (for proton-proton only):

$$V(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)=V^S(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3) +V^Y(\mathbf{r}_1-\mathbf{r}_2)+V^C(\mathbf{r}_1-\mathbf{r}_2)\ , \quad (2)$$

$$V^S=t_1\delta(\mathbf{r}_1-\mathbf{r}_2)+t_2\delta(\mathbf{r}_1-\mathbf{r}_2)\delta(\mathbf{r}_1-\mathbf{r}_3)\ , \quad (3)$$

$$V^Y=t_3\frac{\exp[-\mu|\mathbf{r}_1-\mathbf{r}_2|]}{\mu|\mathbf{r}_1-\mathbf{r}_2|}\ , \quad (4)$$

$$V^C=\frac{Z_1Z_2e^2}{|\mathbf{r}_1-\mathbf{r}_2|}\ . \quad (5)$$

The potential energy of the nucleon is then calculated by integrating the nucleon-nucleon interactions over phase spaces of all interacting partners, i.e.,

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$$U(\mathbf{r}_{0i},t)=\sum_{j,k=1;j\neq i,k\neq i,k\neq j}^{A}\int d\mathbf{r}_i\,d\mathbf{p}_i\,d\mathbf{r}_j\,d\mathbf{p}_j\,d\mathbf{r}_k\,d\mathbf{p}_k\,V(\mathbf{r}_i,\mathbf{r}_j,\mathbf{r}_k)f(\mathbf{r}_i,\mathbf{p}_i,t)f(\mathbf{r}_j,\mathbf{p}_j,t)f(\mathbf{r}_k,\mathbf{p}_k,t)\;. \tag{6}$$

In this paper the nucleons are treated as spin degenerate, and the potential energy is assumed to be isospin independent for simplicity. The initial centroid of nucleon momentum Gaussian distribution ($\mathbf{p}_0$) is determined by the potential energy together with the input binding energy per nucleon of the nucleus. the parameters, t_1, t_2, t_3, and μ, in Eqs. (2)–(5), are adjusted so as to guarantee that the projectile and target nuclei are to be regarded as stable isolated nuclei during the characteristic time interval t_{pass}. Here t_{pass} is defined as the time needed for the projectile nucleus to pass through the target nucleus.

Δ resonances are regarded as transporters (transporting particles) as well as nucleons. No pions are considered explicitly in transport process. The considered reactions are $NN\to NN$, $NN\to N\Delta$, $N\Delta\to NN$, $N\Delta\to N\Delta$, and $\Delta\Delta\to\Delta\Delta$.

The collision between two particles and whether the collision is elastic or inelastic are decided randomly according to the corresponding total, elastic, and inelastic nucleon-nucleon cross section in free space. These cross sections are parametrized in the same energy-dependent way as Refs. [1] and [5]. The cross sections of ΔN and $\Delta\Delta$ interactions are assumed to be the same as the NN interaction at the corresponding center of mass energy. The inelastic cross section of $N\Delta\to NN$ is related to the cross section of $NN\to N\Delta$ via the detail balance [1]. Calculations are performed in a target rest frame, but the particle-particle binary collision is treated in the center of momentum system of the colliding particles.

Particle centroid ($\mathbf{r}_{01}$, for instance) moves along its Newton trajectory

$$\frac{d\mathbf{r}_{0i}}{dt}=\frac{\mathbf{p}_{0i}}{E_i}\;, \tag{7}$$

$$\frac{d\mathbf{p}_{0i}}{dt}=-\nabla_r U(\mathbf{r}_{0i})\;, \tag{8}$$

within the time step δt; here E_i is ith particle energy. Particle collision might happen at the moment $t+\delta t$ if the relative central distance between colliding particles reaches minimum during δt and is less than $\sqrt{\sigma_{\text{tot}}/\pi}$.

Pauli blocking is taken into account by counting the occupied percentage of phase-space volume $\Omega=h^3$ around the scattered state of the colliding particle by the surrounding particles

$$P_i=\frac{1}{(\pi\hbar)^3}\sum_{k=1,k\neq i}^{A_p+A_t}\int_{\Omega_i}\exp[-\alpha^2(\mathbf{r}_i-\mathbf{r}_{0k})^2]\times\exp\left(\frac{(\mathbf{p}_i-\mathbf{p}_{0k})^2}{(\hbar\alpha)^2}\right)d\mathbf{r}_i d\mathbf{p}_i\;. \tag{9}$$

The collision between i and j particles is then unblocked with a probability

$$P_{\text{unblock}}=(1-P_i)(1-P_j)\;. \tag{10}$$

The cluster (fragment) analysis is made at each 15 fm/c until the end of the simulation time ($\sim$ 150 fm/c, which is long enough compared with $t_{\text{pass}}=64$ fm/c or with the time of the preequilibrium emission 70 fm/c [29]). The nuclear cluster is defined as the smallest nucleon assembly, in which any nucleon can be reached from other by sequential skips between nucleons. The length of the skip is assumed to be 3 fm [2,3]. These clusters are reconstructed to be spheres with radius $1.14A_c^{1/3}$, where A_c is the number of constituent nucleons of the cluster. Nonoverlapping, between the clusters, between the nucleons, and between the cluster and the nucleon, is required here.

III. RESULTS AND DISCUSSIONS

In Fig. 1, the calculated results of nuclear fragment charge dispersion in reaction ^{40}Ca(35 MeV/nucleon) $+^{40}$Ca (at $t=150$ fm/c, histogram) and the corresponding experimental data [29] (circles) are shown. Theoretical results in this figure and in Figs. 2–5 are the statistical average of the corresponding variable over 100 nucleus-nucleus collision events. In Figs. 1–4 the theoretical results are also the average over impact parameters $b=0$ and 1 (consistent with the experimental central collisions [29]) with weight 0.111 and 0.889, respectively, and are labeled as a central collision in the figures. The agreement between theory and experiment is reasonably good by considering that the theoretical results are not for the real nuclear fragments but for the nucleon clusters, as usually did in dynamic calculations [1–8].

The corresponding simulated fragment charge dispersions at different time $t=60$, 90, 120, and 150 fm/c are given in Fig. 2. One can see from this figure that after $t\sim 90$ fm/c the fragment charge dispersion seems no longer to change severely with time, which can be regarded as evidence that the nuclear system is going to

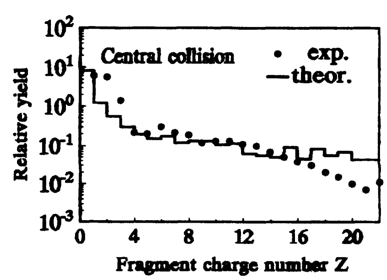

FIG. 1. Charge dispersion of nuclear fragment in reaction ^{40}Ca(35 MeV/nucleon) + ^{40}Ca: circles, the experimental data of Ref. [29]; histogram, the theoretical results.

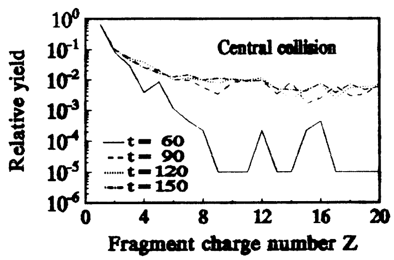

FIG. 2. The time evolution of the fragment charge dispersion.

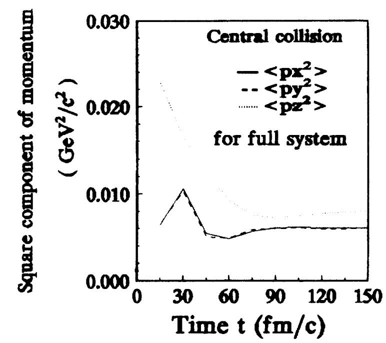

FIG. 4. The same as Fig. 3, but the particles are contained in the whole nuclear system.

approach thermal equilibrium before breakup.

The time evolution of the average square of the component of particle momentum (relative to the momentum of the center of mass) is given in Figs. 3 and 4. Here particle refers to the bound particle in the cluster or to the free particle. That cluster and particle are contained in the sphere with radius 7.8 fm around the center of mass of nuclear system (Fig. 3) or contained in the whole nuclear system (Fig 4). From Fig. 3 one sees that three components $\langle p_x^2 \rangle$, $\langle p_y^2 \rangle$, and $\langle p_z^2 \rangle$ seem to approach each other at $t \sim 90$ fm/c, which means that the nuclear system, inside the corresponding sphere, might be fully thermalized before the breakup. Note that 7.8 fm is twice the radius of the target nucleus, which is consistent with the assumption, adopted in statistical model [13–16], that the hot nucleus is expanded eight times in volume before the breakup. In Fig. 4 one sees that $\langle p_z^2 \rangle$ is larger than $\langle p_x^2 \rangle$ or $\langle p_y^2 \rangle$, which is reasonable, since one cannot ask for the thermal equilibirum in the unrestricted coordinate space.

Figure 5 is plotted as the impact parameter dependence of the time evolution of the average square of the component of particle momentum. One learns from this figure that with the increase of impact parameter the degree of approach to thermal equilibrium is getting worse. The dependence of the time evolution of cluster charge dispersion on the impact parameter is similar.

It is worthwhile here to review the concerned studies existing already. To our knowledge, the first authors who investigated the problem of thermal equilibrium were Cugnon, Mizutani, and Vandermeulen [28]; they looked for the time evolution (up to $t = 12$ fm/c) of baryon rapidity and transverse momentum distributions in reaction ^{40}Ca(2 GeV) $+^{40}$Ca. Due to the remnants of the initial maxima in both the final spectra of y and p_t, they came to the conclusion that the nuclear system is not fully thermalized. We think that 12 fm/c is too early to compare with $t_{\rm pass} \sim 70$ fm/c. Thus their conclusion might be different if their calculation is prolonged to $t > t_{\rm pass}$.

Later on Aichelin *et al.* [2] investigated the problem of thermal equilibrium with quantum molecular dynamics for the reaction Ne(1.05 GeV/nucleon)+ Au. What they looked for was the correlation between initial and final states via the relative probability of the nucleon, which locates initially at position r and finally in a cluster with size A. This relative probability is defined as $P_A = N_A(r)/\sum_{j=1}^{3} N_j(r)$, where N_j denotes the num-

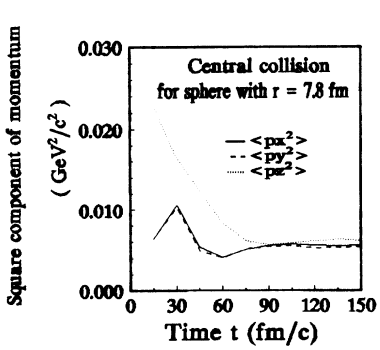

FIG. 3. The time evolution of the average square of the component of particle momentum; particles here are contained in the sphere of radius 7.8 fm around the center of mass of the nuclear system.

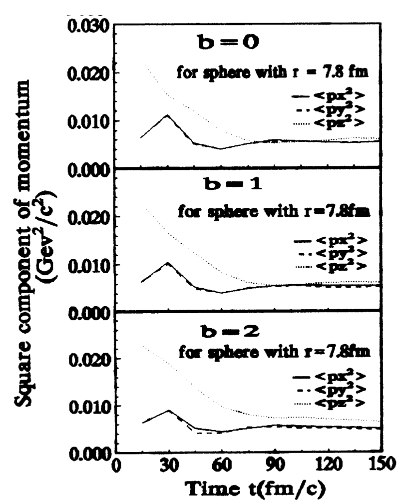

FIG. 5. Impact parameter dependence of the time evolution of the average square of the component of particle momentum; particles here are contained in the sphere of radius 7.8 fm around the center of mass of the nuclear system.

ber of nucleons that are finally in the fragment of class j (the fragments are distinguished into three classes: $A = 1$, $4 < A < 11$, and $A > 60$). They concluded that the equilibrium of the nuclear system is ruled out by the fact that the relative probability as a function of r is nonlinear, which means correlation between initial and final states. We believe that a large amount of free nucleons have been emerged before equilibration; these nucleons, of course, retain a memory of the initial configuration. Therefore, it is not correct to study the problem of thermal equilibrium (via correlation between initial and final states of nuclear system) by invoking free nucleons. Once the result of the $A = 1$ clusters is removed from $P_A(r)$, the relative probability, defined for $4 < A < 11$ and $A > 60$ only, might become linear with respect to r (cf. Fig. 12 in Ref. [2]). Thus their conclusion might be questionable as well.

Recently Donangelo and Wedemann [8] have studied the thermal equilibrium of nuclear system in reaction P(1 GeV)+Ag by an intranuclear cascade model with some mean-field effect. They concluded that the nuclear system reaches thermal equilibrium prior to its fragmentation due to the uniformly distributed kinetic energy of nucleons inside the nuclear system at the time 20 fm/c and at that time the expansion state is practically over.

In summary, in comparing with those existing studies mentioned above, the theoretical evidence supplied in the present paper (Figs. 2–5) reasonably suggests the following conclusion: in intermediate-energy nucleus-nucleus collisions, such as the reaction ^{40}Ca(35 MeV/nucleon)+^{40}Ca, the nuclear system might approach thermal equilibirum in the case of nearly central collisions, but in other cases thermal equilibrium might not occur.

Note added. After submission of our paper, we read the paper "Experimental Signature for Statistical Multifragmentation" written by Morreto, Delis, and Wozniak [30]. They supplied strong evidence of no correlation between entrance and exit channels and came to the conclusion that the dynamics of the nuclear reaction can only describe up to the formation of the source (hot nucleus), and once this source is formed its decay is apparently independent of its formation. Thus one might be fully convinced of the assumption of the thermal equilibrium adopted in the simultaneous multifragmentation models [2–18].

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