Nolen-Schiffer anomaly of mirror nuclei and charge symmetry breaking in nuclear interactions

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The charge symmetry-breaking potentials including the effects of pseudoscalar (π-η) and (π-η'), and vector (ρ-ω) meson in a series of mirror nuclei are examined. The computed Coulomb energy differences along with the present charge symmetry-breaking effects provide a reasonably accurate description of the binding energy differences between mirror nuclei.

I. INTRODUCTION

The study of symmetry properties in nuclear forces and small deviations from symmetries has always revealed important information about the nuclear interactions. In particular, the validity of isospin invariance (after removing the electromagnetic effects mainly the Coulomb force) is of interest. Now it is well known that the charge independence (CI) is violated [1]. This is due primarily to the mass difference of neutral and charged pions. However, the magnitude of deviation of the nuclear forces from charge symmetry (CS) is not well understood.

During the last decade, a considerable amount of work has been carried out regarding the calculation of *Coulomb energy differences* in order to resolve the persistent discrepancy between theory and experiment for a wide range of nuclei, known as the Okamoto-Nolen-Schiffer anomaly. In testing for charge symmetry breaking one may compare the low energy scattering parameters, such as scattering length, for neutron-neutron scattering with those for proton-proton scattering after correction for the electromagnetic effects. The results are not in agreement within their error bars (Table I) [2]. Charge symmetry requires invariance under charge reflection in the x-y plane of isospin space. The charge symmetry operator $P_{\rm CS}$ is defined by [6]

$$P_{\rm CS} = e^{i\pi T_y} = \prod_{i=1}^{A} e^{i\pi T_y(i)}, \tag{1}$$

where

$$T_y = \sum_{i=1}^{A} T_y(i). \tag{2}$$

TABLE I. Experimental results for scattering length in NN scattering.

	Scattering length[a]
nn $^2H(\pi^-,\gamma)2n$	-18.7 ± 0.6[a]
pp	-7.828 ± 0.008[b]
pp, corrected to Coulomb	-17.1 ± 0.2[b]
pp, corrected to Coulomb	~ -17.9[c]

[a] Reference [3].
[b] Reference [4].
[c] Reference [5].

The necessary condition for charge symmetry is

$$[P_{\rm CS}, H] = 0. \tag{3}$$

This implies invariance of interaction through the exchange of neutrons to protons and vice versa. Mirror nuclei provide another source of information in testing of this invariance. Following Henley and Miller [6], the nucleon-nucleon interaction can be divided into four classes. Class I interactions include charge-independent interactions, while chrage-dependent but charge symmetric interactions are categorized in class II. The charge independence and charge symmetry are violated in class III and IV interactions. The latter also causes isospin mixing.

In the present work we investigate the effects of CSB interactions on the binding energy differences of mirror nuclei, ^{15}O-^{15}N, ^{17}F-^{17}O, ^{39}Ca-^{39}K, ^{41}Sc-^{41}Ca. The effects of ρ^0-ω mixing, π^0-η mixing, and two-pion exchange contributions have been studied for a finite nuclei by Blunden and Iqbal [7]. They found that a treatment of the meson exchange picture describes reasonably well the difference in $^1S_0$ scattering length $\triangle a = a_{pp} - a_{nn}$ and $\delta a = a_{nn} - a_{np}$. It can also explain about $\frac{1}{2} - \frac{3}{4}$ of the anomalies of the Coulomb energy differences in mirror nuclei. Coon and Barrett showed that CSB potential due to ρ-ω mixing is about 140% stronger than previous estimates. They also showed that the effect of this source of CSB potential on the scattering length difference $\triangle a =\mid a_{nn} \mid - \mid a_{pp} \mid$ is $\triangle a \sim 1$ fm, in good agreement with the experimental findings of $\triangle a \sim 1.4 \pm 0.8$ fm.

II. CHARGE SYMMETRY-BREAKING INTERACTIONS

It is well known that the binding energy differences in mirror nuclei mainly arise from long-range interactions, namely electromagnetic interactions. There is still a discrepancy between the theory and the experiment which is attributed to the short-range CSB interactions. The total binding energy difference of analog states can be written as

$$\triangle E = \triangle E_{\rm el} + \triangle E_{\rm CSB} \ . \tag{4}$$

The contribution of the electromagnetic portion of the binding energy differences in a series of mirror nuclei are given in Table II [8–12]. The recently calculated results [12] depend on the recent experimental information on nuclear charge densities [13]. The results are given for different sets of Skyrme interactions SII, SIII, and SGII [14] and the Woods-Saxon (WS) potential [15,16].

Charge symmetry-breaking interactions which include the contributions of class III and class IV CSB forces arising from ρ-ω mixing, π-η mixing, and π-η' mixing are considered in our calculations. The main contribution comes from the ρ-ω mixing term. The CSB interaction from ρ-ω, π-η, and π-η' mixing can be expressed as follows:

$$V_{\pi\eta}(r) = -(\tau_1^z + \tau_2^z)\frac{g_\pi g_\eta}{4\pi}\chi_{\pi\eta}\left[V(\mu_\pi, r) - V(\mu_\eta, r)\right] , \tag{5}$$

$$V_{\pi\eta'}(r) = V_{\pi\eta}(r)(\eta \to \eta'), \tag{6}$$

$$\begin{aligned} V_{\rho\omega}(r) = &-(\tau_1^z + \tau_2^z)\frac{g_\rho g_\omega}{4\pi}\chi_{\rho\omega}\left[V_+(\mu_\rho, r) - V_+(\mu_\omega, r)\right] \\ &-(\tau_1^z - \tau_2^z)\frac{g_\rho g_\omega}{4\pi}\chi_{\rho\omega}\left[V_-(\mu_\rho, r) - V_-(\mu_\omega, r)\right], \end{aligned} \tag{7}$$

where

$$\phi(x) \equiv \frac{e^{-x}}{x}, \tag{8}$$

$$\chi(x) \equiv \frac{1}{3}\left[1 + \frac{3}{x} + \frac{3}{x^2}\right]\phi(x), \tag{9}$$

$$\lambda(x) \equiv \left[\frac{1}{x} + \frac{1}{x^2}\right]\phi(x), \tag{10}$$

$$S_{12} \equiv 3(\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_2 \cdot \hat{r}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2, \tag{11}$$

$$\chi_{ab} \equiv \frac{\mu_{ab}^2}{\mu_b^2 - \mu_a^2}. \tag{12}$$

g_π, g_η, g_ρ, and g_ω are the coupling constants and χ_{ab} is the mixing angle.

In Eqs. (5) and (6) for both Henley-Miller (H-M) and Langacker-Sparrow (L-S) pseudoscalar potential we have

$$V(\mu, r) \equiv \frac{\mu^3}{4M^2}\left[\frac{1}{3}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)\phi(\mu r) + S_{12}\,\chi(\mu r)\right], \tag{13}$$

while the ρ-ω interaction for H-M and L-S CSB interactions is given as

$$\begin{aligned} V_{+\mathrm{HM}}(\mu, r) \equiv\ &\mu\left[1 + K^\rho\frac{\mu^2}{4M^2}\right]\phi(\mu r) - \mu\left[\frac{\mu^2}{2M^2}(3 + 2K^\rho)\right]\vec{L}\cdot\vec{S}\lambda(\mu r) \\ &+\mu\left[\frac{\mu^2}{4M^2}(1 + K^\rho)\right]\left[\frac{2}{3}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)\phi(\mu r) - S_{12}\chi(\mu r)\right] , \end{aligned} \tag{14}$$

$$V_{-\mathrm{HM}}(\mu, r) \equiv -\frac{1}{2}\frac{\mu^3}{M^2}K^\rho(\vec{\sigma}_1 - \vec{\sigma}_2)\cdot\vec{L}\lambda(\mu r), \tag{15}$$

TABLE II. Coulomb energy shift and correction terms (in MeV).

Contribution	^{15}O-^{15}N $1p_{\frac{1}{2}}^{-1}$	^{17}F-^{17}O $1d_{\frac{5}{2}}$	^{39}Ca-^{39}K $1d_{\frac{3}{2}}^{-1}$	^{41}Sc-^{41}Ca $1f_{\frac{7}{2}}$
Total (calculated)[a]	3.22	3.25	6.81	6.75
Total (calculated)[b]	3.44	3.32	7.11	6.63
Total (calculated)[c]	3.38	3.23	7.08	6.69
Total (calculated)[d] DME	3.180	3.200	6.895	6.790
Total (calculated)[d] SKII	3.270	3.305	7.000	6.875
Total (calculated)[e] SII	3.347	3.338	6.961	6.754
SIII	3.312	3.433	6.942	6.924
SGII	3.325	3.433	6.990	6.970
WS		3.407		6.954
Experimental	3.536	3.542	7.313	7.278

[a] Reference [8].
[b] Reference [9].
[c] Reference [10].
[d] Reference [11].
[e] Reference [12].

TABLE III. Contribution (in MeV) of Henley-Miller and Langacker-Sparrow CSB potentials to the energy difference of ground states of mirror nuclei.

	$V_{ps}^{\pi\eta}$		$V_{ps}^{\pi\eta'}$		$V_{v}^{\rho\omega}$		Total		
Items	OB	TH	OB	TH	OB	VMD	Case I, II, III		
^{15}O-^{15}N[a]	0.147	0.085			0.314	0.277	0.461	0.424	0.399
^{17}F-^{17}O[a]	0.100	0.058			0.157	0.155	0.257	0.255	0.215
^{39}Ca-^{39}K[a]	0.157	0.092			0.362	0.319	0.519	0.476	0.454
^{41}Sc-^{41}Ca[a]	0.117	0.068			0.201	0.196	0.318	0.313	0.269
^{15}O-^{15}N[b]	0.147	0.085	0.056	0.030	0.343	0.274	0.546	0.477	0.458
^{17}F-^{17}O[b]	0.100	0.058	0.039	0.021	0.180	0.153	0.319	0.293	0.256
^{39}Ca-^{39}K[b]	0.157	0.092	0.061	0.032	0.387	0.308	0.606	0.527	0.511
^{41}Sc-^{41}Ca[b]	0.117	0.068	0.046	0.024	0.223	0.189	0.386	0.352	0.316

[a] Henley-Miller CSB potential [6].
[b] Langacker-Sparrow CSB potential [17].

and

$$
\begin{aligned}
V_{+\mathrm{LS}}(\mu,r) \equiv{}& \mu\left[\left[1+\frac{\mu^2}{8M^2}\right]+K^{\Sigma}\frac{\mu^2}{4M^2}+K^{\Pi}\frac{\mu^4}{16M^4}\right]\phi(\mu r)-\mu\left[\frac{3\mu^2}{2M^2}+K^{\Sigma}\frac{\mu^2}{M^2}+K^{\Pi}\frac{3\mu^4}{8M^4}\right]\vec{L}\cdot\vec{S}\lambda(\mu r)\\
&+\mu\left[\frac{\mu^2}{4M^2}+K^{\Sigma}\frac{\mu^2}{4M^2}+K^{\Pi}\frac{\mu^2}{4M^2}\left[1+\frac{\mu^2}{8M^2}\right]\right]\left[\frac{2}{3}(\vec{\sigma}_1\cdot\vec{\sigma}_2)\phi(\mu r)-S_{12}\chi(\mu r)\right]\\
&+3\mu\left[\frac{\mu^4}{16M^4}+K^{\Sigma}\frac{\mu^4}{4M^4}+K^{\Pi}\frac{\mu^4}{2M^4}\right]Q_{12}\frac{\chi(\mu r)}{\mu^2r^2},
\end{aligned}
\tag{16}
$$

$$
V_{-\mathrm{LS}}(\mu,r) \equiv \frac{1}{2}\frac{\mu^3}{M^2}(K^{\omega}-K^{\rho})(\vec{\sigma}_1-\vec{\sigma}_2)\cdot\vec{L}\lambda(\mu r). \tag{17}
$$

In Eqs. (16) and (17),

$$K^{\Sigma} \equiv K^{\omega}+K^{\rho} \tag{18}$$

$$K^{\Pi} \equiv K^{\omega}K^{\rho}\ , \tag{19}$$

where K^{ω} and K^{ρ} are the isovector magnetic couplings. The quadrature term in our calculation is neglected.

III. RESULTS

Following Langacker and Sparrow [17] we present our results in terms of competing parameter sets, namely *case I*, *case II*, and *case III*. Our results for the HM and LS charge symmetry-breaking potentials are tabulated in Table III and compared with the calculated and experimental values of the *Coulomb displacement energies*. Three different sets of parameters are as follows.

(i) Use of the one-boson (OB) couplings for both pseudoscalar and vector mesons. In this approach the g_{η}^{OB} and $g_{\eta'}^{\mathrm{OB}}$ values are obtained from fits [18–21] of the one-boson exchange potential (OBEP) to the nucleon-nucleon and hyperon-nucleon scattering data (the latter is needed to separate the η and η' contributions). This type of determination, although based directly on experimental results, relies heavily on the validity of specific (OBEP) models for the isospin conserving potential. One of the difficulties is that the parameters extracted include the effects of particle exchanges that have not been explicitly included in the model.

(ii) Use of the one-boson (OB) couplings for the pseudoscalar and vector meson dominance (VMD) for the vector mesons.

(iii) Use of the theoretical (TH) couplings for pseudoscalar mesons and the one-boson (OB) couplings for the vector mesons.

The parameters for the mentioned cases are listed in Table IV. In all of these three cases, we use the OBEP

TABLE IV. The parameters used in the isospin violating potential for the cases I, II, and III. TH, VMD, and OB refer to estimates derived from theory [SU(3), SU(6), Zweig rule], vector meson dominance, and one-boson exchange potential fits (to the isospin conserving potential), respectively.

$\mu^2_{\pi\eta}$ (GeV^2)	$\mu^2_{\pi\eta'}$ (GeV^2)	$\mu^2_{\rho\omega}$ (GeV^2)	$\chi_{\pi\eta}$	$\chi_{\pi\eta'}$	$\chi_{\rho\omega}$
-0.0036	-0.0035	-0.0037	-0.013	-0.0039	-0.1837
Case	$\frac{g_\pi g_\eta}{4\pi}$	$\frac{g_\pi g_{\eta'}}{4\pi}$	$\frac{g_\rho g_\omega}{4\pi}$	K^{ρ}	K^{ω}
I	10.32^{OB}	11.12^{OB}	2.80^{OB}	6.60^{OB}	0.655^{OB}
II	10.32^{OB}	11.12^{OB}	2.80^{OB}	3.7^{VMD}	-0.12^{VMD}
III	6.01^{TH}	5.87^{TH}	2.80^{OB}	6.60^{OB}	0.655^{OB}

TABLE V. Contribution (in MeV) of HM and LS CSB potential to the energy difference of ground states of mirror nuclei and dominant discrepancies.

Items	Discrepancies[a]				Discrepancies[b]	
	SII	SIII	SGII	WS	DME	SKII
^{15}O-^{15}N	0.189	0.224	0.211		0.356	0.266
^{17}F-^{17}O	0.204	0.109	0.109	0.135	0.342	0.237
^{39}Ca-^{39}K	0.352	0.371	0.323		0.418	0.313
^{41}Sc-^{41}Ca	0.524	0.354	0.308	0.324	0.488	0.403

Items	CSB Contribution HM			CSB Contribution LS		
	Case I	II	III	Case I	II	III
^{15}O-^{15}N	0.46	0.42	0.40	0.55	0.48	0.46
^{17}F-^{17}O	0.26	0.26	0.22	0.32	0.29	0.26
^{39}Ca-^{39}K	0.52	0.48	0.45	0.61	0.53	0.51
^{41}Sc-^{41}Ca	0.32	0.31	0.27	0.39	0.35	0.32

[a] Reference [12].
[b] Reference [11].

result for g_ρ and g_ω [18–21], which is compatible with vector meson dominance (VMD) for the electromagnetic charge form factors when the small ϕ_{NN} coupling is taken into account [19–21].

Table V presents a comparison between our results and the persistent discrepancies. In the evaluation of the CSB effects in mirror nuclei we have used harmonic oscillator wave functions. All the contributions from the interactions except the quadrature term are considered in our calculations. Suzuki *et al.* [12] have considered contributions only from the central part of the ρ-ω interaction. Including other terms gives rise in some cases to an increment of 75% to the ρ-ω contribution. From this table it can be seen that reductions of Coulomb energy differences with the use of SGII interaction and WS potential compared with the values quoted before [8–10] are considerable. This provides a better consistency between the results of CSB interactions and the present discrepancies. The discrepancies are given for DME and SKII theories [11] and for three different sets of Skyrme interactions, SII, SIII and SGII and the WS potential [12]. In the nuclei with one particle above the closed shell SIII and WS calculations produce discrepancies smaller than those for the SII calculation, while this situation is reversed for the states with one hole below the closed shell.

IV. CONCLUSION

We examined the charge symmetry-breaking potentials due to the strong interaction proposed by Henley-Miller and Langacker-Sparrow in a series of mirror nuclei. Different sets of parameters are considered in our calculations for HM and LS CSB potentials [17].

A comparison between the CSB contribution and the discrepancies shows that the best agreement occurs in SKII and SGII [12] cases. The results are in general encouraging. We have also checked that the exclusion of the noncentral term may considerably affect the results. Much better agreement is expected to be obtained with correlated wave functions. Furthermore, there are considerable differences in the calculated *Coulomb energy differences* carried out by different authors [8–12]. It is clear, therefore, that any investigation of the charge dependence of a specific nuclear interaction must use refined methods to peer behind the masking effects of the electromagnetic contribution to the nucleon-nucleon interaction.

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