## Approach for calculating multistep direct reactions of continuum and discrete levels

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A method for calculating multistep direct reactions for both continuum and discrete levels is proposed. The energy-angle correlation scattering kernel is adopted for continuum and discrete levels in the semiclassical approach, in which the angular momentum and parity conservations are considered. Then, following the Feshbach, Kerman, and Koonin quantum multistep direct reactions theory, the Legendre coefficients of the angular distributions are calculated based on onestep distorted-wave Born approximation instead of nucleon-nucleon scattering expressions in nuclear matter. Since the quantum effects are properly considered, we call it the quasiquantum multistep direct reactions theory. For the specific reaction considered here, the calculated discrete level neutron angular distributions of the  ${}^{11}{\rm B}(p,n_0){}^{11}{\rm C}$  reaction and the cross sections of the  ${}^{11}{\rm B}(p,n_0){}^{11}{\rm C}$  and the  ${}^{11}\mathrm{B}(p,n){}^{11}\mathrm{C}$  reactions reproduce the experimental data reasonably. This approach can also be used to composite particle emissions.

PACS number(s): 24.50.+g, 24.10.-i, 25.40.Hs, 21.65.+f

## I. INTRODUCTION

In recent years, the quantum-mechanical preequilibrium theory of Feshbach, Kerman, and Koonin (FKK) [1] has been applied with considerable success to describe nucleon-induced reactions [2-6]. This theory describes the reaction as passing through a series of particle-hole excitations, caused by nucleon-nucleon interactions as the nuclear system evolves towards equilibrium. Quantum-mechanical preequilibrium theory distinguishes between multistep compound (MSC) processes (i.e., all particles remain bound throughout the successive stages of the reaction) and multistep direct (MSD) reactions (i.e., at least one particle remains in the continuum). The calculated results with both mechanisms as well as subsequent Hauser-Feshbach (HF) equilibrium emission show that the MSC processes are less important than previously thought and the MSD mechanism dominates preequilibrium emission even for incident neutron energies as low as 14 MeV [6]. The calculated nucleon double-differential cross sections (DDCS's) by quantum-mechanical preequilibrium theory are clearly better than semiclassical model calculations, which underpredict backward-angle emission [5,6]. However, so far the particle emissions of the discrete levels and the composite particles  $(d, t, {}^{3}$  He,  $\alpha)$  emissions have not been calculated by the FKK theory. This situation limits this theory in applications.

The DDCS's of a single particle in nucleon-induced reactions around several tens of MeV were described by the improved exciton model [7]. Later, Fermi motion and Pauli exclusion principle were taken into account [8,9], so that the behavior of the backward-angle scattering was improved at low incident energies. Then the energy-angle correlation effect was considered [10-12], and the calculated angular distributions for single-nucleon emissions were improved in agreement with the experiments for incident energies up to several tens of MeV. The effect of

anisotropic intranuclear nucleon-nucleon scattering was also studied [13]. For composite-particle emissions, the pickup niechanism in precompound reactions was studied to search for a useful tool to calculate the cross sections and spectra of emitted composite particles  $(\alpha, d, t,^3 \text{ He})$ [14-17], and this model was extended to DDCS calculations for  $\alpha$ -particle emissions [11]. Afterwards, another method for calculating the DDCS's of the emitted composite particle in precompound reactions was proposed [18-21]. Meanwhile, a semiclassical model (SCM) considering the angular momentum and parity conservations was presented [22,23], with which the nuclear data of the discrete levels can be calculated for preequilibrium reaction processes. Following the FKK model, the angular momentum factor of the spin- $\frac{1}{2}$  nucleon is given in the SCM [24]. However, the calculated nucleon DDCS's with the SCM at backward angles give much lower values than the experimental data [13]. It is mainly caused by using the semiclassical nucleon-nucleon scattering kernel in nuclear matter.

In terms of the comparison between the FKK quantum theory [5,6] and the semiclassical theory [12,22,23], we find that the final equations of the FKK quantum model are very similar to the semiclassical theory. For improving the semiclassical method, the one-step distorted-wave Born approximation (DWBA), which automatically accounts for interference, refraction, and finite-size effects, is employed to calculate the angular factor instead of the nucleon-nucleon scattering expressions in nuclear matter, so that a quasiquantum MSD (QMSD) method both for continuum and discrete levels is proposed. The DDCS's of composite-particle emissions can also be calculated with this QMSD theory. Since the MSD mechanism dominates preequilibrium emission [6], the nuclear reactions can generally be described by the optical model, direct reaction theory, QMSD preequilibrium emission theory, and HF equilibrium emission theory.

In Sec. II the formulations of the quasiquantum MSD

theory both for continuum and discrete levels are presented. The calculated results and a discussion are given in Sec. III.

## II. FORMULATIONS OF THE QUASIQUANTUM MSD THEORY FOR BOTH CONTINUUM AND DISCRETE LEVELS

The quasiquantum MSD theory for both continuum and discrete levels has been proposed to consider the angular momentum and parity conservations and to calculate the energy-angle correlation angular factor on the basis of the one-step distorted-wave Born approximation.

The formula for the cross section for the emission of particle b reads [22,23]

$$\sigma_b = \sum_{I\Pi} \sigma_a^{J\Pi} \sum_n \tau^{J\Pi}(n) T_b^{J\Pi}(n, E) , \qquad (1)$$

where E refers to the excitation energy and n stands for the exciton number. J and  $\Pi$  are the total angular momentum and parity of the system.  $\sigma_a^{J\Pi}$  is the absorption cross section of the  $J\Pi$  channel, which formula is as same as that in HF theory.

The excitation states of the residual nucleus are divided into continuum and discrete levels. We have

$$T_b^{J\Pi}(n,E) = \int T_{bc}^{J\Pi}(n,E,\varepsilon) d\varepsilon + \sum_{\pmb{k}} T_{b\pmb{k}}^{J\Pi}(n,E,\varepsilon_{\pmb{k}}) \ , \ (2)$$

where

$$T_{bc}^{J\Pi}(n, E, \varepsilon) = \frac{1}{2\pi\hbar} \sum_{I'\pi'} \hat{I}^{'2} \sum_{j'=|J-I'|}^{J+I'} \sum_{l'=|j'-s_b|}^{j'+s_b} T_{l'j'}^{b}(\varepsilon) \sum_{\lambda} F_{\lambda\nu}(\varepsilon) Q_{\lambda\nu}(p, h) \omega^{I'\pi'} \left( p - \lambda, h, E - B_b - \frac{M_C}{M_R} \varepsilon \right) \times \delta(\Pi, \pi'(-1)^{l'}) ,$$

$$(3)$$

$$T_{bk}^{J\Pi}(n, E, \varepsilon_{k}) = \frac{1}{2\pi\hbar} \hat{I}_{k}^{2} \sum_{j'=|J-I_{k}|}^{J+I_{k}} \sum_{l'=|j'-s_{b}|}^{j'+s_{b}} T_{l'j'}^{b}(\varepsilon_{k}) \sum_{\lambda} F_{\lambda\nu}(\varepsilon_{k}) Q_{\lambda\nu}(p, h) \omega^{I_{k}\pi_{k}} \left(p - \lambda, h, E - B_{b} - \frac{M_{c}}{M_{R}} \varepsilon_{k}\right) \times \delta(\Pi, \pi_{k}(-1)^{l'}) ,$$

$$(4)$$

$$\varepsilon_{\mathbf{k}} = \frac{M_R}{M_C} (E - E_{\mathbf{k}} - B_{\mathbf{b}}) , \qquad (5)$$

where  $B_b$  is the binding energy and  $E_k$  is the energy of kth level.  $M_R$  and  $M_C$  are the masses of the residual and compound nuclei, respectively.  $T^b_{l'j'}(\varepsilon)$  and  $T^b_{l'j'}(\varepsilon_k)$  are the transmission coefficients calculated by the optical model.  $F_{\lambda\nu}(\varepsilon)$  and  $F_{\lambda\nu}(\varepsilon_k)$  are pickup factors of the emitted composite particle [14–17], and  $\lambda + \nu = A_b$ , where  $A_b$  is the mass number of the emitted particle b. The symbol  $\hat{x}$  is defined as  $\hat{x} = \sqrt{2x+1}$ .

If the pickup configuration is taken into account, the expression of the combination factor can be found [25]:

$$Q_{\lambda\nu}(p,h) = \left(\frac{A}{Z}\right)^{Z_b} \left(\frac{A}{N}\right)^{N_b} \begin{bmatrix} p \\ \lambda \end{bmatrix}^{-1} \begin{bmatrix} A-h \\ \nu \end{bmatrix}^{-1} \begin{bmatrix} A_b \\ Z_b \end{bmatrix}^{-1} \sum_{i=0}^{h} \begin{bmatrix} h \\ i \end{bmatrix} \left(\frac{Z}{A}\right)^i \left(\frac{N}{A}\right)^{h-i}$$

$$\times \sum_{j=0}^{\lambda} \begin{bmatrix} Z_a+i \\ j \end{bmatrix} \begin{bmatrix} N_a+h-i \\ \lambda-j \end{bmatrix} \begin{bmatrix} Z-i \\ Z_b-j \end{bmatrix} \begin{bmatrix} N-h+i \\ N_b-\lambda+j \end{bmatrix},$$
(6)

where p and h represent the particle and hole numbers, respectively. A, Z, and N ( $A_a$ ,  $Z_a$ , and  $N_a$ ;  $A_b$ ,  $Z_b$ , and  $N_b$ ) are the mass, proton, and neutron numbers of the target (incident particle, emitted particle). In above expression, the following symbol is used:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{y!}{(y-x)!x!} \ . \tag{7}$$

Based on the group method to account for the exact Pauli exclusion effect in the exciton state densities [26], for the nucleon-induced nuclear reaction processes, the exciton state density was proposed as

$$\omega^{J\Pi}(n,E) = P(\Pi)R_n(J)\omega(n,E) , \qquad (8)$$

where

$$P(\Pi) = \frac{1}{2} \ , \tag{9}$$

$$R_n(J) = \frac{\hat{J}^2}{2\sqrt{2\pi}\sigma_n^3} \exp\left(-\frac{(J+1/2)^2}{2\sigma_n^2}\right),$$
 (10)

$$\omega(n,E) = g \frac{(gU)^h [gU - A(p,h)]^{p-1}}{p!h!(n-1)!} , \qquad (11)$$

with n = p + h and the spin cutoff factor  $\sigma_n^2 = 0.24nA^{2/3}$  [27].  $U = E - \Delta$ , where  $\Delta$  is the pair correction.  $g = \frac{6a}{\pi^2}$  is the single-particle level density around the Fermi surface, and a is the level density parameter [28].

The Pauli exclusion correction A(p,h) is approximately equal to

$$A(p,h) = \frac{1}{2}[p(p-1) + h(h-1)]. \tag{12}$$

The lifetime  $\tau^{J\Pi}(n)$  of exciton state n in the  $J\Pi$  channel can be obtained by solving the  $J\Pi$ -dependent master equation [22,23]. The total emission rate  $W_t^{J\Pi}(n)$  in the  $J\Pi$  channel can be obtained as follows:

$$W_t^{J\Pi}(n) = \frac{T_b^{J\Pi}(n, E)}{\omega^{J\Pi}(n, E)} . \tag{13}$$

The transition rates of the  $J\Pi$ -dependent exciton model are given by [24]

$$\lambda_{\mu}^{J}(n) = \lambda_{n}^{\mu} X_{\mu}^{J}(n), \quad \lambda_{n}^{\mu} = \frac{2\pi}{\hbar} |\langle M \rangle^{2}| Y_{\mu}(n), \quad \mu = +, 0, -.$$
(14)

The final state probabilities are as follows:

$$Y_{+} = g \frac{[gU - A(p,h)]^{2}}{2(n+1)}$$
,

$$Y_0 = g[gU - A(p,h)] \frac{A(p,h) + 2ph}{n} , \qquad (15)$$

$$Y_{-}=g(n-2)\frac{ph}{2}.$$

The averaged radial transition matrix element  $|\langle M \rangle^2|$  is given as

$$|\langle M \rangle^2| = \frac{K}{EA^3} , \qquad (16)$$

where K is the exciton model parameter.

The form factor of the angular momentum is obtained in terms of the FKK theory [1,3], but a spin  $\frac{1}{2}$  was used instead of a spin of 0. Their final expressions are as follows [24]:

$$X_0^J(n) = \frac{1}{32\pi^2} \frac{1}{R_n(J)} \sum_{sj} R_{n-2}(s) F_0(j) \Delta(Jjs) . \quad (17)$$

$$F_0(j) = \sum_{j_a j_b} \hat{j}_a^2 \hat{j}_b^2 R_1(j_a) R_1(j_b) G_0(j_a j_b j) , \qquad (18)$$

$$G_0(j_a j_b j) = \sum_{j_c j_d} \hat{j}_c^2 \hat{j}_d^2 R_1(j_c) R_1(j_d) [1 - (-1)^{j - j_c - j_d}] \times [C_{j_c(1/2)j_d - (1/2)}^j]^2,$$
(19)

$$X_+^J(n) = \frac{1}{32\pi^2} \frac{1}{R_n(J)} \sum_{sj_a} R_1(j_a) R_{n-1}(s) F_+(j_a) \Delta(j_a J s) ,$$

$$F_{+}(j_{a}) = \sum_{j_{3}j_{c}} \frac{\hat{j}_{c}^{2}}{\hat{j}_{3}^{2}} R_{1}(j_{c}) G_{+}(j_{a}j_{c}j_{3}) , \qquad (21)$$

$$G_{+}(j_{a}j_{c}j_{3}) = \sum_{j_{b}j_{d}} \hat{j}_{3}^{2} \hat{j}_{d}^{2} R_{1}(j_{3}) R_{1}(j_{d}) \begin{bmatrix} C_{j_{a}(1/2)j_{c}}^{j_{3}} & 0 \\ j_{a}(1/2)j_{c}^{-}(1/2) \end{bmatrix}$$

$$\times C_{j_{b}(1/2)j_{d}-(1/2)}^{j_{3}} ,$$

$$+(-1)^{j_{c}+j_{d}} C_{j_{a}(1/2)j_{d}-(1/2)}^{j_{3}}$$

$$\times C_{j_{b}(1/2)j_{c}-(1/2)}^{j_{3}} \Big]^{2} , \qquad (22)$$

$$X_{-}^{J}(n) = \frac{1}{32\pi^{2}} \frac{1}{R_{n}(J)} \sum_{sj_{a}} R_{1}(j_{a}) R_{n-3}(S) F_{+}(j_{a}) \Delta(j_{a}Js) ,$$
(23)

where the triangular function  $\Delta(abc)$  is defined as unity if  $|a-b| \leq c \leq a+b$  and zero otherwise.  $C_{j_1m_1}^{j_3}{}^{m_3}{}_{j_2m_2}$  is a Clebsch-Gordan (CG) coefficient.

The energy spectra formulation of the continuum reads

$$\frac{d\sigma_b}{d\varepsilon} = \sum_{J\Pi} \sigma_a^{J\Pi} \sum_n \tau^{J\Pi}(n) T_{bc}^{J\Pi}(n, E, \varepsilon) . \qquad (24)$$

The DDCS of single-particle emission of the continuum is given by the expression

$$\frac{d^2\sigma}{d\Omega\,d\varepsilon} = \sum_{n} \frac{d\sigma_n(n)}{d\varepsilon} A(n\Omega\varepsilon) , \qquad (25)$$

where

$$\frac{d\sigma_b(n)}{d\varepsilon} = \sum_{III} \sigma_a^{JII} \tau^{JII}(n) T_{bc}^{J\pi}(n, E, \varepsilon) . \qquad (26)$$

By using the energy-angle correlation scattering kernel approach [10-13], the angular factor of a single particle for the continuum reads

$$A(n\Omega\varepsilon) = \frac{1}{\zeta_0^k(n\varepsilon)} \sum_{l} \frac{2l+1}{4\pi} \zeta_l^k(n\varepsilon) P_l(\cos\theta) , \qquad (27)$$

which satisfies

$$\int A(n\Omega\varepsilon)d\Omega = 1. (28)$$

Under the "never-come-back" assumption, the following expression is obtained [11–13]:

$$\zeta_{l}(n\varepsilon) = \eta(n) \int d\varepsilon_{2} \int d\varepsilon_{3} \cdots \int d\varepsilon_{(n+2-n_{0})/2} \\
\times \mu_{l}(E, \varepsilon_{2}) \mu_{l}(\varepsilon_{2}, \varepsilon_{3}) \cdots \mu_{l}(\varepsilon_{(n+2-n_{0})/2}, \varepsilon) . (29)$$

The mean lifetime is given by

$$\eta(n) = \frac{1}{\lambda_n^+ + W_n} \prod_{\substack{i = n_0 \\ \Delta i = 2}}^{n-2} \frac{\lambda_i^+}{\lambda_i^+ + W_i}.$$
 (30)

(20)

(39)

The  $J\Pi$ -independent total emission rate  $W_n$  of the composite system reads

$$W_n = \sum_b \int W_n^b(\varepsilon) d\varepsilon , \qquad (31)$$

$$W_{n}^{b}(\varepsilon) = \frac{\hat{I}_{b}^{2}}{\pi^{2}\hbar^{3}}\mu_{b}\varepsilon\sigma_{b}(\varepsilon)\sum_{\lambda}F_{\lambda\nu}(\varepsilon)Q_{\lambda\nu}(p,h)$$

$$\times \frac{\omega(p-\lambda,h,E-B_{b}-(M_{C}/M_{R})\varepsilon)}{\omega(p,h,E)}, \quad (32)$$

where  $\mu_b$  and  $\sigma_b(\varepsilon)$  are the reduced mass and inverse cross section of the emitted particle b, respectively.

Following the FKK quantum MSD theory [5,6], the expression for  $\mu_1(\varepsilon'', \varepsilon')$  is obtained as follows:

$$\mu_l(arepsilon'',arepsilon') = rac{1}{\overline{\sigma}(arepsilon'')} \int d\Omega' rac{d^2 \sigma(arepsilon'',arepsilon')}{d\Omega' darepsilon'} igg|_{ ext{one step}} P_l(\cos heta') \; ,$$

$$\overline{\sigma}(\varepsilon'') = \int d\varepsilon' d\Omega' \frac{d^2 \sigma(\varepsilon'', \varepsilon')}{d\Omega' d\varepsilon'} \bigg|_{\text{one of the }}, \qquad (34)$$

$$\left. rac{d^2\sigma(arepsilon'',arepsilon')}{d\Omega'darepsilon'} 
ight|_{ ext{one step}} = \sum_{I'\pi'} \hat{I}^{'2} \omega^{I'\pi'} (1p1h,arepsilon')$$

$$\times \left[ \frac{d\sigma(\varepsilon'', \varepsilon', I'\pi')}{d\Omega'} \right]_{\text{DWBA}}$$
 . (35)

The spin I' and parity  $\pi'$  of the residual nucleus are considered in Eq. (35).

The angular distribution of a single particle for discrete levels reads

$$\frac{d\sigma_b^k}{d\Omega} = \sum_n \left[ \sum_{J\Pi} \sigma_a^{J\Pi} \tau^{J\Pi}(n) T_{bk}^{J\Pi}(n, E, \varepsilon_k) \right] A(n\Omega \varepsilon_k) ,$$
(36)

$$A(n\Omega\varepsilon_{k}) = \frac{1}{\zeta_{0}^{k}(n\varepsilon_{k})} \sum_{l} \frac{2l+1}{4\pi} \zeta_{l}^{k}(n\varepsilon_{k}) P_{l}(\cos\theta) , \quad (37)$$

which satisfies

$$\int A(n\Omega\varepsilon_{k})d\Omega = 1.$$
 (38)

According Eq. (29), one has

$$\zeta_l(n\varepsilon_k) = \eta(n) \int d\varepsilon_2 \int d\varepsilon_3 \cdots \int d\varepsilon_{(n+2-n_0)/2} \mu_l(E,\varepsilon_2) \mu_l(\varepsilon_2,\varepsilon_3) \cdots \mu_l(\varepsilon_{(n-n_0)/2},\varepsilon_{(n+2-n_0)/2}) \nu_l(\varepsilon_{(n+2-n_0)/2},\varepsilon_k) \ .$$

where

$$\nu_{l}(\varepsilon_{(n+2-n_{0})/2}, \varepsilon_{k}) = \frac{1}{\overline{\sigma}_{k}(\varepsilon_{(n+2-n_{0})/2})}$$

$$\int d\Omega' \left[ \frac{d\sigma(\varepsilon_{(n+2-n_{0})/2})}{d\Omega'} \right]_{\text{DWBA}}^{k} P_{l}(\cos\theta') , \qquad (40)$$

$$\overline{\sigma}_{k}(\varepsilon_{(n+2-n_{0})/2}) = \int d\Omega' \left[ \frac{d\sigma(\varepsilon_{(n+2-n_{0})/2})}{d\Omega'} \right]_{\text{DWBA}}^{k} . \tag{41}$$

The  $\mu_1(\varepsilon'', \varepsilon')$  in Eq. (39) is still calculated by Eq. (29). If only the  $n = n_0$  one-step preequilibrium process is considered, then we have

$$\zeta_l(n_0 \varepsilon_k) = \eta(n_0) \nu_l(E, \varepsilon_k) , \qquad (42)$$

$$\nu_l(E, \varepsilon_k) = \frac{1}{\overline{\sigma}_k(E)} \int d\Omega' \left[ \frac{d\sigma(E)}{d\Omega'} \right]_{\text{DWBA}}^k P_l(\cos\theta') , \qquad (43)$$

$$\overline{\sigma}_{k}(E) = \int d\Omega' \left[ \frac{d\sigma(E)}{d\Omega'} \right]_{\text{DWBA}}^{k} , \qquad (44)$$

$$\eta(n_0) = [\lambda_{n_0}^+ + W_{n_0}]^{-1} . (45)$$

For composite-particle emissions, the angular factors  $A(n\Omega\varepsilon)$  and  $A(n\Omega\varepsilon_k)$  can also be calculated by using the energy-angle correlation approach [21] with the formulations of  $\mu_l(\varepsilon'', \varepsilon')$  and  $\nu_l(\varepsilon', \varepsilon_k)$  given in this paper.

## III. CALCULATED RESULTS AND DISCUSSION

The low-energy nuclear reactions can generally be described by optical model, direct reaction theories, preequilibrium emission theory, and the equilibrium emission theory. The preequilibrium emission theory is only used for the continuum previously, whereas the preequilibrium emission theory QMSD can describe the continuum and discrete levels. Various direct reaction DWBA theories and coupled-channels optical model as well as equilibrium emission HF theory can be included in our calculations. The low-lying state excitation function is mainly contributed by statistical theory (i.e., HF theory before and QMSD+HF theory now) in the low-energy region and by direct reaction theory in the high-energy region. Generally speaking, a straightforward calculations with direct reaction theory must be included for the low-lying excitation state, but for a specific low-energy light nucleus  $p+^{11}$ B reaction, for which there exist only low-energy discrete level angular distribution and cross section experimental data, the calculated results show that using only QMSD and HF theories is suitable.

We calculated the reaction  $p+^{11}\mathrm{B}$  in the energy region 1–25 MeV using the QMSD and HF theories. The various cross sections and the discrete level angular distributions of the  $(p,n_0)$  reaction were calculated and compared with the experimental data. Since the first step contribution in MSD theory is dominant [6], only the one-step process is considered in our calculations. The exciton model parameter K in Eq. (16) is taken as 300 MeV<sup>3</sup>.

Figure 1 shows the calculated value and the experimental datum [29] comparison of the neutron angular distributions of the  ${}^{11}{\rm B}(p,n_0){}^{11}{\rm C}$  reaction ( ${}^{11}{\rm C}$  in the ground state) at incident proton energies 7.1 and 8.2 MeV. The calculated values represented by the dashed lines with the SCM [22,23] and HF theory are near the isotropic angular distributions and slightly forward, which cannot give the experimental data changing tendency with the angle. The solid lines represent the calculated values obtained using the QMSD and HF theories, which are basically in agreement with the experimental data. The backward tendencies both of the experimental and calculated angular distributions at incident proton energy 7.1 MeV are all stronger than those at incident proton energy 8.2 MeV. The DWBA angular distributions of the direct (p, n) knock-out reaction [30] were calculated by the code KORP [31]. The theoretically predicted neutron angular distributions obtained using the QMSD and HF theories of the  $^{11}\text{B}(p,n_0)^{11}\text{C}$  reaction at incident proton energies 4.0–11 MeV are shown in Fig. 2. The backward tendency of the neutron angular distributions is gradually reduced with the incident proton energy increasing, as same as the measured data. What kind of reactions (e.g., inelastic scattering) can be successfully analyzed to discrete level angular distribution experimental data in the low-energy region by the QMSD + HF theory should be studied in the future.

Since QMSD theory formulations are theoretically very close to the FKK quantum MSD theory [5,6], good calculated DDCS's of the continuum would be obtained with the QMSD theory, but there exist no double-differential cross section experimental data of the continuum for the  $p+^{11}$ B reaction to be compared with theory. We can see clearly that for calculating the various cross sections with Eq. (1) and the continuum energy spectra with Eq. (24), the angular factor  $A(n\Omega\varepsilon)$  in Eq. (25) and corresponding DWBA calculations are not needed. Therefore the various cross sections given in this paper as well as the

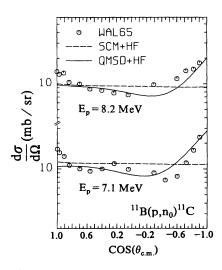


FIG. 1. Calculated value and the experimental datum [29] comparison of the neutron angular distributions of the  $^{11}$ B( $p, n_0$ ) $^{11}$ C reaction ( $^{11}$ C in the ground state) at incident proton energies 7.1 and 8.2 MeV.

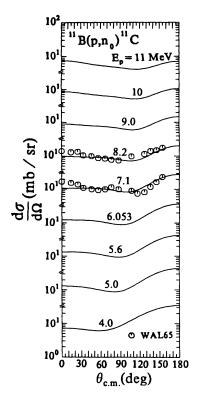


FIG. 2. Theoretically predicted neutron angular distributions obtained using quasiquantum MSD and HF theories of the  $^{11}\text{B}(p,n_0)^{11}\text{C}$  reaction at incident proton energies 4.0–11 MeV.

angular distributions are all calculated by the QMSD + HF theory.

Figure 3 shows the calculated value and the experimental datum [29,32] comparison of the neutron cross sections of the  $^{11}\mathrm{B}(p,n_0)^{11}\mathrm{C}$  reaction. The calculated values obtained using the QMSD and HF theories are basically in agreement with the experimental data. Especially, the theoretical curve passes the experimental points at incident proton energies 7.1 and 8.2 Mev. The dashed and

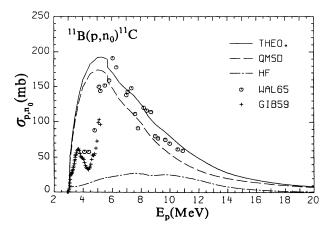


FIG. 3. Calculated value and the experimental datum [29,32] comparison of the neutron cross sections of the  $^{11}\mathrm{B}(p,n_0)^{11}\mathrm{C}$  reaction.

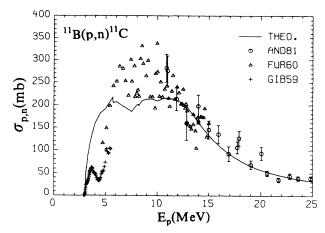


FIG. 4. Calculated value and the experimental datum [32-34] comparison of the neutron cross sections of the  ${}^{11}\mathrm{B}(p,n){}^{11}\mathrm{C}$  reaction.

dot-dashed lines represent the calculated values obtained using the QMSD and HF theories, respectively. The calculated results show that the preequilibrium emission contribution given by the QMSD theory dominates the discrete level excitation function of the  ${}^{11}\mathrm{B}(p,n_0){}^{11}\mathrm{C}$  reaction. The relative magnitudes of the contributions to the discrete level excitation function obtained using the QMSD and HF theories should be studied for different reactions (e.g., inelastic scattering) and different target (e.g., heavy nuclei) in the future. In our calculations the straightforward contributions of the direct reactions were not included. If we include the straightforward (p, n) direct knock-out reaction calculations, the calculated angular distributions at  $E_p = 7.1$  and 8.2 MeV would not be affected obviously because the contribution of the direct reaction is small at low energies.

The calculated results for the  $^{11}\mathrm{B}(p,n_0)^{11}\mathrm{C}$  reaction indicate that, using the direct knock-out reaction theory, the data of neutron angular distributions can be fitted reasonably, but the excitation function cannot be fitted; using semiclassical model, the excitation function can be fitted satisfactorily, but the neutron angular distributions cannot be fitted; using the quasiquantum MSD theory, both of them can all be fitted reasonably.

The calculated value and the experimental datum [32-34] comparison of the neutron cross sections of the

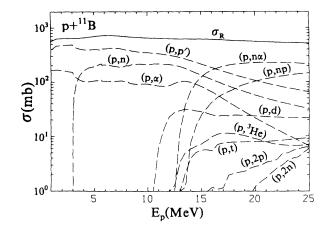


FIG. 5. Theoretically predicted various cross sections obtained using the quasiquantum MSD and HF theories of the  $p+^{11}$ B reaction in the energy region 1–25 MeV.

 $^{11}\mathrm{B}(p,n)^{11}\mathrm{C}$  reactions are shown in Fig. 4. The calculated values obtained using the QMSD and HF theories are basically in agreement with the experimental data. The theoretically predicted various cross sections obtained using the QMSD and HF theories of the  $p+^{11}\mathrm{B}$  reaction in the energy region 1–25 MeV are shown in Fig. 5.

In summary, the method for calculating multistep direct reactions for both continuum and discrete levels is proposed. The energy-angle correlation scattering kernel is employed to calculate the angular factor in the generalized exciton model, in which the angular momentum and parity conservations are considered. Then, following the FKK quantum MSD theory, we calculate the Legendre coefficients of the angular distributions on the basis of one-step DWBA cross sections instead of the nucleon-nucleon scattering expression in nuclear matter. Since the quantum effects are properly considered, it is a quasi-quantum MSD theory. For the specific reaction considered here, the calculated discrete level neutron angular distributions of the  $^{11}$ B( $p, n_0$ ) $^{11}$ C reaction and the cross sections of the  $^{11}$ B( $p, n_0$ ) $^{11}$ C reactions with this method fit the experimental data reasonably.

The authors would like to thank Prof. Xixiang Bai for helpful discussions and suggestions.

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