Anisotropic alpha decay from oriented odd-mass isotopes of some light actinides

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Half-lives and anisotropies in the α decay of ^{205,207,209}Rn, ^{219}Rn, ^{221}Fr, ^{227,229}Pa, and ^{229}U have been calculated using the reaction-theoretical formalism proposed by Jackson and Rhoades-Brown and adapted for axially symmetric deformed nuclei by Berggren and Olanders. The possibility of octupole deformation has been taken into account. In addition, a variant of triaxial octupole deformation has been considered tentatively in the case of ^{227}Pa and ^{229}Pa.

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I. INTRODUCTION

Alpha decay has traditionally been treated semiclassically (see, e.g., [1]) with remarkable success for spherical nuclei with long half-lives. The semiclassical methods have also been extended to deformed nuclei, e.g., by Fröman [2]. However, these methods turned out to be difficult to apply with any confidence to anisotropic α decays of high-spin states [3] and we decided that a fully quantal formulation on a firm reaction-theoretical foundation [4] would be more suitable for the analysis. The resulting program was also used to analyze the Leuven data [5] for odd-mass At and Po isotopes [6]. This program used harmonic oscillator wave functions; these were later [7] replaced by Woods-Saxon wave functions.

Recently, however, other formulations of alpha-decay theories have been proposed and applied [8,9]. In both Refs. [8,9] the reaction is treated semiclassically, while the main difference between them lies in the approach to the structure of the parent and daughter nuclei. Rowley *et al.* [8] treat the alpha as an elementary particle in a potential generated by the (possibly deformed) core, similar to our approach. Delion *et al.* [9] describe the alpha cluster microscopically, but they invoke the Fröman theory [2] for calculating the angular distribution of the ejected alpha particles. In order to put all these theories to test, experimental programs have been proposed [10] at ISOLDE (CERN) and LISOL (Louvain-la-Neuve) involving odd-mass nuclei of both spherical (^{205,207,209}Rn), transitional (^{219}Rn and ^{221}Fr), and well-deformed (^{227,229}Pa and ^{229}U) shape. The following is a report on the results obtained using one of the models to be tested: the alpha-plus-rotor model constructed on the basis of the Jackson–Rhoades-Brown theory [4]. The term "rotor" should, strictly speaking, be replaced by "core" in applications involving daughters whose deformation β (which can be related to their quadrupole moment) is rather small ($|\beta| < 0.1$, say). We should keep in mind that odd-mass nuclei always have a spin so that a pure spherical potential model for the alpha-core interaction may in any case be questionable for these nuclei.

In the α-particle-plus-rotor model [11] we have assumed an α-core interaction of quadrupole-quadrupole type which is equivalent to a deformed α-core potential whose symmetry axis coincides in every instance with the symmetry axis of the core (= the daughter nucleus). This potential then generates α orbitals. For an infinitely inert core each of these would have a well-defined angular momentum component $\hbar K_\alpha$ along the symmetry axis. If the potential is symmetric with respect to reflection in a plane orthogonal to the symmetry axis, then the energy eigenvalues will depend on $|K_\alpha|$. As it is, the core is not infinitely inert, and it has an angular momentum $\mathbf{J}_C$. The spin $\mathbf{I}$ of the parent nucleus is the sum $\mathbf{I} = \mathbf{J}_C + \mathbf{L}_\alpha$. By diagonalizing the quadrupole-quadrupole term in the weak-coupling basis of eigenfunctions to $\mathbf{L}_\alpha$ and $\mathbf{J}_C$ we obtain $(2I+1)/2$ solutions, which are the finite-inertia analogs of the $|K_\alpha|$-dependent α orbitals mentioned above. (They are also analogous to the Nilsson nucleon orbitals [12] corrected for the Coriolis force.)

The interpretation of the experimental data that was proposed to explain the results of Ref. [5] is that an increase of the neutron number N by two units necessitates a shift from one solution to the next one. From the point of view of the Pauli principle, this is most natural. An α particle in a nucleus contains two protons and two neutrons which need unoccupied nucleon orbitals having quantum numbers compatible with those of the α orbital. In the oscillator case the Moshinsky bracket formalism [13] is one way to establish such compatibility. In practice, however, especially in the case of deformed nuclei and for other forms of the mean field, the exact correspondence between an α orbital and the orbitals of its constituents may be very complicated. It is, however, not too difficult to show that only a finite number $(j+1/2)$ of nucleon pairs from a given shell (j) can exist at the same time. For deformed potentials, the orbitals are usually at most doubly degenerate. Thus, if N is so large that a given degenerate pair of neutron orbitals is occupied by neutrons in the core, then an α orbital is blocked and thus not available as a possible state for the α particle.

II. BRIEF ACCOUNT OF THE FORMALISM

We define the α-plus-core model by the Hamiltonian (where the H's are intrinsic energies)

$$H_A = H_C + H_\alpha + T_\alpha + U(r) + \kappa \sum_{m=-2}^{2} (-1)^m Q^2_m(\mathbf{r}) Q^2_{-m}(\Omega). \quad (1)$$

The Q's are quadrupole operators. For the α-particle orbital we write

$$Q^2_m(\mathbf{r}) = f(r)\sqrt{\frac{4\pi}{5}}Y_{2m}(\hat{r}),$$

where $f(r)$ is a form factor and κ is a strength parameter to be determined in principle from the particle-particle interaction underlying Eq. (1). In order to obtain a rotationally invariant theory it is necessary that the form factor $f(r)$ be entirely independent of angles.

Provided the core can be treated as a rigid axial-symmetric rotor, its quadrupole moment can be written

$$Q^2_m(\Omega) = Q_0 \mathcal{D}^{2*}_{m0}(\Omega) ,$$

where $\mathcal{D}^I_{MK}(\Omega)$ is a rotation matrix as defined, e.g., by Brink and Satchler [14], and $\Omega \equiv (\alpha, \beta, \gamma)$ denotes the Euler angles defining the spatial orientation of the rotor. The quantity Q_0 is the intrinsic quadrupole moment of the rotor.

From the first-order term of a Taylor expansion in β_2 of an axial-symmetric, quadrupole-deformed Woods-Saxon potential, assuming that its half-value radius may be written

$$R(\theta) = R_{\rm WS}\{1 + \beta_2 Y_{20}(\cos\theta)\},$$

we find that the form factor $f(r)$ is given by the derivative of the Woods-Saxon shape function. The coupling constant κ is given by

$$\kappa = V_{\rm WS} R_{\rm WS}/\{2a_{\rm WS}\langle r^2\rangle_{\rm core}\}.$$

The angle between the symmetry axis of the core and the position vector $\mathbf{r}$ of the α particle relative to the center of mass of the core is θ, while $a_{\rm WS}$ and $V_{\rm WS}$ are the diffuseness and the strength, respectively. We express β in terms of the Nilsson [12] deformation parameter $\delta = (3\beta_2/2)\sqrt{5/4\pi}$ and relate this parameter to the intrinsic quadrupole moment Q_0 and mean square radius $\langle r^2\rangle_{\rm core}$ of the core by

$$\delta = 3Q_0/[4\zeta\langle r^2\rangle_{\rm core} + 2Q_0].$$

Here ζ is the charge parameter for the core ($=Z_{\rm core}$ for charge quadrupole moment, $=A_{\rm core}$ for nuclear quadrupole moment, $= 1$ for geometrical quadrupole moment).

We now define a *model* Hamiltonian H_0 by

$$H_0 = H_C + H_\alpha + T_\alpha + \theta(r_b - r)U(r) + \theta(r - r_b)U_b + \kappa\mathbf{Q}\cdot\mathbf{Q}, \quad (2)$$

where the last term, the quadrupole-quadrupole interaction, is defined more explicitly in Eq. (1). The potential term here is constant $= U_b$ from $r = r_b$, the barrier top radius, to infinity. Therefore H_0 may have bound states ϕ_E with energies in the interval $0 < E \le U_b$. These states may be used as "initial states" in a process leading to eigenstates Ψ_E of the true Hamiltonian (1), in accordance with the Jackson-Rhoades-Brown formalism [4,11]. Since the states Ψ_E are unbound and asymptotically free outgoing Coulomb waves, they describe the decay of the initial state ϕ_E. Because of this definition, the interaction $V = H_A - H_0$ that causes the decay of the initial state will be *independent* of the quadrupole field from the core and thus independent of angles: *there is no anisotropic barrier penetration.*

The solutions to $H_0\phi^I_M = E\phi^I_M$ with well-defined I and M are obtained by diagonalizing H_0 in the weak-coupling basis

$$\phi^{(\ell J_C)I}_{(nK_C)M}(\mathbf{r},\Omega) = [\chi_{n\ell}(\mathbf{r})X^{J_C}_{K_C}(\Omega)]^I_M,$$

resulting in coefficients $a^{K_C I}_{n\ell J_C}$. Here $\chi_{n\ell m}(\mathbf{r})$ is the α orbital and $X^{J_C}_{K_C M_C}$ is an eigenfunction of the core Hamiltonian. The parameter K_C is the projection of the core spin $\mathbf{J}_C$ on the symmetry axis of the core.

Using the formalism of Refs. [4,11] with the model eigenstates ϕ^I_M as initial states we obtain states Ψ^I_M which asymptotically have only outgoing waves. The radial current at large distances can easily be deduced and may be written

$$j_{r,J_C}(\mathbf{r}) = \frac{\mu}{\pi\hbar^3 k_{K_C J_C}} \sum_\lambda B_\lambda P_\lambda(\cos\theta_r) \sum_{n\ell n'\ell'} (-1)^{I+J_C+\ell+\ell'} a^{K_C I}_{n\ell J_C} a^{K_C I*}_{n'\ell' J_C} T_{n\ell} T^*_{n'\ell'} \times\sqrt{(2\ell+1)(2\ell'+1)(2\lambda+1)(2I+1)} \begin{pmatrix} \ell & \ell' & \lambda \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} I & I & \lambda \\ \ell' & \ell & J_C \end{Bmatrix} . \quad (3)$$

Here the coefficients

$$B_\lambda(I) = \sum_M w_I(M)\sqrt{2\lambda+1}\langle I, M, \lambda, 0|I, M\rangle$$

are the orientation parameters [15]. In axially symmetric cases these parameters characterize the initial spin distribution $w_I(M)$, where M is the projection on the z axis of the spin I. The quantity

$$T_{n\ell} = \exp[i(\sigma_\ell - \ell\pi/2)] \int F_\ell V u_{n\ell} dr \quad (4)$$

is the α transition amplitude ("transmission coefficient") in the $(n\ell)$ channel. Here $u_{n\ell}(r)$ is the radial wave function for the α particle.

We should note that the term with $\lambda = 0$ equals the total decay rate $\Gamma/(4\pi\hbar)$ per steradian and thus

$$j_{r,J_C}(\mathbf{r}) = (\Gamma/4\pi\hbar)\sum_{\lambda} B_\lambda A_\lambda P_\lambda(\cos\theta_r).$$

Here the (normalized) directional distribution coefficient A_λ (cf. Ref. [15]) is the coefficient of $B_\lambda P_\lambda(\cos\theta_R)$ in Eq. (3) divided by the total decay rate per steradian. Note also that $B_0(I) = 1$ and $A_0 = 1$. We may finally add that the influence of temperature T, substitutional fraction f, and angular resolution can be taken into account in the usual way [15,16].

III. RESULTS

A. General calculational procedures and inputs

The radial α-orbital wave functions are generated in a spherical potential well of Woods-Saxon shape with a depth V_0 determined by means of an iterative procedure in such a way that the selected solution to the secular problem has the relative α-core kinetic energy that follows from the Q value of the decay studied. This is a condition that must be satisfied if the wave function shall describe the decay in a reasonably realistic way. The Q values are calculated from the masses of the nuclides involved using the 1983 Atomic Mass Table [17]. The Woods-Saxon parameters are taken to be $r_0 = 1.05$ fm, $a_{\mathrm{WS}} = 0.65$ fm, writing the half-value radius $R_{\mathrm{WS}} = r_0(A_{\mathrm{core}}^{1/3} + 4^{1/3})$. The radial quantum number n_α and the orbital angular momentum $\hbar L_\alpha$ of the α orbitals are usually taken to satisfy the so-called "oscillator rule" [4]

$$2n_\alpha + L_\alpha = 2(2n_p + l_p) + 2(2n_n + l_n)\,, \tag{5}$$

where (n_p, l_p) and (n_n, l_n) are the corresponding parameters for the valence nucleon shells in the parent nucleus. This will to some extent take the Pauli principle into account with regard to the nucleon constituents of the α particle.

When possible, the deformation parameter δ has been calculated from measured ("spectroscopic") quadrupole moments Q_s of the *daughter* nuclides as reported in [18,19]. Following [18] we use the relation

$$Q_s = I(2I-1)Q_0/[(I+1)(2I+3)]$$

between the spectroscopic and the intrinsic quadrupole moments. (This specializes Eq. (17) of Ref. [12] to the ground bandhead state which usually also is the ground state.) In some cases the δ values are calculated from the quadrupole moment of the *parent*, not the daughter, when no value pertinent to the daughter nuclide is reported in Refs. [18,19]. Thus since a value *is* reported for ^{205}Po, we calculated the results for ^{209}Rn using both alternatives, in order to see the difference. For other nuclides structure-theoretical analyses [20–22] can be used to find realistic values for the deformation parameters.

Some of the nuclei studied here, principally those with $A > 220$, are considered to be octupole deformed. We can easily extend the Hamiltonian (1) so as to include octupole-octupole core-alpha coupling by simply adding a term

$$\kappa_3 \sum_{m=-3}^{3} (-1)^m Q_m^3(\mathbf{r}) Q_{-m}^3(\Omega), \tag{6}$$

where κ_3 is the appropriate coupling constant for this multipole, which we expect to be

$$\kappa_3 = \frac{7V_{\mathrm{WS}}R_{\mathrm{WS}}}{6a_{\mathrm{WS}}R_{\mathrm{Coul}}^3}.$$

It should be noticed that since the octupole operator $Q_{-m}^3(\Omega)$ has nonzero matrix elements only between core states of opposite parity, the term (6) added to the Hamiltonian does not produce any change in the *static* alpha-core potential. The interaction (6) is a nondiagonal one which induces a parity change in the core state as well as in the alpha orbital.

The theory used here does not include any formation probabilities for the α particles (these probabilities are always put = 1) so the predicted transition rate must be greater (the predicted half-life must be shorter) than the experimental one in order to be acceptable. If one defines the *overall hindrance factor* as OHF = $T_{1/2}(\mathrm{expt})/T_{1/2}(\mathrm{pred})$, then OHF should always exceed 1. The reader should notice that whereas the conventional hindrance factor (HF) is the ratio of the decay rate of a single-alpha state with appropriate angular momentum in a spherical potential to the observed partial decay rate populating a given daughter state, the overall hindrance factor OHF defined here compares the alpha-plus-core approximation for the total decay rate with the observed total decay rate of the true initial state. The HF for a given daughter state is very roughly (because the theoretical models involved are different) the product of OHF and the relative intensity for that state. It seems reasonable to assume that OHF is inversely proportional to the alpha formation probability. A more specific discussion of half-lives and predicted spectroscopic quadrupole moments is given in Sec. III H.

B. Results for the lighter Rn isotopes

For the lighter radon isotopes (A = 205, 207, 209) we put the α particle in an orbital with $N_\alpha \equiv 2n_\alpha + L_\alpha = 20$, since the spins and parities of the odd-mass nuclear ground states in this mass region are consistent with $N_p = N_n = 5$. The daughter spectra as seen in α decay are usually simple and rather similar, with a $5/2^-$ state (which is the ground state for A = 203 and 205, the first excited state for A = 201) taking nearly all of the decay strength in all three cases [23]. The operator H_0, Eq. (2), then leads to a matrix diagonalization with six to eight eigenvalues, depending on the number of core levels included and the assumptions made concerning the projection of the spins on the core symmetry axis. Since the α-decay relative intensities are measured, we can use the agreement between the relative intensities predicted by each solution and the experimental ones as a quality criterion for the solutions. The other proper-

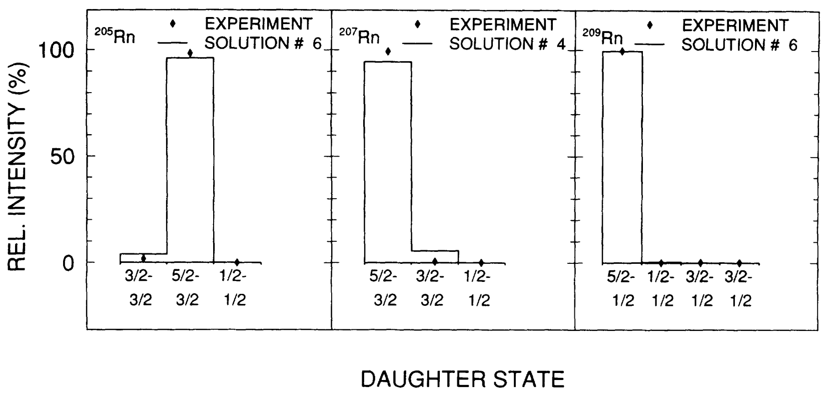

FIG. 1. Experimental and calculated relative intensities (in %) for α decay of ^{205,207,209}Rn; best fits using $\delta = 0.003$, $\delta = 0.011$, and $\delta = 0.016$, respectively. The uppermost row of fractions below the histogram gives the spin and parity of the daughter states populated. The lowest row of fractions shows the corresponding K_C values assumed for the daughter state.

ties such as half-life, quadrupole moment, and anisotropy $W(0)/W(\pi/2)$ calculated from the "best" solution according to this criterion are then taken as the theoretical predictions provided by the model used. The degree of agreement between calculated and experimental relative intensities can be quantified in terms of the (root) mean square deviation of theory from experiment. This usually leads to a unique choice of theoretical solution.

A comparison between experimental relative intensities and the best-fitting solutions is shown in Fig. 1. The daughter states are labeled by their spins and parities which are experimentally determined quantities (the upper row below the graph) and by the values of the assumed spin projection K_C on the symmetry axis of the daughter nucleus (lower row). In well-deformed nuclei the latter quantities are identical to the spins of the bandhead states. These Rn isotopes are all nearly spherical and the K_C values are in practice not well defined. Technically they must be specified, being inputs for the theoretical prediction; they can be chosen in various ways but cannot be larger than the corresponding spins. Furthermore, since in Eq. (2) the coupling term $\kappa \mathbf{Q} \cdot \mathbf{Q}$ is diagonal in K_C for axially symmetric nuclear shapes, each state of the parent nucleus will be built from core states with the same K_C value and thus the decay will only branch to daughter states with this K_C value. We have tried various possible sets of K_C values; the figure shows the choice that minimizes the rms deviation of the calculated relative intensities from the experimental ones. Note that although the $5/2^-$ and $3/2^-$ states have been assigned the same K_C values, the decay to the $5/2^-$ state is strongly favored in these solutions.

The corresponding anisotropies $W(0)/W(\pi/2)$ are shown in Fig. 2. A remarkable feature of the angular α distribution according to this calculation is that its principal component in ^{205}Rn is nearly isotropic (anisotropy ≈ 1) whereas in ^{207}Rn and in ^{209}Rn it is rather strongly peaked in the spin direction.

As already mentioned, it is only for ^{209}Rn that we have experimental information [18,19] on the spectroscopic quadrupole moment of both parent and daughter. We have therefore here an opportunity to compare the results of choosing the deformation of the daughter (which is what our model actually needs) instead of that of the parent as input parameter in the calculation. As the deformation parameters deduced from these data in this case are very small both for the parent ($\delta = 0.016$) and the daughter ($\delta = 0.009$) the comparison is, however, not a particularly severe test. This is seen in Figs. 3 and 4. In the lowest row below the histogram the assumed values of K_C are shown; from this point of view the comparison should be done between the left-hand and the right-hand diagrams. However, at so low a value of δ the direction of the symmetry axis, and thus the projection K_C of the core spin, would be quite uncertain. The middle and right-hand diagrams are performed for the same solution number and may give an impression of how the K_C value influences the result. The decay to the $5/2^-$ state is favored since the ground state of ^{209}Rn is a $5/2^-$ state. Only if a daughter state has the same K_C as the

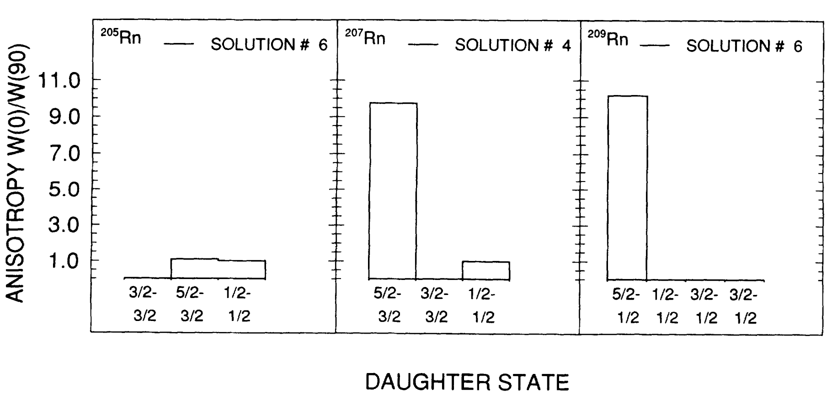

FIG. 2. Calculated anisotropies $W(0)/W(\pi/2)$ for α decay of ^{205,207,209}Rn for the solutions shown in Fig. 1.

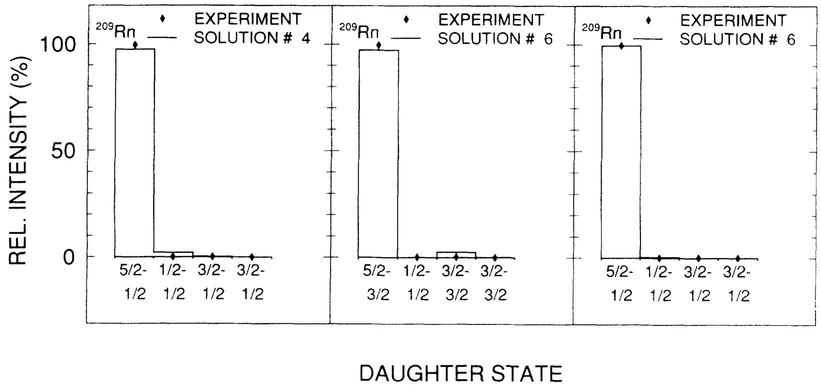

FIG. 3. Comparison of relative intensities for α decay of ^{209}Rn; best fits for slightly different deformation parameters. The value $\delta = 0.009$ is used for the cases shown in the left-hand and middle panels. It is calculated from the spectroscopic quadrupole of the parent (0.311 b). The value $\delta = 0.016$ used for the case shown in the right-hand panel is calculated from the quadrupole moment of the daughter ^{205}Bi (0.17 b).

favored state will there be any branching to it.

The corresponding comparison of the predicted anisotropies shown in Fig. 4 confirms the conclusion reached in earlier work [6,7] that the anisotropy is not primarily a function of the deformation but is strongly dependent on the structure of the state. The difference between the anisotropies in the right-hand (10.19 from computer output) and left-hand (0.0234 from computer output) diagrams is *not* due to the difference in the values of the deformation parameter δ (interchanging the δ values yields 10.36 and 0.0107, respectively) but to the fact that the solution number is different: the two states have a different place in the sequence of solutions to the Schrödinger equation. Thus the difference in anisotropy reflects a *structural* difference between the states which is not caused by the difference in deformation.

C. Results for ^{219}Rn

We have also calculated relative intensities and anisotropies for ^{219}Rn. The spectrum of the daughter ^{215}Po is considerably more complex than those of the lighter Po isotopes. In particular, the α-decay relative intensities for populating the ^{215}Po states have been measured and found to be 81%, 11%, and 7.5%, respectively, for populating the 9/2, 7/2, and 5/2 positive parity states [24,25]. No value is reported for the quadrupole moment; we have assumed a small prolate deformation ($\delta = 0.055$). At so small a deformation it is difficult (perhaps even meaningless) to assign an effective value of K_C, the bandhead spin of a rotor state, since the spectrum does not show any band structure. (We may note in passing that if we put $\delta = 0$ then there will be no branching of the decay to different final states: in each of the nine solutions one of the states mentioned above is populated 100%.) In order to be able to apply the axially symmetric particle-rotor model we have provisionally assumed that the strongly populated ^{215}Po states all have the same value of K_C so that they can couple to each other via the quadrupole-quadrupole interaction with the α particle. This makes it possible to calculate the theoretical α decay relative intensities from the dynamics of the particle-rotor model. That solution which for a given choice of deformation and bandhead spin K_C most closely reproduces the observed relative intensities is then, provisionally, taken as the best approximation to the ^{219}Rn ground state. In practice this means that for each assumed K_C value we choose that solution for which the rms deviation of the theoretical relative intensities from the experimental ones is the smallest (Fig. 5). Using this solution we then predict the anisotropies (see Fig. 6) for each branch of the decay. Comparing the half-life thus predicted with the observed one we get for each acceptable solution the overall hindrance factor OHF $= T_{1/2}(\text{expt})/T_{1/2}(\text{pred})$ reported in Table I.

A look at Fig. 5 gives the impression that $K_C = 3/2$ describes the low-lying ^{215}At spectrum better than $K_C = 5/2$ does. A measurement of the anisotropy of the ground-state–to–ground-state decay might give a clearer

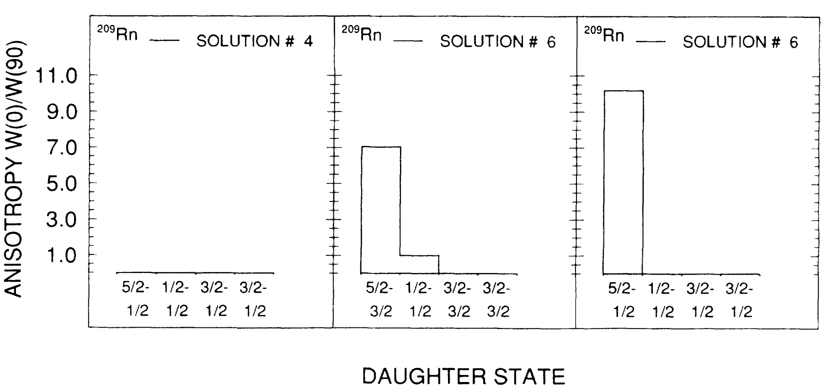

FIG. 4. Comparison of anisotropies for α decay of ^{209}Rn corresponding to Fig. 3.

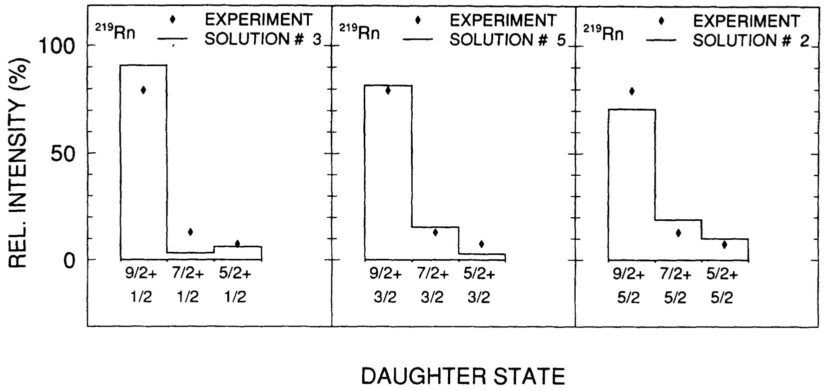

FIG. 5. Experimental values of the relative intensities (in %) for α decay of ^{219}Rn, with calculated values based on different assumptions concerning the spin projection K_C on the symmetry axis. The deformation parameter $\delta = 0.055$. For further explanations see caption of Fig. 1.

indication which of the K_C values is the most realistic one. We should mention that for $K_C = 5/2$, solution 5 reproduces the experimental branching ratios almost as well as solution 2 shown here. It predicts very large anisotropy (> 5) in the decays to the $9/2^+$ as well as to the $5/2^+$ state.

D. Results for ^{221}Fr

The same formalism has also been applied to ^{221}Fr. The ground-state–to–ground-state transition goes from $I^\pi = 5/2^-$ to $I^\pi = 9/2^-$ [26]. We have used α wave functions generated in a Woods-Saxon well with $R_{\rm WS} = 7.976$ fm, $a_{\rm WS} = 0.7$ fm. The experimental relative intensities [24,26] are 83.4% to the $9/2^-$ ground state, 1.34% to the $7/2^-$ state at 99.5 keV, and 15.1% to the $5/2^-$ state at 218.19 keV in the decay ^{221}Fr (α) ^{217}At.

It has been suggested [27] that the nuclide ^{221}Fr is octupole deformed [20] as shown by the existence of parity doublet states or bands in this nucleus. This might have consequences for the α decay to the various states of ^{217}At. The present formulation of the decay formalism can, however, only consider octupole deformation if it refers to the *daughter* nuclide *and* is accompanied there by low-lying opposite-parity states having the same spin projection K_C on the intrinsic symmetry axis. Data compilations available at present [24,26] show no states with even-parity assignment in ^{217}At. In order to test the model, calculations were performed assuming quite arbitrarily that a state at 577 keV can be given the assignment $1/2^+$, $3/2^+$, or $5/2^+$. (The exact value of the excitation energy of the hypothetical even parity state is not important for these calculations, which must not be quoted as a support for such an assignment.) An octupole moment due to a β_3 deformation of about 0.08 [27] would couple such a state with odd-parity alpha orbitals so as to form a component of the $5/2^-$ ground state of ^{221}Fr.

As one might expect, with a value of 577 keV for the assumed excitation energy of the positive parity state we only obtain barely discernible effects of its presence or absence on the relative intensities, anisotropies, and transition rates. In order to provide odd-parity α orbitals we must prescribe both odd and even values of the principal quantum number $N \equiv N_\alpha$. The octupole moment operator can have nonzero matrix elements only between α orbitals having opposite parity to each other. It is not always obvious which value to choose for the odd N; for the even parity orbitals the oscillator rule (5) seems to give a reasonable N value (viz., $N = 22$). It may be instructive to present two sets of optimal solutions (in the sense that each solution minimizes the mean square difference between experimental and theoretical relative intensities) for $K_C = 1/2$, 3/2, and 5/2, the maximum possible values compatible with $J_C = 1/2^+, 3/2^+, 5/2^+$, respectively. The first set is calculated with the odd-parity alpha orbitals in the $N = 21$ shell. The relative intensities to the four states considered are shown in Fig. 7, the corresponding anisotropies $W(0)/W(\pi/2)$ in Fig

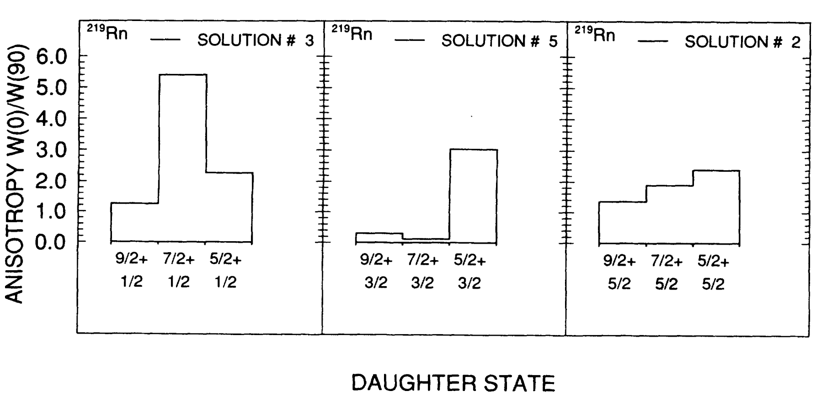

FIG. 6. Calculated anisotropies $W(0)/W(\pi/2)$ for α decay of ^{219}Rn corresponding to the results shown in Fig. 5.

TABLE I. Predicted overall hindrance factors (OHF), formation probabilities, half-lives, and spectroscopic quadrupole moments. Experimental quadrupole moments are quoted from Ref. [19].

Solution No.	K_C	Overall hindrance factor	Formation probability	T_{half}(th) s	Q_{spect} b
		^{205}Rn. Expt. half-life: 170 s. $Q_s = +0.62$ b			
6	3/2,1/2	121.7	8.218×10^{-3}	1.397	−0.010
		^{207}Rn. Expt. half-life: 9.3 m. $Q_s = +0.220$ b			
4	3/2,1/2	71.99	1.389×10^{-2}	7.751	−0.022
		^{209}Rn. Expt. half-life: 28.5 m. $Q_s = 0.311$ b. $Q_s(^{205}\mathrm{Po}) = 0.17$ b			
6	3/2,1/2	110.8	9.023×10^{-3}	15.43	0.012
4	1/2	158.9	6.292×10^{-3}	10.76	−0.142
6	1/2	88.88	1.125×10^{-2}	19.24	−0.019
		^{219}Rn: Expt. half-life: 3.96 s. $Q_s = +0.93$ b, +1.15 b			
3	1/2	156.9	0.006373	2.5235×10^{-2}	−0.363
5	3/2	6.458	0.1548	0.6132	−0.091
2	5/2	150.4	6.649×10^{-3}	2.6336×10^{-2}	−0.077
		^{221}Fr: Expt. half-life: 288 s. $Q_s = -0.96$ b			
$N = 21$ and $N = 22$ shells					
5	1/2	74.8	1.34×10^{-2}	3.850	0.251
9	3/2	4.62	0.216	62.28	0.248
2	5/2	4037	2.48×10^{-4}	7.133×10^{-2}	-0.125
$N = 22$ and $N = 23$ shells					
9	1/2	5.40	0.185	53.29	0.502
8	3/2	4.74	0.211	60.75	0.106
7	5/2	16.3	6.14×10^{-2}	17.68	0.251
$N = 22$ and $N = 23$ shells at approx. equal energies					
5	1/2	74.8	1.33×10^{-2}	3.850	0.251
10	3/2	3.92	0.255	73.44	0.110
8	5/2	7.21	0.139	39.92	0.216
		^{227}Pa: Expt. half-life: 2298 s			
Using ^{223}Ac spectrum of Refs. [30,31]					
27	3/2,5/2	1.102	0.908	2086	−0.142
3	3/2,5/2	77840	1.285×10^{-5}	2.952×10^{-2}	−0.245
Using ^{223}Ac spectrum of Ref. [25]					
4	3/2,5/2	587	1.705×10^{-3}	3.918	0.220
		^{229}Pa: Expt. half-life: 1.4 d = 121 000 s			
6	3/2,5/2	5.01	0.20	24139	0.116
4	3/2,5/2	194	5.15×10^{-3}	622.5	1.322
10	3/2,5/2	(0.423)	(2.37)	286270	(0.108)
		^{229}U: Expt. half-life: 3480 s			
1	3/2	$1.26\times10^{+7}$	7.92×10^{-8}	2.7563×10^{-4}	1.187
1	3/2	$1.75\times10^{+8}$	5.71×10^{-9}	1.9864×10^{-5}	1.127
4	3/2	1277	7.83×10^{-4}	2.725	0.218

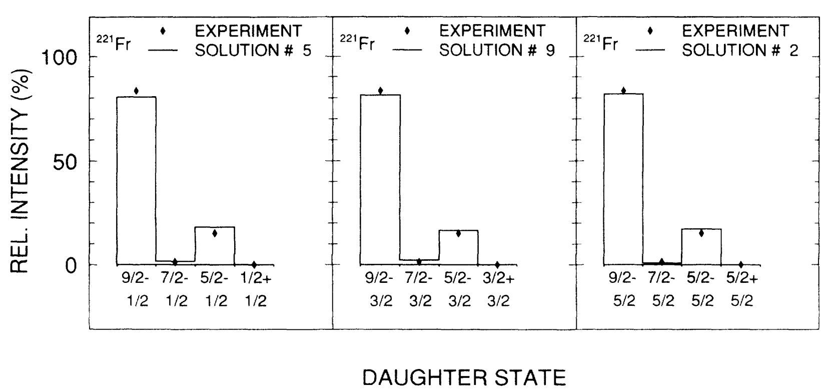

FIG. 7. Experimental and calculated relative intensities (in %) for α decay of ^{221}Fr, best fits for three choices of K_C with the α orbitals of odd-parity chosen from the shell $N = 21$, those of even parity from $N = 22$. Deformation parameter values are $\beta_2 = -0.052$ and $\beta_3 = 0.08$. For further explanations see caption of Fig. 1.

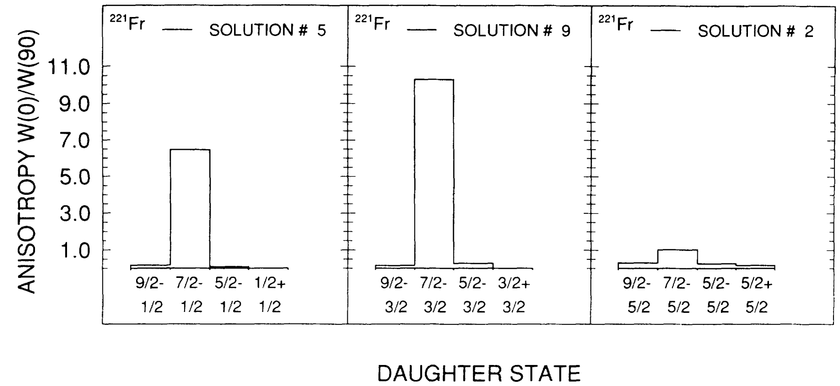

FIG. 8. Anisotropies $W(0)/W(\pi/2)$ for α decay of ^{221}Fr, for the same cases as in Fig. 7.

8. The second set, with results shown in Figs. 9 and 10, is calculated with the odd-parity alpha orbitals in the $N = 23$ shell. A clear difference between the two choices of odd-N shell is seen in the much stronger population of the opposite-parity 577 keV state for $N = 23$.

It is remarkable that a considerably better fit to the relative intensity distribution is obtained if the alpha orbitals are taken from the $N = 21$ and $N = 22$ shells instead of the $N = 22$, $N = 23$ alternative. This is true for all the three values of $J_C^{(\mathrm{even})} = K_C$ used here. A simplified calculation including only the $N = 22$ shell yields relative intensity distributions which are in fair agreement with experiment but not quite as good as those of Fig. 7.

The suppression of the decay rate to the first excited $7/2^-$ state is well reproduced in all these solutions although the same K_C value is assigned to all of the three odd-parity states. It is true that with our criterion for choosing solutions we only accept solutions with this property; among those rejected there are several solutions in which the $7/2^-$ state is strongly populated. Therefore one should not attach too much importance to this fact except that it demonstrates that we need not explicitly impose this suppression on calculations performed using this model.

As explained above, in this application of the decay model we can only expect small effects of the octupole degree of freedom. The results we have obtained must therefore be taken as a rather clear indication that the octupole-octupole interaction is important for the description of the ^{221}Fr ground state. As further explained in Sec. III E, we have found that in order to reproduce the alpha decays in an octupole-deformed nucleus, we should modify the search procedure for the depth of the central alpha-core potential so that it will in lowest order reproduce the energy for the decay to the least-excited daughter state with parity opposite to that of the daughter ground state. Thus the even-L and odd-L alpha orbitals are generated in potential wells of different depth such that the asymptotic radial behavior of both types of orbital is approximately the same. If we calculate the decay using this procedure with $N = 22$ and $N = 23$, we obtain as "best fit" solutions those presented in Figs. 11 and 12. We see that in this solution the decay to the opposite-parity state is strongly suppressed. Despite this suppression of the *decay*, the opposite-parity state can have a large amplitude in the parent nucleus.

E. Results for ^{227}Pa

The spectrum of the daughter ^{223}Ac presented in Tables of Isotopes [24] was based on work by Subrahmanyam [28]. Experimentally, the ground state is populated in about 50% of the α-decay events. In Ref. [24] the spin and parity assignment for the ground state is $(5/2^-)$, and one tentatively identifies the ground state with the 5/2[523] Nilsson state. The $7/2^-$, $9/2^-$, and $11/2^-$ members of the ground band are then found at 42 keV, 91 keV, and 141 keV populated with relative inten-

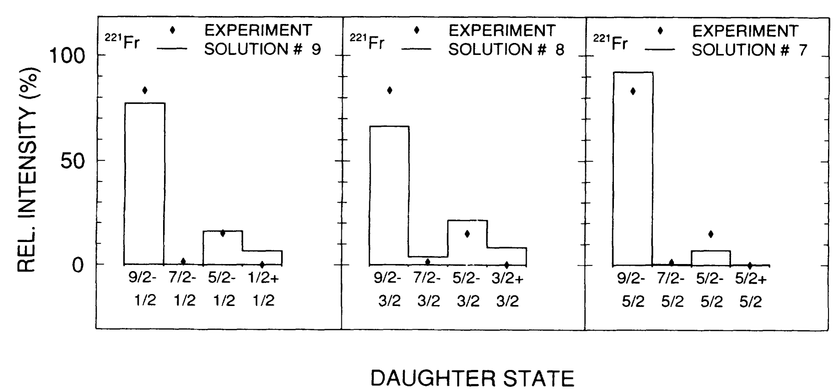

FIG. 9. Experimental and calculated relative intensities (in %) for α decay of ^{221}Fr, best fits for three choices of K_C with the α orbitals of odd-parity chosen from the shell $N = 23$, those of even parity from $N = 22$. Deformation parameters as in Fig. 7.

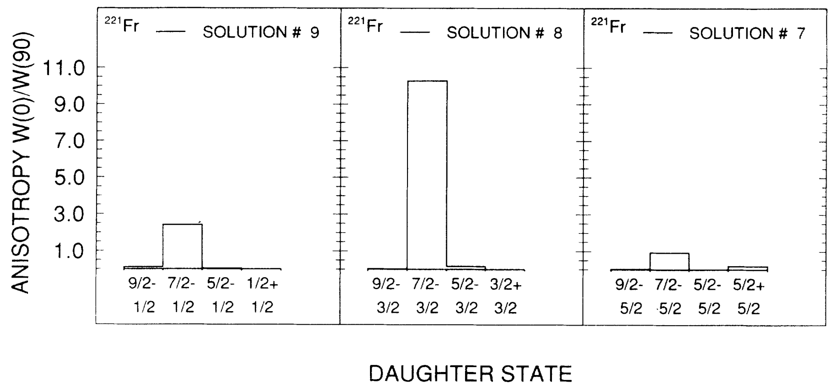

FIG. 10. Anisotropies $W(0)/W(\pi/2)$ for α decay of ^{221}Fr, for the same cases as in Fig. 9.

sities 11.8%, 2.6%, and 0.4%, respectively. A state of unknown spin and parity at 50 keV excitation is populated in 15.2% of the decays, while 9.6% go to a state at 65 keV assigned $(5/2^+)$. It would be tempting to make the assignment $3/2^+$ to the state at 50 keV, with 3/2[651] as a Nilsson candidate for this state, consistent with a deformation $\varepsilon \approx 0.12$. (See the appropriate Nilsson diagram in Ref. [24].) However, as we have already emphasized, in the axially symmetric deformation model used here K_C is a good quantum number, and the α-core interaction can only couple states (possibly of opposite parity if an octupole-octupole interaction is included) with the *same* projection of the core spin on the symmetry axis. As a consequence, in this model the parent state (^{227}Pa g.s.) cannot be a mixture of, say, odd-parity α orbitals times ($K_C = 3/2$, even parity) core states and even-parity α orbitals times ($K_C = 5/2$, odd parity) core states.

An alternative ^{223}Ac spectrum was suggested by Ahmad *et al.* [29] on the basis of experiments performed at the LISOL isotope separator, Louvain-la-Neuve. These authors invoke the octupole model of Leander and Sheline [27,20] implying the presence of parity doublets in the low energy spectrum of ^{223}Ac. In their scheme the state at 50.5 keV is interpreted as a $5/2^+$ member of a $K_C = 3/2$ band (their parity assignments are generally opposite to those of [24], so the parent and daughter ground states are assumed to be $5/2^+$ states). They are also able to resolve the 110 keV state into a 107.0 keV $7/2^+$ state and a 110.4 keV $7/2^-$ state.

Two recent papers reporting experiments performed at the ISOCELE Orsay mass separator by Sheline and co-workers [30,31] reverse the parity assignments of Ahmad *et al.* [29]. They present a spectrum of ^{223}Ac which partly differs from that of Ref. [29]. In particular they report a $3/2^-$ state at 4.1 keV excitation. Since this state can have at most $K_C = 3/2$, the existence of parity doublet bands in ^{223}Ac with at least two different K_C values would be rather well confirmed. The recent evaluation by Browne [25] endorses the parity assignments of Sheline *et al.* for the well-established daughter states as well as the parent ground state.

It is therefore inevitable that the decay model used so far must be extended beyond the axial-symmetry approximation in order to describe the decay of ^{227}Pa. This extension may involve the inclusion of triaxial degrees of freedom in the quadrupole-quadrupole interaction and/or the octupole-octupole interaction. As the multipole-multipole interaction terms do not affect the *intrinsic* structure or the symmetries of the *core* states, only the core-plus-alpha system, it would involve only moderate changes in the decay model if we try to tackle the problem by introducing an additional *octupole* component, viz. a term of the form

$$\kappa_3 f(r) \sum_{\mu=-3}^{3} \{Q^3_{1\mu}(\mathbf{\Omega})Y_{3,\mu}(\hat{r}_\alpha) + Q^3_{-1\mu}(\mathbf{\Omega})Y_{3,\mu}(\hat{r}_\alpha)\}\,,$$

which is rotationally invariant but couples even-parity core states having a given K_C value to odd-parity core states with $K'_C = K_C \pm 1$, provided also the alpha or-

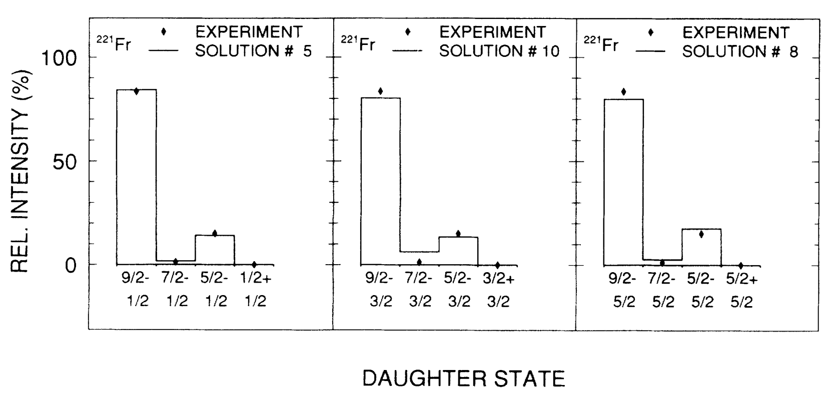

FIG. 11. Experimental and calculated relative intensities (in %) for α decay of ^{221}Fr, best fits for three choices of K_C with the α orbitals of odd-parity chosen from the shell $N = 23$, those of even parity from $N = 22$. Here the potential depths for odd-L and even-L orbitals are different as explained in the text. The "binding energies" of the orbitals are, however, more equal than in the previous case.

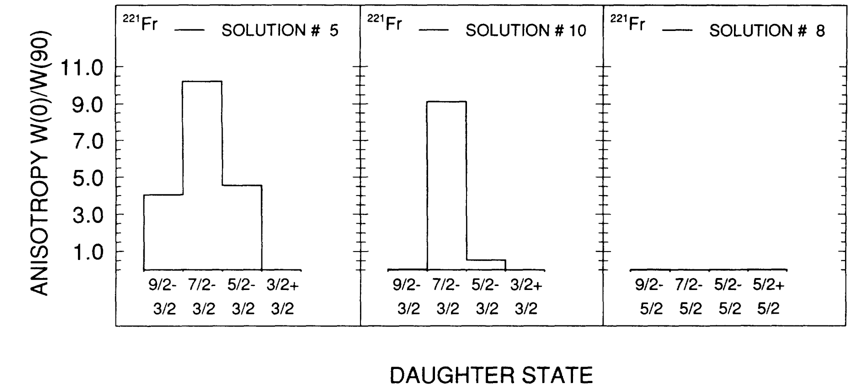

FIG. 12. Anisotropies $W(0)/W(\pi/2)$ for α decay of ^{221}Fr, for the same cases as in Fig. 11.

bitals change parity and intrinsic angular momentum projection. Generalizing from the quadrupole case, we parametrize the general multipole moment tensors in terms of deformation coordinates $\beta_{\lambda,\mu}$ as

$$Q_{\lambda,\mu} = \frac{3ZR_0^\lambda \beta_{\lambda,\mu}}{\sqrt{\pi(2\lambda+1)}}.$$

Here R_0 is the radius of the "equivalent uniform charge distribution," the uniform distribution which has the same rms radius as the actual nucleus.

However, when the model thus modified was applied to the possibly octupole-deformed nuclei it was found to be virtually impossible to use an alpha-core potential independent of the core state to reproduce the observed relative intensities for populating states with parity opposite to that of the ground state of the daughter nucleus. A closer examination of the intermediate calculational results showed that the reason for this was that the transmission amplitudes (4) involving orbitals from the odd-N_α shell below the even-N_α "valence" shell were an order of magnitude smaller than those of the ground-state–to–ground-state decays. These odd-N_α orbitals decrease much faster for large distances because they are more strongly bound (by roughly $1\hbar\omega \sim 7$ MeV) than the valence orbitals whenever they are generated in the same potential. But since the core producing the potential *is* in different states for odd N_α and even N_α it is only natural that the potential at least depends on the parity of the core. An obvious and successful remedy for this shortcoming of the model was to make a separate determination of the central potential depth V_0 for the odd-parity alpha orbitals. This depth is thus chosen so that the orbital that has the lowest odd L_α value (satisfying $|I - J_C^{\rm op}| \leq L_\alpha \leq I + J_C^{\rm op}$) gets an energy eigenvalue equal to the kinetic energy of the alphas emitted in a decay that leaves the daughter in the lowest opposite-parity state, which has spin $J_C^{\rm op}$. This prescription is similar to the well-depth procedure employed in direct stripping and pick-up reactions. It is necessitated by the pronounced sensitivity of the decay amplitude to the asymptotic behavior of the radial wave function of the ejected particle, analogous to that observed in direct particle transfer. This sensitivity is not apparent in the WKB treatment of alpha decay or if harmonic oscillator wave functions are used to describe the alpha orbitals. (Cf. also Fig. 10 in Ref. [7] and the related discussion.) The use of different well depths depending on the parity of the core is clearly associated with the occurrence of parity doublets.

In Fig. 13 we show the "best fit" to each of three experimental relative intensity distributions, viz., the spectrum of Sheline *et al.* [30,31] assuming that the $5/2^-$ ground state is populated about 9 times as often as the first excited $3/2^-$ state; the spectrum of Sheline *et al.* [30,31] assuming that the first excited $3/2^-$ state is populated about 9 times as often as the $5/2^-$ ground state; the spectrum given by Browne [25].

The first two versions of the experimental spectrum are introduced because in Refs. [30,31] the hindrance factor

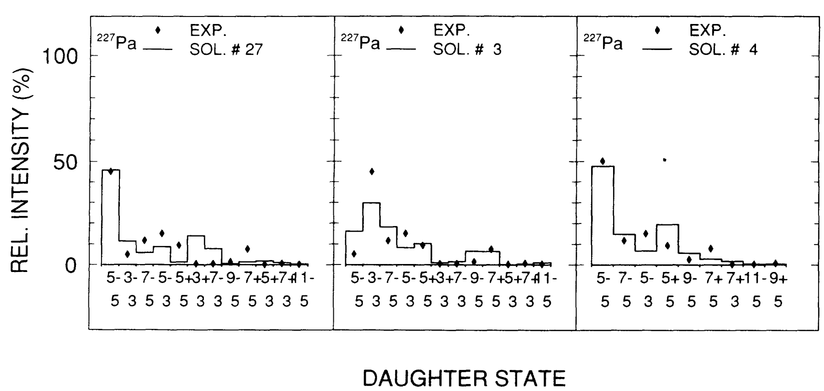

FIG. 13. Experimental and calculated relative intensities (in %) for α decay of ^{227}Pa. Deformation parameters: $\beta_2 = 0.12$, $\beta_{30} = 0.08$, $\beta_{31} = 0.05$. Best fits using data of Refs. [30,31] (left-hand and middle panel) and Ref. [25] (right-hand panel). In order to save space in the diagram we have here labeled the daughter states with $2J_C$, parity, and $2K_C$.

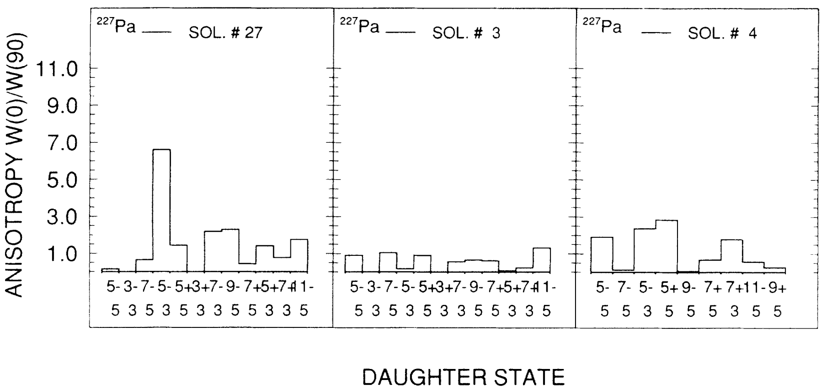

FIG. 14. Predicted anisotropies for α decay of ^{227}Pa for the same cases as in Fig. 13.

for populating the $3/2^-$ state at 4.1 keV is not reported. The possibility therefore exists that some of the decays registered as ground-state decays may actually populate the 4.1 keV state. The second version tests the assumption that most of the alleged ground-state decay might actually go to the slightly excited state. We note that in the "best fit" obtained using that assumption the upper state takes about 2/3 of the added decay strength to the two lowest states.

We have used $\beta_2 = 0.12$ which is a reasonable average of the values for ^{223}Ac used in Ref. [22]. Furthermore we have used $\beta_{30} = 0.08$ (a value used by Sheline [27] for these nuclei) and $\beta_{31} = 0.05$. We have not tried to vary these values in order to improve the fits to the alpha decay data. We have only intended to show here that it is possible to obtain transition rates with realistic strength to states of opposite parities and different K_C quantum numbers using the kind of model proposed here. It should be noted that the usual quadrupole triaxiality described by $\beta_{2\pm2}$ is not sufficient to account for the observed population of bands in ^{223}Ac in the α decay of ^{227}Pa.

The anisotropies $W(0)/W(\pi/2)$ corresponding to the model calculations itemized above and shown in Fig. 13 are presented in Fig. 14. At the time of writing no experimental results for the anisotropies are known.

F. Results for ^{229}Pa

In response to a private communication from Paul Schuurmans [10] an attempt was made to predict the anisotropy in the α decay of ^{229}Pa. The first calculation was based on the ^{225}Ac spectrum reported in Tables of Isotopes [24]. It was obvious also from these data that those states in ^{225}Ac which are strongly populated in the α decay of ^{229}Pa cannot be characterized by a single value of the core spin projection K_C. In the spectrum of ^{225}Ac there are two $5/2^+$ states which are strongly populated (the ground state of ^{229}Pa is supposed to be a $5/2^+$ state; see [32]). One of them may be a member of a $K_C = 3/2$ rotational band, but probably not both. We therefore have to make a calculation of the same type as in the ^{227}Pa case, i.e., we try to reproduce the data by introducing a triaxial component in the octupole deformation tensor. That an *octupole-octupole interaction* is necessary follows already from the fact that both odd and even parity daughter states are populated with comparable probability in the alpha decay of ^{229}Pa. Grafen *et al.* [33] have argued that this nucleus should not be considered octupole deformed since their ^{230}Th$(p,2n)$ and ^{231}Pa(p,t) reaction data cast doubt on the previously proposed [34] existence of a close $I = 5/2$ ground-state parity doublet in ^{229}Pa. However, Sheline [35] refutes this argument on the basis of the systematics of the alpha-decay hindrance factors in this mass region which clearly indicates octupole deformation. The opposite parity partner of the ^{229}Pa may yet be found at a higher excitation energy.

The population of daughter states having different K_C values (to the extent that such values can be assigned with some precision) indicates that ^{229}Pa is triaxial or at least rather soft with respect to that type of deformation. We show three relative intensity distributions in Fig. 15

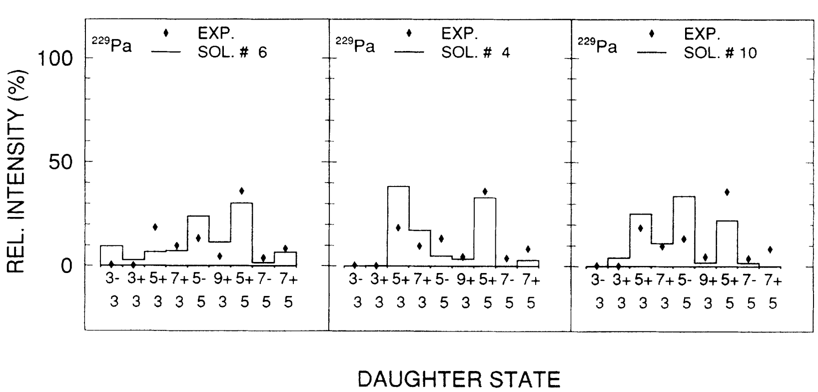

FIG. 15. Experimental and calculated relative intensities (in %) (three best fits) for α decay of ^{229}Pa. In order to save space in the diagram we have here labeled the daughter states with $2J_C$, parity, and $2K_C$.

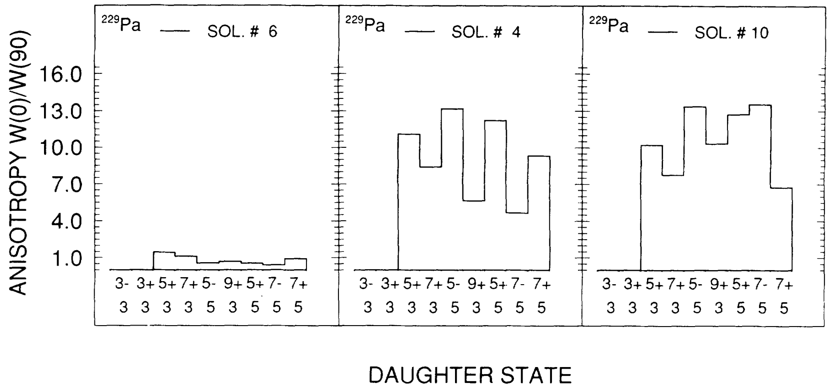

FIG. 16. Predicted anisotropies for α decay of ^{227}Pa for the same solutions as in Fig. 15.

which are in reasonable agreement with the observed one, and the corresponding anisotropies in Fig. 16. (However, as can be seen in Table I solution 10 yields a half-life that is almost twice as large as the one observed so this solution can almost certainly be discarded.) Preliminary results [10] indicate a large anisotropy in the favored decay channel $5/2^+$ to $5/2^+$. It should be pointed out that we have not spent much effort on searching for deformation parameters that might improve the fit. The results shown are sufficient to demonstrate that the alpha-decay model used is dynamically valid also in the case of octupole-octupole coupling that is not axially symmetric.

G. Results for ^{229}U

The alpha decay of ^{229}U populates nearly always the $K_C = 3/2$ ground band in ^{225}Th. (See [24,32]; compare also Fig. 17 of Ref. [21] and the relevant comments in the text.) We have included the state at 178 keV as the $11/2^+$ member of this band and, in our first run, ignored the state at 102 keV since with the K_C value 5/2 suggested in Ref. [21] it would not couple to the $K_C = 3/2$ ground band unless there is a $\beta_{2\pm1}$ term if its parity is even, or $\beta_{3\pm1}$ if its parity is odd. There is at present no definite experimental information concerning the parity of this state so we treat ^{225}Th as an axially symmetric rotor. We expect (cf. Ref. [22]) a quadrupole deformation of $\beta_2 \approx 0.14$. The relative intensities in the decay ^{229}U$(\alpha)^{225}$Th have thus been calculated for the ten solutions obtained in the simplest rotor-plus-particle model of ^{229}U. The lowest-energy solution (i.e., solution 1) turns out to be the one that reproduces the relative intensities most closely in terms of rms deviation (see the left-hand panel of Fig. 17). The predicted half-life is about 0.27 ms compared to the observed value 58 min.

For comparison we have also performed calculations which include the 102 keV state with the assignment $J_C = 3/2^-, K_C = 3/2$ (one of the possibilities suggested by Leander and Chen [21]). We can couple such a state to the $K_C = 3/2$ band assuming a core octupole moment corresponding to $\beta_{30} = 0.08$. In accordance with what we found necessary in the ^{227}Pa case, we also renormalize the central potential for the "un-normal" parity states so that the $N = 21$ or the $N = 23$ shell will have approximately the same energy as the $N = 22$ shell. In the case that the odd alpha orbitals are chosen from the $N = 23$ shell solution 1 is the "best" one (see the middle panel of Fig. 17). On the other hand, if we take the odd alpha orbitals from the $N = 21$ shell, then we find that the "best" solution is 4 (see the right-hand panel of Fig. 17). The predicted half-life for this solution is as long as 2.725 s, which still is almost 1300 times faster than the observed transition rate. (Solution 4 in the other cases yields similar half-life but less good fits to the relative intensities.) The anisotropies $W(0)/W(\pi/2)$ for these three solutions are shown in Fig. 18. The anisotropy distributions are all considerably larger than 1, i.e., the angular distribution is peaked in the direction of the nuclear spin which coincides with the nuclear symmetry axis when the nu-

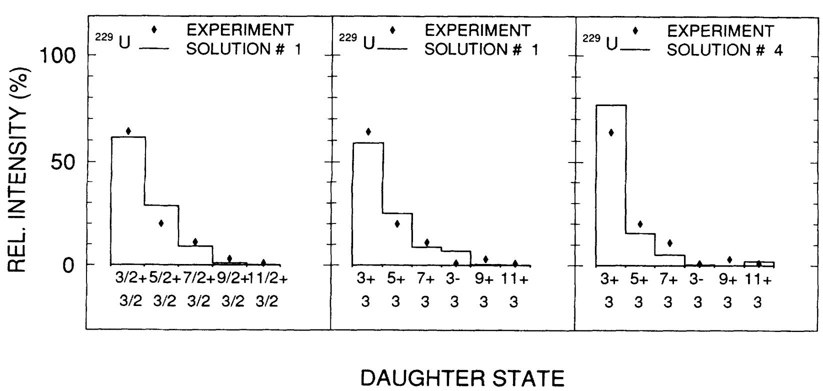

FIG. 17. Experimental and calculated relative intensities (in %) for α decay of ^{229}U. The left-hand panel shows the best fit when the 102 keV state, presumably of odd parity, is excluded. The middle panel shows the best fit obtained assuming contributions from $N = 22$ and $N = 23$, and the right-hand panel shows best fit using $N = 21$ and $N = 22$.

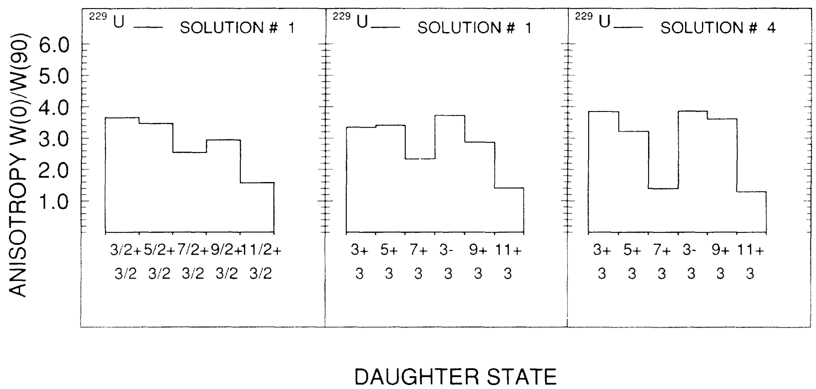

FIG. 18. Predicted anisotropies for α decay of ^{229}U, for the solutions shown in Fig. 17.

cleus is completely aligned along the magnetic field (at temperature 0 K).

The alpha decay of ^{229}U is a typical example of the favoured decay of a well-deformed prolate nucleus. Our results are in good agreement with the expectation that in the decay of a prolate odd-mass nucleus the alpha particles are ejected more or less along the symmetry axis. We emphasize, however, that these results are *not* due to anisotropic barrier penetration but follow from the properties of the rotationally invariant coupling between the α orbitals and the core, i.e., they reflect the state of the system prior to its disintegration.

H. Summary of half-life predictions

In the first applications [11] of the alpha-decay model we used harmonic oscillator functions for the α orbitals. This resulted in half-life predictions which were surprisingly close to those observed. When the more realistic Woods-Saxon shape of the potential describing the interaction between the alpha particle and the residual nucleus was introduced [7], the gap between predicted and observed half-life increased by several orders of magnitude. The reason is obvious: the longer tail of the Woods-Saxon radial wave function increases both the range and the magnitude of the integrand in the transmission amplitude (4), so the calculated transition rate increases. The resulting loss of agreement must be considered a blessing in disguise, because it leaves room for a physical improvement of the theory, viz., it becomes possible to consider the probability for the formation of the α particle from the nucleons in the parent nucleus and take the discrepancy between predicted and observed transition rate as a measure of this probability.

We shall not attempt to solve the formation problem here. In formulations such as that of Ref. [9] it is taken as the starting point. Here we limit ourselves to introducing in a phenomenological way the overall hindrance factor (OHF) $T_{1/2}(\text{expt})/T_{1/2}(\text{pred})$, which we interpret as the inverse of the formation probability. We present in Table I a summary of the predicted half-lives, quadrupole moments, and OHF's for the cases shown in the figures. We are not yet ready to present a dynamical theory for the formation probabilities, but we are investigating the problem carefully. There is little doubt that the question of how to take fermion antisymmetry into account when constructing the alpha-plus-core wave function is also involved in this problem.

IV. CONCLUDING REMARKS

The alpha-decay relative transition rates (relative intensities) and relative angular distributions $W(0)/W(\pi/2)$ (anisotropies) have been calculated for some nuclei proposed for an on-line nuclear orientation experiment. Branching ratios when known from earlier measurements have been used to select the appropriate theoretical solutions. The anisotropies of the decays have not yet been compared to any experimental anisotropy data. Half-lives are known for all parent nuclei considered here and have been calculated, but the model used in this work is not equipped to predict this property with any precision.

ACKNOWLEDGMENTS

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