Can one simultaneously describe the deuteron properties and the nucleon-nucleon phase shifts in the quark cluster model?

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Deuteron properties have been calculated in the framework of the constituent quark model using a quark-quark interaction which incorporates σ and π chiral mesons besides the one-gluon exchange between quarks. This interaction is able to describe simultaneously the nucleon-nucleon phase shifts. In contrast with earlier calculations, in which equivalent meson potentials between nucleons have been used, this description includes only interactions between quarks.

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Our present understanding of hadrons as extended objects containing colored quarks and gluons suggests that the nuclear dynamics may be derived directly from these fundamental degrees of freedom. There are several advantages of such a description: First, quark antisymmetrization effects, which turn out to be very important to describe the short-range interaction, can be implemented in the model in a natural way. Second, this scheme works with a fundamental vertex at the quark level and therefore it is not necessary to use different vertex parameters for each baryon-baryon interaction. And finally, it allows the attempt of a unified description of the baryon structure and the baryon-baryon interaction. A problem appears to develop in this program: Despite the progress made in understanding the consequences of QCD, the complexity of the theory forces us to use QCD-inspired quark models. Among them, the nonrelativistic quark model (NRQM) has been quite successful in its description of single baryon properties. Presently, it is the only model that can be easily extended to study nuclear phenomena in terms of quark degrees of freedom [1–4].

This model includes one-gluon-exchange quark-quark interactions to simulate the perturbative contribution of the quark-gluon dynamics. The nonperturbative features of QCD have been included in different ways: Oka and Yazaki [1] and Takeuchi *et al.* [2] used the one-pion exchange between nucleons and a phenomenological intermediate-range attraction also between nucleons. The parameters of these interactions were fitted to the experimental data. Yamauchi, Yamamoto, and Wakamatsu [3] used a more realistic meson-exchange potential between nucleons with a configuration space cutoff. However, all these models are inconsistent because, as stated by Yamauchi *et al.* [3], they ignore the contribution of the meson cloud to the Δ-N mass difference. Introducing this contribution, the value of α_s becomes smaller (typically one-half of the primitive value), and the repulsion coming from the one-gluon-exchange (OGE) interaction could not be enough to account for the experimental data. Using meson exchanges at the level of quarks instead of at baryonic level allows to introduce in a natural way the effect of the mesonic cloud in the baryon self-energy. Furthermore, it was shown that the new effects connected with quark antisymmetrization on the mesonic exchanges are important [5]. This approach has been partially adopted by the Tübingen group [4]. They included the one-pion exchange between quarks but still used a phenomenological σ-like potential between baryons. Introducing the medium-range attraction in this way, several new parameters (m_σ, $g^2_{\sigma NN}$, ...) appear. These were unrelated to the other quark model parameters and, thus, they were fitted to the experimental data. However, previous calculations were not able to describe the deuteron binding energy and the scattering phase shifts using the same set of parameters, specifically with the same scalar coupling constant $g^2_{\sigma NN}/4\pi$ [4,6]. Moreover, two different σ-nucleon coupling constants were necessary to reproduce the S and higher partial wave phase shifts [4].

Recently, some progress has been made in formulating a microscopic σ-quark interaction that is based on the chiral symmetry and its dynamical breaking by the physical vacuum [7,8]. The starting point of the model is the assumption that the quark condensate, or conversely the dynamical quark mass, appears at a scale $\Lambda_{\rm CSB}(\equiv \Lambda)$ which is smaller than the confinement scale Λ_C [9]. Quarks acquire the dynamical mass as a consequence of the chiral symmetry breaking. Then, quark dynamics can be described by using a Nambu–Jona-Lasinio-type Lagrangian [10]

$$\mathcal{L} = \bar{q}\,(i\,\partial\!\!\!/ - m)\,q + G(q^2)\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2\right] , \qquad (1)$$

m being the current quark mass. After bosonization, the quark-quark Hamiltonian interaction can be written as

$$H_{\rm ch} = m_q\,\bar{q}q \,+\, g_{\rm ch}\,F(q^2)\,\bar{q}\,\left(\sigma + i\gamma_5\tau\pi\right) q\,, \qquad (2)$$

where m_q is the constituent quark mass and $g_{\rm ch}$ is the vertex coupling constant. Assuming that $F(q^2)$ can be parametrized as [11]

$$F(q^2) = \left[\frac{\Lambda^2}{\Lambda^2 + q^2}\right]^{1/2} , \qquad (3)$$

and working in the spirit of the linear sigma models, one is able to write immediately the one-pion-exchange (OPE) and one-sigma-exchange (OSE) potentials generated by the Hamiltonian of Eq. (2) as [8]

$$V_{\mathrm{OPE}}(\mathbf{r}_{ij}) = \frac{1}{3}\,\alpha_{\mathrm{ch}}\frac{\Lambda^2}{\Lambda^2 - m_\pi^2}\,m_\pi \left\{\left[Y(m_\pi\,\mathbf{r}_{ij}) - \frac{\Lambda^3}{m_\pi^3}\,Y(\Lambda\,\mathbf{r}_{ij})\right]\vec{\sigma}_i\cdot\vec{\sigma}_j + \left[H(m_\pi\,\mathbf{r}_{ij}) - \frac{\Lambda^3}{m_\pi^3}\,H(\Lambda\,\mathbf{r}_{ij})\right]S_{ij}\right\}\vec{\tau}_i\cdot\vec{\tau}_j\,, \tag{4}$$

$$V_{\mathrm{OSE}}(\mathbf{r}_{ij}) = -\alpha_{\mathrm{ch}}\,\frac{4\,m_q^2}{m_\pi^2}\,\frac{\Lambda^2}{\Lambda^2 - m_\sigma^2}\,m_\sigma\left[Y(m_\sigma\,\mathbf{r}_{ij}) - \frac{\Lambda}{m_\sigma}\,Y(\Lambda\,\mathbf{r}_{ij})\right]\,, \tag{5}$$

where the chiral coupling constant α_{ch} is related to the π-nucleon coupling constant $g^2_{\pi NN}$ [12] by

$$\alpha_{\mathrm{ch}} = \frac{g^2_{\mathrm{ch}}}{4\pi}\frac{m_\pi^2}{4m_q^2} = \left(\frac{3}{5}\right)^2\frac{g^2_{\pi NN}}{4\pi}\frac{m_\pi^2}{4m_N^2}\,. \tag{6}$$

Therefore, the scalar σ-quark coupling constant is automatically determined from the pseudoscalar π-quark one. Besides, the cutoff is the same for both mesons $\Lambda_\sigma = \Lambda_\pi$, and the mass of the sigma is determined through $m_\sigma^2 \approx (2m_q)^2 + m_\pi^2$ [7,8]. The cutoff mass Λ is bound to the range 1 GeV–600 MeV [13]. With these values, the Δ-N mass difference due to the OPE interaction ranges between 150 and 200 MeV, far from the experimental one (300 MeV). This means that the nonperturbative contributions are not enough to reproduce the Δ-N mass difference. The rest of the mass difference must have its origin in a perturbative process, namely the well-established one-gluon exchange (OGE) [14]

$$V_{\mathrm{OGE}}(\mathbf{r}_{ij}) = \frac{1}{4}\,\alpha_s\,\vec{\lambda}_i\cdot\vec{\lambda}_j\left\{\frac{1}{\mathbf{r}_{ij}} - \frac{\pi}{m_q^2}\left[1 + \frac{2}{3}\vec{\sigma}_i\cdot\vec{\sigma}_j\right]\delta(\mathbf{r}_{ij}) - \frac{3}{4m_q^2\,\mathbf{r}_{ij}^3}\,S_{ij}\right\}. \tag{7}$$

Fitting the constant α_s to reproduce the remaining mass difference between the Δ and the nucleon, one finds $\alpha_s \sim 0.4-0.5$. This is in much better agreement with the standard value for this energy regime than the value obtained by those quark models where the entire Δ-N mass difference is attributed to the OGE interaction ($\alpha_s \approx 1$). This procedure guarantees that the OPE and OGE interactions contribute to the hyperfine splitting of the hadrons in a fixed way. This avoids double counting problems in the $3q$ and $6q$ system. Finally, the quark Hamiltonian includes a confinement potential defined by

$$V_{\mathrm{con}}(\mathbf{r}_{ij}) = -a_c\,\vec{\lambda}_i\cdot\vec{\lambda}_j\,\mathbf{r}_{ij}^2\,, \tag{8}$$

where a_c is the confinement strength.

It is important to note that our model does not contain any massive vector meson exchange potentials (ρ, ω). These are known to be important in one-boson-exchange models. In these models, the ω meson provides the short-range repulsion of the NN interaction. In our model, the OGE combined with quark antisymmetrization takes over this task. Besides, the ρ meson reduces the strength of the tensorial pionic interaction. In our model, the quark exchange terms of the one-pion-exchange potential produce similar effects. This has been checked in charge-exchange reactions [15]. The inclusion of vector meson exchanges between quarks could lead to problems with double counting whereas there is no problem with exchanging scalar and pseudoscalar mesons between quarks [16].

The bound state and scattering problems of the two-nucleon system are formulated using the resonating group method (RGM) (see Refs. [1] and [8] for details). In the present study we work in the one-channel approximation.

In this framework we have studied the deuteron properties and the nucleon-nucleon phase shifts. To our knowledge this is the first quark-model calculation of the deuteron and the nucleon-nucleon phase shifts in the resonating group method that does not make use of a phenomenological σ-meson exchange potential $V_{\sigma NN}$ or an effective NN potential. It is therefore essential to test if in this model one can describe the deuteron, the NN scattering phase shifts and under which conditions one can do it simultaneously.

The way the parameters of the model are determined has been discussed in the literature. We follow the method given in Refs. [8,11], but instead of fixing b arbitrarily to the value 0.5 fm, we move it to reproduce the binding energy of the deuteron. The obtained parameters are shown in Table I.

In Table II we present numerical results for the low energy observables of the deuteron obtained with this parametrization. The agreement with the experimental data is very good and comparable to that obtained with sophisticated NN meson-exchange potentials using a lot of fitted parameters [17,18]. The S and D wave components of the renormalized wave function are shown in Fig. 1. For comparison, the wave functions obtained by the Paris group [18] are also shown. The wave functions of the two theories are very similar. The contribution of the nonlocal terms in the σ-exchange interaction allows, in contrast with former calculations, to reproduce simultaneously the NN phase shifts. The slight modification in the harmonic oscillator parameter b with respect to previous calculations does not affect these observables much, and we obtain nearly the same results as before in all the partial waves. Still some deviation is seen at low energies in the ${}^1S_0$ partial wave [8]. On the other hand, the present choice of the harmonic oscillator parameter allows us to describe the static deuteron properties. In Fig. 2 we show the ${}^3S_1$ and ${}^1P_1$ NN phase shifts again

TABLE I. Quark model parameters.

m_q (MeV)	313
b (fm)	0.518
α_s	0.485
a_c (MeV fm^{-2})	46.938
α_{ch}	0.027
m_σ (fm^{-1})	3.421
m_π (fm^{-1})	0.70
Λ_{CSB} (fm^{-1})	4.2

with a good agreement. Moreover, we are able to reproduce both phase shifts simultaneously using the same coupling constant for the scalar interaction in spite of being an S and a P wave [8].

Finally, we study the electromagnetic structure functions of the deuteron. These observables are of considerable interest because they contain information on the off-shell properties of the NN system not accessible by NN scattering phase shift measurements. Our description contains the impulse, isoscalar one-pion-exchange and one-gluon-exchange currents [6]. The main difference between the present microscopic calculation and previous ones in the quark cluster model lies in the deuteron wave function which has been calculated using only interactions between quarks. We reiterate that previous calculations used a deuteron wave function based on hybrid models [1–3,6]. We have calculated the structure functions $A(q^2)$ and $B(q^2)$. These two structure functions are experimentally available and can be obtained from the differential cross section of the unpolarized elastic e-d scattering

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left[A(q^2) + B(q^2)\tan^2\left(\frac{\theta}{2}\right)\right], \quad (9)$$

where

$$A(q^2) = F_C^2(q^2) + \frac{q^4}{18} F_Q^2(q^2) + \frac{q^2}{6M_N^2} F_M^2(q^2), \quad (10)$$

and

$$B(q^2) = \frac{q^2}{3M_N^2}\left(1 + \frac{q^2}{16M_N^2}\right) F_M^2(q^2). \quad (11)$$

Here, q denotes the four-momentum transfer of the virtual photon and θ the electron-scattering angle. The nucleon mass is denoted by M_N. A detailed description of this calculation together with the definition of the deuteron charge $F_C(q^2)$, quadrupole $F_Q(q^2)$, and magnetic dipole $F_M(q^2)$ form factors can be found in Ref. [6]. In Fig. 3 we show the calculated structure functions $A(q^2)$ and $B(q^2)$ using the parameters of Table I. The results obtained with the present model are again of the

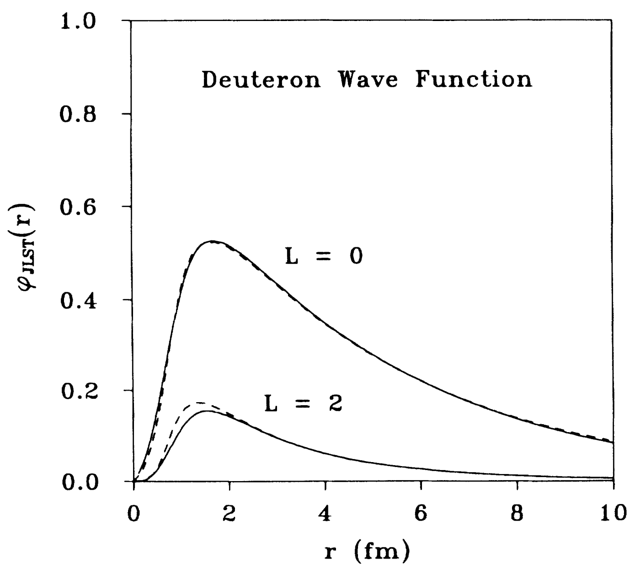

FIG. 1. The quark model wave functions of the deuteron (solid line) compared with those of the Paris group (dashed line) Ref. [18].

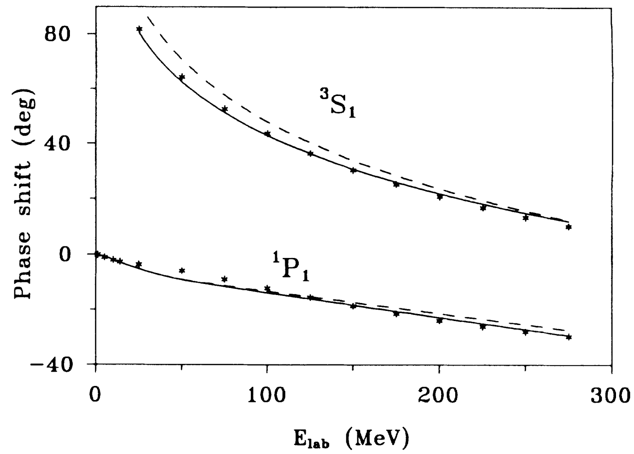

FIG. 2. $^3S_1$ and $^1P_1$ NN phase shifts as a function of the laboratory energy. The dashed line represents the results of Ref. [4] using a baryonic σ-like potential with two different scalar coupling constant fitted to the S waves ($g^2_{\sigma NN}/4\pi = 3.7$) or to the higher partial waves ($g^2_{\sigma NN}/4\pi = 1.9$). The solid line corresponds to the results of the present model using the chiral symmetry value ($g^2_{\sigma NN}/4\pi = 9\,\alpha_{\text{ch}}\,4m_q^2/m_\pi^2 = 5.0$). Experimental data are from Ref. [19].

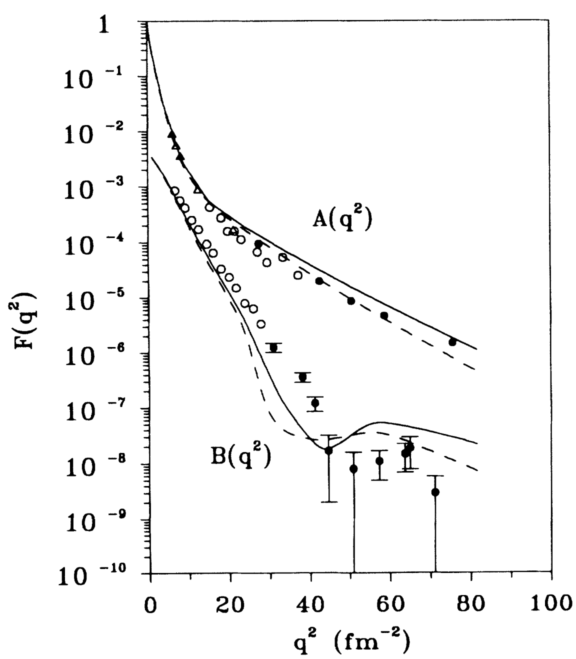

FIG. 3. Deuteron structure functions $A(q^2)$ and $B(q^2)$. The dashed line represents the results of Ref. [6] using a baryonic σ-like potential with the scalar coupling constant fitted to the deuteron binding energy ($g^2_{\sigma NN}/4\pi = 2.5$). The solid line corresponds to the results of the present model using the chiral symmetry value ($g^2_{\sigma NN}/4\pi = 9\,\alpha_{\text{ch}}\,4m_q^2/m_\pi^2 = 5.0$). Experimental data are taken from Ref. [6] and references therein.

TABLE II. Deuteron observables in comparison with the OBEP results [17], the Paris potential [18], and the experimental data [20]. For a recent determination of the deuteron matter radius see Ref. [22].

Deuteron	NRQM	Paris	OBEP	Exp.
E_D (MeV)	2.2246	2.2249	2.2246	2.22457
P_D (%)	4.91	5.77	4.99	···
A_S (fm$^{-\frac{1}{2}}$)	0.8765	0.8871	0.8860	0.8846
η	0.0261	0.0261	0.0264	0.0256
r_d (fm)	1.9657	1.9717	1.9688	1.963

same quality as those obtained using baryonic potentials with parameters fitted to the experimental data [6]. The quark exchange terms connected with $V_{\rm OSE}$ are mainly responsible for the improvement in the structure function B. The overestimation in structure function A is a well known problem connected with the strength of the pion-pair current [21].

In summary, the deuteron properties have been analyzed using a parametrization of the nucleon-nucleon interaction in terms of interactions between quarks, containing a minimal set of parameters. Unlike previous calculations where the intermediate-range part has been parametrized using available experimental input, we start here from a microscopic σ-quark interaction whose parameters are fixed by chiral symmetry arguments. The nonlocal scalar interaction generated by the quark exchange terms of the sigma-exchange potential together with the chiral symmetry requirements allow one to reproduce simultaneously to the properties of the deuteron, the NN phase shifts with the aforementioned accuracy. We are aware that most of the static properties of the deuteron only depends on the OPE and could be reproduced with an OPE-like potential with some fitted parameters [23]. However, one needs more ingredients to reproduce the scattering phase shifts and the form factors. These are provided by other mesons (like in the Bonn potential [17]) or by the combined effects of pion, sigma, and quark exchange [8]. This description of the nucleon-nucleon interaction in terms of interactions between quarks will open the possibility of studying quark effects in a wide spectrum of baryon-baryon interactions and electronuclear processes.

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