Finite 3π cut approximation for the $\pi N\bar{N}$ form factor

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(Received 4 January 1994)

Assuming the length of the 3π cut to be finite and approximating the integrated 4π amplitude by a constant, we derive an expression for the $\pi N\bar{N}$ form factor which is very close to that given by a simple pole. The specific predictions of the obtained form factor for the region of small momentum transfer are compared with existing effective pole formulas and discussed along the lines of the Goldberger-Treiman relation.

PACS number(s): 13.75.Cs, 13.75.Gx, 14.20.Dh, 14.40.Aq

I. INTRODUCTION

The problem of determining the $\pi N\bar{N}$ vertex with the pion off its mass shell has been open for about 30 years and as yet has had no reliable solution. There exist a few model calculations for zero-mass pions or very close to this limit. Being designed for the spacelike region only, these models have little chance for any extrapolation.

In the spacelike region it is legitimate to parametrize the form factor (FF) by means of an effective pole, dipole, or even an exponential. The effective mass λ, as extracted from variety of models [1], reflects a strong model dependence: 0.6 GeV $\leq \lambda \leq$ 1.5 GeV.

To some extent, such an inconclusive situation is understandable from the point of view of analytic functions. The very fact that a cut is approximated by a pole has different impact on models dealing with different sectors in the momentum transfer plane. What seems obvious is that the position of the effective pole should move to lower values as the momentum transfer range under consideration approaches the cut along the no-cut region. In other words, smaller effective mass corresponds to models like those dealing with chiral pions, whereas larger mass is consistent with, e.g., N-N potential models. As extensively discussed in Ref. [1], a clear-cut conclusion cannot be drawn.

The increased accuracy of determination of the pion-nucleon coupling constant [2–7] raises also expectations for improvements in determining the entire FF as well. Therefore, any attempt to clarify the structure of the $\pi N\bar{N}$ form factor is timely and of importance.

In what follows we present the derivation of a modified pion-nucleon FF based on its analyticity properties in the timelike region where the form factor develops an imaginary part (Sec. II). While performing the integration over the (dominant) three-pion cut, we assume that the off-shell behavior of 4π and $3\pi N\bar{N}$ vertices is determined by the existence of an effective cut of finite length. Then approximating the structure of the 4π vertex simply by a constant, we arrive at the expression for the form factor (Secs. III and IV). Its specific features are mostly of kinematical origin. Instead of a simple pole, a more general one-parameter formula is obtained. It reduces to the effective pole when the length of the cut shrinks to zero. Its properties are discussed in Secs. V and VI.

II. TIMELIKE REGION

In the timelike region there is no experimental information about the pion-nucleon form factor. Therefore the main formula of dispersion theory,

$$G(\tau) = \frac{1}{\pi}\int_{\tau_0}^{\infty} d\tau' \frac{\operatorname{Im} G(\tau')}{\tau' - \tau} \tag{1}$$

(as well as any variant with the suitable number of subtractions), was practically of no use.

Already the first unitarity diagram for the imaginary part contains an insurmountable difficulty in handling the inelastic amplitudes. However, the situation might seem less desperate if we observe that the $N\bar{N}$ pair may be considered as an effective pion very far away from its mass shell ($m_{\pi^*} \approx 2$ GeV). Intermediate pions are physical pions by the very fact that in unitarity diagrams the intermediate set of states is always on the mass shell. So, in essence, the evaluation of the imaginary part will include the product of two pion-pion elastic scattering amplitudes, each of them containing one pion off the mass shell.

Let us now attack the 3π cut along these lines. The conventional expression for the $\pi N\bar{N}$ vertex in the $N\bar{N}$ annihilation region has the form

$$\begin{aligned}\langle N_\alpha^{(\mu)}(p)\ \bar{N}_\beta^{(\nu)}(p')|S-1|\pi^d(q)\rangle &= i(2\pi)\delta^{(4)}(p+p'-q) \\ &\quad\times \bar{v}^{(\nu)}(p') i\gamma_5 u^{(\mu)}(p)\tau_{\alpha\beta}^d\, G_{\pi N\bar{N}}(\tau),\end{aligned} \tag{2}$$

where

$$G_{\pi N\bar{N}}(\tau) = g_{\pi N} G(\tau), \quad \tau \equiv q^2\,, \tag{3}$$

with $g_{\pi N}$ being the pion-nucleon coupling constant, and the form factor $G(\tau)$ is normalized by

$$G(m_\pi^2) = 1; \tag{4}$$

other notation is obvious.

The unitarity condition for $S = 1 + iT$, after straightforward manipulations, in the 3π-cut approximation has the form

$$\begin{aligned}
&2\pi\delta^{(4)}(p+p'-q)\bar{v}^{(\nu)}(p')i\gamma_5 u^{(\mu)}(p)\tau^d_{\alpha\beta}\left[G_{\pi N\bar{N}}(\tau) - G^*_{\pi N\bar{N}}(\tau)\right] \\
&\quad = \sum \langle N\bar{N}|T|n\rangle\langle n|T^\dagger|\pi^d\rangle \\
&\quad \approx \int \prod \frac{d^3k_i}{(2\pi)^3 2k_{0i}} \langle N^{(\mu)}_\alpha(p)\bar{N}^{(\nu)}_\beta(p')|T|\pi_a(k_1)\pi_b(k_2)\pi_c(k_3)\rangle \\
&\qquad \times \langle \pi_a(k_1)\pi_b(k_2)\pi_c(k_3)|T^\dagger|\pi^d(q)\rangle \\
&\quad = \int \prod \frac{d^3k_i}{(2\pi)^3 2k_{0i}} (2\pi)^4\delta^{(4)}(p+p'-k_1-k_2-k_3) M^{\alpha\beta,\mu\nu}_{abc}(N\bar{N}\to 3\pi) \\
&\qquad \times (2\pi)^4\delta^{(4)}(q-k_1-k_2-k_3) M^*_{abcd}(\pi\to 3\pi) \\
&\quad = (2\pi)^4\delta^{(4)}(p+p'-q)\int d\Omega_{3\pi} M^{\alpha\beta,\mu\nu}_{abc}(N\bar{N}\to 3\pi) M^*_{abcd}(\pi\to 3\pi),
\end{aligned} \tag{5}$$

where

$$\int \prod \frac{d^3k_i}{(2\pi)^3 2k_{0i}} \delta^4(q-k_1-k_2-k_3) \equiv \int d\Omega_{3\pi}. \tag{6}$$

After dropping the δ functions, the unitarity condition in the 3π intermediate-state approximation becomes

$$2\bar{v}^{(\nu)}(p')i\gamma_5 u^{(\mu)}(p)\tau^{\alpha\beta}_d \operatorname{Im} G_{\pi N\bar{N}}(\tau) = \int d\Omega_{3\pi} M^{\alpha\beta,\mu\nu}_{abc}(N\bar{N}\to 3\pi) M^*_{abcd}(\pi\to 3\pi). \tag{7}$$

In order to form an isospin pion state in the left-hand-side (LHS), we have to multiply it by the factor $\tau^{\beta\alpha}_{d'}$. The antiparallel-spin nucleon wave functions should be used for representing the pion state. For this purpose, we simply multiply both sides by

$$\bar{u}^{(\mu)}(p)i\gamma_5 v^{(\nu)}(p') \tag{8}$$

and perform a summation over μ,ν to remove the spinor structure. Denoting

$$\begin{aligned}
M_{abcd'} \equiv \frac{1}{\tau}\frac{1}{2}\sum_{\beta,\alpha}\frac{1}{4}\sum_{\mu,\nu} M^{\alpha\beta,\mu\nu}_{abcd}(N\bar{N}\to 3\pi) \\
\times \bar{u}^{(\mu)}(p)i\gamma_5 v^{(\nu)}(p')\tau^{\beta\alpha}_{d'},
\end{aligned} \tag{9}$$

we finally obtain

$$\operatorname{Im} G_{\pi N\bar{N}}(\tau)\delta_{dd'} = \frac{1}{2}\int d\Omega_{3\pi} M_{abcd'} M^*_{abcd}. \tag{10}$$

Here we should note that the most important property of our FF comes from the $(1/\tau)$ multiplier in the expression for $\operatorname{Im} G_{\pi N\bar{N}}$. One should stress that it is not the factor $(1/\tau)$ in expression (9). Indeed, the latter must be contracted in the major spinor structure of the vertex $M^{\alpha\beta,\mu\nu}_{abcd}(N\bar{N}\to 3\pi)$ in the explicit form, and in all the other structures after the $d\Omega_{3\pi}$ integration.

III. INTEGRATION OVER 3π PHASE SPACE

Let us now consider the integration over the 3π phase space. Following the standard definitions and conventions, we first convert the integration over the internal pion momenta into the integration over the scalar invariant variables [8]:

$$\int d\Omega_{3\pi} = \frac{1}{(2\pi)^5}\frac{\pi}{2^4\lambda^{\frac{1}{2}}(\tau,m_N^2,m_N^2)}\int \frac{ds_1 ds_2 dt_1 dt_2}{\sqrt{-\Delta_4}}, \tag{11}$$

where Δ_4 is the Gram determinant,

$$\Delta_4 \equiv \Delta_4(p,p',k_1,k_3) = \begin{vmatrix} p\cdot p & p\cdot p' & p\cdot k_1 & p\cdot k_3 \\ p'\cdot p & p'\cdot p' & p'\cdot k_1 & p'\cdot k_3 \\ k_1\cdot p & k_1\cdot p' & k_1\cdot k_1 & k_1\cdot k_3 \\ k_3\cdot p & k_3\cdot p' & k_3\cdot k_1 & k_3\cdot k_3 \end{vmatrix}, \tag{12}$$

and the scalar variables are those used in Ref. [8],

$$\begin{aligned}
s_1 = (k_1+k_2)^2, \quad s_2 = (k_2+k_3)^2, \quad \tau = (p+p')^2, \\
t_1 = (p-k_1)^2, \quad t_2 = (p'-k_3)^2.
\end{aligned} \tag{13}$$

The region of integration is limited to that where $\Delta_4 \le 0$ and $q^2 \equiv \tau \ge 9m_\pi^2$.

We shall take advantage of introducing two other variables and renaming the rest. In terms of the relative nucleon momentum $P = p - p'$, the suitable invariant variables are

$$t_1' = P(k_1 + k_3), \quad t_2' = P(k_1 - k_3), \quad t \equiv s_1, \quad s \equiv s_2, \quad \tau. \tag{14}$$

Here s and t are the usual Mandelstam variables of the 4π vertex and τ is the mass of the heavy pion.

There are two important properties of the t_1',t_2' variables. First, the 4π vertex does not depend on them. There are also grounds to consider the dependence of the amplitude $M(N\bar{N} \to 3\pi)$ on these variables to be negligible: The full kinematics data [9, 10] on the cross reaction $\pi^+p \to \pi^+\pi^-n$ show no dependence on these variables apart from the dependence coming from the phase space. The analysis [11] of recent measurements also confirms this observation.

Thus we can rely upon the fact that the integrated expression in (11) is free of explicit t_1',t_2' dependence.

Second, the integration domain of t_1',t_2' is an ellipse in the t_1',t_2' plane, and so the above integration can be easily performed.

Finally, our 3π-phase-space integral (11) takes the form

$$\int d\Omega_{3\pi} = \frac{\text{const}}{\tau} \int_{S_0}^{S_1} ds \int_{t^-}^{t^+} dt, \tag{15}$$

where

$$S_0 = 4m_\pi^2, \tag{16}$$

$$t^\pm = \frac{1}{2}\left[\tau + 3m_\pi^2 - s \pm \frac{\sqrt{(s - 4m_\pi^2)(s - S_1)(s - S_2)}}{\sqrt{s}}\right], \tag{17}$$

$$S_{1,2} = (-\sqrt{\tau} \pm m_\pi)^2. \tag{18}$$

This result contains all $q^2 \equiv \tau$ dependence in the approximation when the amplitude M in the integral (10) is taken to be a constant.

IV. BEYOND THE EFFECTIVE POLE APPROXIMATION

If the effective pole formula [see Eq. (23) below] tolerates $\lambda^2 \approx \tau_0 \equiv 9m_\pi^2$, it means that the form-factor behavior is determined by the very fact of the presence of the cut from τ_0 to infinity. If the structure of the amplitude $M(\pi_* \to 3\pi)$ in Eq. (10) is essential too, then $\lambda^2 \gg \tau_0$.

In this case it might be convenient to introduce the notion of the effective cutoff τ_m in the amplitude $M(\pi_* \to 3\pi)$ such that any integration of the amplitude squared is equivalent to the finite-range integration of some mean value. If this holds, then a meaningful approximation could be obtained in the following way:

$$\begin{aligned} G_{\pi N\bar{N}}(\tau) &= K\int_{\tau_0}^{\infty} \frac{d\tau'}{\tau' - \tau}\frac{1}{\tau'} \\ &\quad \times \int\int_{D(\tau')} ds dt\, |M(\pi_* \to 3\pi)|^2 \\ &\approx K\int_{\tau_0}^{\tau_m} \frac{d\tau'}{\tau' - \tau}\frac{1}{\tau'}K' \\ &= \frac{KK'}{\tau}\ln\frac{1 - \tau/\tau_m}{1 - \tau/\tau_0}, \end{aligned} \tag{19}$$

where the overall constant KK' must be determined by the normalization, $G_{\pi N\bar{N}}(m_\pi^2) = g_{\pi N}$.

If the double integral including the factor $1/\tau'$ is approximated by the constant, we obtain the (normalized) form factor

$$G(\tau) = \frac{\ln[(\tau_m - \tau)/(\tau_0 - \tau)]}{\ln[(\tau_m - m_\pi^2)/(\tau_0 - m_\pi^2)]}. \tag{20}$$

If the value of the cutoff parameter τ_m is close to τ_0, then the FF (20) and the FF we shall be dealing with later on are practically the same and are very close to the form of a simple pole.

In the limit $\tau_m \to \infty$, expression (20) is a constant $(= 1)$. When approximating the inner integral in (19) by a sequence of Θ function times some simpler expansion, the weight of the term giving rise to expression (20) must vanish at large cutoff. Otherwise it will give rise to the form factor containing a hard core. In our opinion, one would like to exclude this possibility.

The approximation we have made is very rough, but it leaves us with a single free parameter τ_m. One can realize that inclusion of further dynamical details immediately converts the form-factor problem into a multiparametric one.

Therefore, in the approximation of a constant amplitude over the cut of finite length, our normalized form factor assumes the form

$$G(\tau) = \frac{g_0}{\tau}\ln\frac{1 - \tau/\tau_m}{1 - \tau/\tau_0}, \tag{21}$$

with

$$g_0 = \frac{m_\pi^2}{\ln\frac{1 - m_\pi^2/\tau_m}{1 - m_\pi^2/\tau_0}}. \tag{22}$$

The most important properties of the expression (21), to be pointed out immediately, are (1) $\tau = 0$ is not a singularity; (2) for $\tau \to -\infty$, it decreases as $1/\tau$, as in the case of a simple pole.

In what follows we investigate the properties of the FF (21) mainly by comparing it with those of the well-known form of a (simple) pole,

$$P(\tau) = \frac{\lambda^2 - m_\pi^2}{\lambda^2 - \tau}. \tag{23}$$

We leave aside the detailed comparison with other broadly used kinds of FF's such as a dipole

$$D(\tau) = \frac{(\Lambda^2 - m_\pi^2)^2}{(\Lambda^2 - \tau)^2} \tag{24}$$

or an exponential

$$E(\tau) = \exp\left(\frac{\tau - m_\pi^2}{\Lambda_e^2}\right). \tag{25}$$

It is a simple algebraic exercise to identify the parameters of the FF's in question at $\tau \approx 0$. On the other hand, of the quoted FF's, only (21) and the monopole are alike in the asymptotic region $\tau \to -\infty$.

The origin of the similarity of these two FF's might be clarified by considering the limiting case $\tau_m \to \tau_0$. In this limit, because of the normalization condition (4), the FF (21) assumes the form

$$\lim_{\tau_m \to \tau_0} \frac{g_0}{\tau} \ln \frac{1 - \tau/\tau_m}{1 - \tau/\tau_0} = \frac{\tau_0 - m_\pi^2}{\tau_0 - \tau}. \tag{26}$$

So the effective monopole shape of the pion-nucleon FF might indeed be the natural one and its effective mass is

$$\lambda = \sqrt{\tau_0} = 3m_\pi \approx 0.414 \text{ GeV}. \tag{27}$$

The most important regions for comparison of the FF's in question when $\tau_m \neq \tau_0$ are the region of small τ and the asymptotic region when $\tau \to -\infty$. Let us now proceed along these lines.

V. $\pi N\bar{N}$ FORM FACTOR AT SMALL MOMENTUM TRANSFERS

The region of small τ is of great importance since the experimental quantities measured here meet the theoretical predictions of chiral dynamics. Therefore, we compare the FF's (21) and (23) in terms of the Goldberger-Treiman relation (GTR) [12] .

Let us first briefly recall the usual derivation of the GTR. Identifying the $G_{\pi N\bar{N}}$ form factor in the matrix element of the pion-field source j^a ,

$$\langle N(p')|j^a(0)|N(p)\rangle = iG_{\pi N\bar{N}}(q^2)\bar{u}(p')\gamma_5\tau^a u(p), \tag{28}$$

$$p' = (p+q), \quad p'^2 = p^2 = m_N^2, \quad q^2 = 2p\cdot q, \tag{29}$$

one converts, in the standard way, the matrix element (28) of the pionic source $j^a(0)$ into the pion field and then into the partially conserved axial current as follows:

$$\begin{aligned}\langle N(p+q)|j^a(0)|N(p)\rangle &= (-q^2 + m_\pi^2)\langle p+q|\phi^a(0)|p\rangle \\ &= \frac{-q^2+m_\pi^2}{m_\pi^2 f_\pi\sqrt{2}}\langle p+q|\partial j^a_{5\mu}|p\rangle \\ &= \frac{-q^2+m_\pi^2}{m_\pi^2 f_\pi\sqrt{2}}\bar{u}(p+q)i\gamma_5\tau^a\left[2m_N g_A(q^2) + q^2 h_A(q^2)\right]u(p),\end{aligned} \tag{30}$$

where the notation is straightforward and conventional.

After comparing (28) with (30), we obtain

$$g_{\pi N}G(q^2) = \frac{-q^2+m_\pi^2}{m_\pi^2 f_\pi\sqrt{2}}\left[2m_N g_A(q^2) + q^2 h_A(q^2)\right]. \tag{31}$$

This relation holds in a limited region of q^2 where the pion-field-to-axial-current identity holds. Provided that the axial form factors $g_A(q^2)$ and $h_A(q^2)$ can be measured or calculated, it defines the off-shell behavior of the pion-nucleon coupling. At $q^2 = 0$ the Goldberger-Treiman relation follows, $g_{\pi N} f_\pi G(0) = \sqrt{2}\, m_N m_\pi^2 g_A(0)$. We rewrite it for the intercept $G_0 \equiv G(0)$:

$$G_0 = \frac{m_N}{F_\pi}\frac{G_A}{g_{\pi N}}, \tag{32}$$

where $G_A \equiv 2g_A(0) = 1.261 \pm 0.004$ [13] and $F_\pi \equiv f_\pi\sqrt{2} = (92.6 \pm 0.2)$ MeV [14] .

As $g_A(q^2)$ receives no contribution from the pion pole, we can evaluate the slope of the form factor $G(q^2)$ at $q^2 = 0$ by using the pion-pole contribution to $h_A(q^2)$:

$$h_A(q^2)|_{\pi\ \text{pole}} = \frac{f_\pi g_{\pi N}\sqrt{2}}{q^2 - m_\pi^2}, \tag{33}$$

resulting in

$$G'(0) = \frac{1}{m_\pi^2 f_\pi\sqrt{2}}\left[2m_N g_A(0) + m_\pi^2 h_A(0)\right] + \frac{m_N\sqrt{2}}{f_\pi}g'_A(0). \tag{34}$$

The GT relation and $h_A(0)$ from (33) make the brackets vanish, so that we finally obtain

$$\frac{G'(0)}{G(0)} = \frac{g'_A(0)}{g_A(0)}. \tag{35}$$

Expressions (32) and (35) are the tools for comparison of FF's at $\tau = 0$ since any physically acceptable FF must provide a reasonable value for the discrepancy of the GTR as well as for the nucleon size. Let us now expand the FF's in question around $\tau = 0$:

$$P(\tau) = \frac{\lambda^2 - m_\pi^2}{\lambda^2 - \tau} = \frac{\lambda^2 - m_\pi^2}{\lambda^2}\left[1 + \tau/\lambda^2 + \cdots\right], \tag{36}$$

$$G(\tau) = g_0\frac{\tau_m - \tau_0}{\tau_m\tau_0}\left[1 + \tau\frac{1}{2}\frac{\tau_m + \tau_0}{\tau_m\tau_0} + \cdots\right], \tag{37}$$

$$g_0 = \frac{m_\pi^2}{\ln\frac{1 - m_\pi^2/\tau_m}{1 - m_\pi^2/\tau_0}}. \tag{38}$$

The terms of zeroth order then give the intercepts P_0 and

G_0 of the monopole and the modified FF, respectively, entering the GTR:

$$P_0 = \frac{\lambda^2 - m_\pi^2}{\lambda^2}, \tag{39}$$

$$G_0 = \frac{\tau_m - \tau_0}{\tau_m \tau_0} \frac{m_\pi^2}{\ln\left(\frac{\tau_0}{\tau_0 - m_\pi^2}\frac{\tau_m - m_\pi^2}{\tau_m}\right)}. \tag{40}$$

Here the first difference in the features of the considered FF's appears. The monopole FF can in principle tune any, even unphysical, value of P_0 from zero to unity. In contrast to this, expression (40) predicts a very narrow interval of G_0 values from $G_0^0 = 8/9$ to $G_0^\infty = [9\ln(8/9)]^{-1}$:

$$0.889 \le G_0 \le 0.943, \tag{41}$$

spanned under the variation of τ_m from τ_0 to infinity.

The 6% discrepancy of GTR deduced in Refs. [15, 16] by analyzing the chiral limits of all four quantities entering the RHS of Eq. (32) corresponds to $G_0 \approx 0.94$. This at least means that there is no obvious disagreement of assumptions underlying the FF (21) with the approach of chiral perturbation theory. When compared with the value of $G_0 = 0.954 \pm 0.011$ given by the GTR (32) at the standard value of $g_{\pi N} = 13.40 \pm 0.08$, the interval (41) is found to be at the edge of consistency. The discrepancy of GTR is lower at lower coupling — this has been already discussed in Ref. [17] . Thus, in terms of the GT relation, the lower value [2–7] of the coupling constant at the present level of precision probably reflects the importance of contributions beyond the 3π cut.

By virtue of relations (39) and (40) it is easy to express the positions of the effective pole in terms of the cutoff parameter

$$\lambda^2 = m_\pi^2 \left(1 - g_0 \frac{\tau_m - \tau_0}{\tau_m \tau_0}\right)^{-1} \tag{42}$$

and to find the interval of λ describing the same region (41):

$$3m_\pi \le \lambda \le 4.20 m_\pi \tag{43}$$

or

$$0.414 \text{ GeV} \le \lambda \le 0.580 \text{ GeV}. \tag{44}$$

We would like to postpone the discussion how this meets the present experimental information on the position of the effective pole, in order to gather more details to be discussed.

Now let us compare the FF's (21) and (23) in terms of relation (35), which requires

$$\frac{G'(0)}{G(0)} = \frac{P'(0)}{P(0)}. \tag{45}$$

The solution of the latter equation for λ,

$$\lambda^2 = \frac{2\tau_m \tau_0}{\tau_m + \tau_0}, \tag{46}$$

is again remarkable since it maps all the allowed regions of τ_m, $\tau_0 \le \tau_m \le \infty$, into the small enough interval of the positions of the effective pole:

$$3m_\pi \le \lambda \le 3\sqrt{2} m_\pi \tag{47}$$

or

$$0.414 \text{ GeV} \le \lambda \le 0.585 \text{ GeV}. \tag{48}$$

This interval is practically the same as (43). The analogous calculation for the effective mass Λ_A of the dipole (24) describing the axial vector FF which enters the RHS of (35) provides

$$\Lambda_A^2 = \frac{4\tau_m \tau_0}{\tau_m + \tau_0}, \tag{49}$$

$$3\sqrt{2} m_\pi \le \Lambda_A \le 6 m_\pi, \tag{50}$$

or

$$0.585 \text{ GeV} \le \Lambda_A \le 0.828 \text{ GeV}. \tag{51}$$

A straightforward comparison of this interval with the experimental value [13]

$$\Lambda_A = (1.032 \pm 0.036) \text{ GeV}, \tag{52}$$

results in the original impression that the prediction (50) is inconsistent with the present experimental information. To get some ideas on a possible interpretation of this contradiction, let us consider the properties of FF's in the asymptotic spacelike region.

VI. LARGE MOMENTUM TRANSFERS

In the asymptotic region the exponential and the dipole FF's do not allow a comparison with the FF (21). When $|\tau| \gg \lambda$ and $|\tau| \gg \tau_m$, by equating the coefficients of the leading $1/(-\tau)$ terms of expressions (21) and (23), one obtains

$$\lambda^2 = \frac{m_\pi^2 \ln[(\tau_m - m_\pi^2)/(\tau_0 - m_\pi^2)]}{\ln\{[\tau_0/(\tau_0 - m_\pi^2)][(\tau_m - m_\pi^2)/\tau_m]\}}. \tag{53}$$

Now the range of λ values spanned under the variation of τ_m from τ_0 to infinity is

$$3m_\pi \le \lambda \le \infty. \tag{54}$$

Contrary to the previous relations (46) and (42), where the singular character of mapping did not allow us to estimate the cutoff parameter τ_m from the value of the effective mass exceeding the allowed regions (43) and (47), relation (53) is helpful for getting an impression how large the values of τ_m might be. For the purpose of quick reference, we present a simple table reproducing relation (53) at some points:

λ (GeV)	0.414	0.450	0.500	0.600	0.700	0.800
$\sqrt{\tau_m}$	$3m_\pi$	$3.6m_\pi$	$4.5m_\pi$	$7.3m_\pi$	$12m_\pi$	$20m_\pi$

The straightforward use of the common value [3, 18–20]

$\lambda \approx 0.800$ GeV provides the value of the cutoff τ_m as large as $\tau_m \approx (20m_\pi)^2$.

The more moderate value $\lambda \approx 0.730$ GeV deduced in Ref. [15] from the axial vector FF $g_A(q^2)$ [13] corresponds to $\tau_m \approx (13.7m_\pi)^2 \approx (2m_N)^2$. Probably the lowest possible effective mass is given in Ref. [21]. The value

$$\lambda \approx 3.16m_\pi \;\Rightarrow\; \tau_m \approx 3.3m_\pi \tag{55}$$

is obtained by fitting the data on the $pp \to pp\pi$ reaction at 800 MeV. It is precisely pointing to the region where the monopole approximation (26) is being approved.

It is premature to make a definite statement on the value of τ_m as an experimental quantity: This should rather be made by independent groups possessing the original data. In this paper we would like to outline the ambiguities of the direct translation of any form-factor parameter to the effective mass of the monopole to be used in (53).

The origin of a possible confusion might be clarified by examining the properties of the FF (21), which has a real chance to be closer to reality than the other parametrizations discussed here. Namely, when approximating expression (21) with a monopole ansatz (23), in different regions one inevitably obtains larger values for the effective mass at larger scale of $(-\tau)$ than that obtained at small τ .

The above mentioned value $\lambda \approx 0.730$ GeV is obtained in Ref. [15] from the dipole parameter Λ_A of the axial vector FF $g_A(q^2)$ as

$$\lambda = \Lambda_A/\sqrt{2}. \tag{56}$$

In view of relation (35), this is quite correct for values obtained for small τ. However, this is not the case for the special example in question: The value (52) is obtained by fitting the data at the scale of τ up to $\tau \approx -(1\text{ GeV})^2$.

Certainly, the principal way is to make separate fittings for every choice of the FF. Since this has not yet been done, one might look for an approximate rule for quick estimations.

To do this, let us examine the dispersion representation (1). Suppose that the integral of the imaginary part can be approximated by a dipole expression with the effective mass Λ placed somewhere nearby the left end of the cut and that the remainder allows the approximation of the same kind, but with the position of its effective mass somewhere twice further.

Then the fitting by a single-term dipole at $(-\Lambda^2) \leq \tau \leq (-\tau_0)$ should reproduce the value of the effective mass corresponding to the closest term and the value will become larger at larger scale.

With the rough empirical rule that the effective mass Λ is obtained at the scale $\tau = -\Lambda^2$, the prescript for a comparison of the dipole and monopole effective masses

$$\frac{[\Lambda^2 - m_\pi^2]^2}{[\Lambda^2 - (-\Lambda^2)]^2} \approx \frac{\lambda^2 - m_\pi^2}{\lambda^2 - (-\Lambda^2)} \tag{57}$$

should be

$$\lambda \approx \Lambda_A/\sqrt{3} \tag{58}$$

rather than (56).

Following the conjecture of Ref. [15] that $g_A(q^2)$ and the $\pi N\bar{N}$ form factor should be of the same shape and substituting the value $\Lambda_A/\sqrt{3}$ from (52) into relation (53), one obtains

$$\lambda \approx 0.596\text{ GeV}, \quad \tau_m \approx (7.3m_\pi)^2. \tag{59}$$

The continuation into the small τ region then results in the relative slope of $g_A(q^2)$ in (35) provided by the dipole effective mass

$$\Lambda_A^0 \approx 0.765\text{ GeV} \quad (\lambda^0 \approx 0.540\text{ GeV}). \tag{60}$$

This is almost by 25% smaller than (52) and practically consistent with the derivations of theoretical models, such as the Skyrme model (see, for example, Ref. [22]).

It goes without saying that only the direct fittings of the data as precise as possible at small τ can help to avoid the uncertainties of the presented speculations.

VII. CONCLUSIONS

In this paper we have derived a one-parameter expression for the $\pi N\bar{N}$ form factor, the only input information being the existence of the cut in the timelike region, the known position of its branch point corresponding to the 3π intermediate state, and the assumption that the FF has no hard core and that the cut has finite length.

The remarkable feature of the discussed FF in the region of small momentum transfers is the stability of its prediction in respect to the variation of the parameter (effective cutoff): $G(0)$ is allowed to vary only within 5.5%. This means in particular that the departure from the interval (41) might be used to probe the physics going beyond the assumptions listed above (provided the effect sufficiently exceeds experimental errors).

Another property of the FF (21) is better formulated in terms of the effective mass of the simple pole interpolating the FF (21) in the given range of momentum transfer. Namely, the effective mass determined in the region of large momentum transfers is significantly larger than the one at small momentum transfers. This property might help to understand how the predictions of theoretical models (usually derived from the theory of chiral pions and, therefore, restricted to rather small momenta) can meet the experimental information (which is more accurate at larger energies and momentum transfers).

The incomplete list of problems being at present under study and strongly relying on the $\pi N\bar{N}$ coupling constant and/or on the $\pi N\bar{N}$ form factor includes (i) pion-nucleon elastic and inelastic scattering, (ii) nucleon-nucleon one-pion-exchange potential, (iii) few-nucleon bound states (the S/D ratio of deuterium), (iv) pion photoproduction and electroproduction, and (v) weak πN coupling.

The FF (21), discussed in this paper, can in principle help to avoid some of the known ambiguities in the above mentioned problems or at least to outline the relevance of the $\pi N\bar{N}$ coupling and/or FF to the problem in question.

ACKNOWLEDGMENTS

Correspondence and discussions with I.I. Strakovsky, R. Timmermans, and R.L. Workman are gratefully acknowledged. This research was supported by the Croatian Ministry of Science and Technology. One of us (A.A.B.) thanks the Open Society Foundation for a travel grant.

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