Multichannel dynamical symmetry and heavy ion resonances

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The concept of the multichannel dynamical symmetry is introduced. This symmetry may show up in an atomic nucleus due to its different cluster configurations, and connects the interactions of distinct reaction channels. The correlated distribution of different cluster states at low and high energies can serve as a signature of it. An application to the ^{28}Si nucleus is performed in terms of the ^{24}Mg+α and ^{12}C+^{16}O fragmentations.

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The concept of dynamical symmetry proved to be a fruitful tool of analyzing complex nuclear spectra [1]. A dynamical symmetry holds, if the Hamiltonian of the system can be expressed in terms of the invariant operators of a group chain

$$G \supset G' \supset G'' \supset \cdots . \tag{1}$$

Then the energy-eigenvalue problem has an analytical solution, and the expectation values of the other physical quantities can also be obtained in a simple way.

The purpose of this Brief Report is to point out the possibility of a new type of dynamical symmetry in atomic nuclei that is related to the cluster configurations, and has a multichannel character, i.e., it connects different fragmentations. The multichannel dynamical symmetry can be formulated in the framework of the semimicroscopic algebraic cluster model [2, 3]. In this approach the relative motion of the clusters is treated in terms of the vibron model [4], while their internal structures are accounted for by the SU(3) shell model [5]. The model space is constructed to be free from the Pauli forbidden states and from the spurious excitations of the center of mass; practically it is identical with that of the microscopic harmonic oscillator cluster model [6]. The interactions are phenomenological ones, which are obtained in terms of the generators of the dynamical group of the system. When only one of the clusters has internal structure (the other one is a doubly magic nucleus), the model has a $\mathrm{U}_C^{ST}(4)\otimes\mathrm{U}_C(3)\otimes\mathrm{U}_R(4)$ group structure, where C refers to cluster, R to relative motion, and $\mathrm{U}_C^{ST}(4)$ is Wigner's spin-isospin group [7]. Further simplification takes place if the cluster with internal structure is also a spin-isospin scalar. Then the dynamical symmetry of the model is characterized by the group chain:

$$\begin{array}{lllllll} \mathrm{U}_C(3) & \otimes\ \mathrm{U}_R(4) \supset \mathrm{U}_C(3)\otimes\mathrm{U}_R(3) & \supset & \mathrm{U}(3) \supset \mathrm{SU}(3) \supset \mathrm{O}(3) \supset \mathrm{O}(2) \\ |[n_1^C, n_2^C, n_3^C], [N,0,0,0], & [n_\pi,0,0,], & [n_1,n_2,n_3], & (\lambda,\mu), K\ , L & , M\ \rangle . \end{array} \tag{2}$$

This dynamical symmetry, when applied to a single cluster configuration, i.e., in a single channel form, seems to be a reasonable approximation to some core-plus-alpha-particle systems [2, 3]. A similar, but slightly more complicated U(3) dynamical symmetry fits well to some nonscalar [with respect to $\mathrm{U}_C^{ST}(4)$ or $\mathrm{U}_C(3)$] two-cluster configurations [8–10].

This symmetry can easily be generalized to a multichannel one, which has interesting new features. Not only that it associates seemingly different cluster configurations to the same nuclear state, but it gives a well-defined and simple relation between the phenomenological cluster-cluster interactions of different channels as well. The generalization is based on the following consideration.

The basis states defined by the quantum numbers of (2) are harmonic oscillator eigenfunctions. Since the Hamiltonians of the harmonic oscillator shell model and the harmonic oscillator cluster model can be transformed into each other exactly [6], their basis states have also a simple relation: any state of one of these models can be expressed as a linear combination of those basis states of the other model that belong to the same oscillator quantum number. This theorem can be used to get rid of the forbidden states in the nonantisymmetric cluster model basis, by requiring the matching of the quantum numbers of the states (2) with those of the totally antisymmetric shell model basis. For two-cluster systems, considered here, this means a matching between the representation labels of the combined U(3) group of (2), with the corresponding shell-model quantum numbers.

The interaction of the dynamical symmetry splits the degenaracies of the harmonic oscillator states, in fact the spectrum can be very far from being harmonic. Nevertheless, the U(3) representation labels are still good quantum numbers. Let us now consider the case of states that have U(3) quantum numbers with a single multiplicity both in the shell-model and in different cluster model bases. Due to the shell-model connection mentioned in the preceding paragraph, and the fact that the basis states of different U(3) labels are orthogonal, these wave functions are identical with each other. Therefore,

when the U(3) dynamical symmetry holds in a certain cluster configuration, and some (or all) of the single valued U(3) quantum numbers match with those of another cluster basis, then the corresponding cluster bands have to be present in the second configuration as well.

The energy eigenvalues corresponding to the dynamical symmetry (2) up to linear terms in n_π can be written as

$$E = \epsilon + \gamma n_\pi + \beta L(L+1) + \theta n_\pi L(L+1) + F(\lambda, \mu, L), \tag{3}$$

where ϵ, γ, β, and θ are parameters to be fitted to the experimental data, while $F(\lambda, \mu, L)$ is a function of the indicated quantum numbers. When we consider two different cluster configurations of a nucleus, both consisting of two clusters, one of them is inert, then the n_π quantum number depends on the fragmentation: n_{π_i}, $i = 1, 2$, while the λ, μ, and L quantum numbers do not. E and its parameters also carry the channel index i. If, however, several states are common in the two configurations because of the reasons discussed above, then their energies have to be the same. This circumstance establishes a straightforward transformation between the phenomenological interactions in Eq. (3). Let the relation of the relative motion quantum numbers be $n_{\pi_1} = n_{\pi_2} + n_{\pi_0}$. Then the

$$\gamma_1 = \gamma_2 = \gamma\ ,\ \epsilon_1 = \epsilon_2 + \gamma n_{\pi_0}\ ,\ \theta_1 = \theta_2 = \theta\ ,$$
$$\beta_1 = \beta_2 + \theta n_{\pi_0}\ ,\ F_1(\lambda, \mu, L) = F_2(\lambda, \mu, L) \tag{4}$$

equations have to hold. They guarantee the identical energies for the common states of the two fragmentations, and give a unique relation between the cluster-cluster interactions of two different configurations. Similar relations can be obtained between the matrix elements of other physical quantities.

When comparing two cluster configurations that need somewhat different formalisms [e.g., in one of them there are two non-$U_C(3)$-scalar clusters, or more than two clusters], the relations of the interactions are not so complete, as in Eq. (4), but they still provide us with nontrivial constraints. In such a case the multichannel dynamical symmetry is only a partial one [11], i.e., valid only for a subset of the states, not for all of them.

The multichannel dynamical symmetry is a consequence of the combination of two other symmetries, namely, the single channel U(3) dynamical symmetry of a certain cluster configuration and the antisymmetry of the total wave function. The latter transforms the wave function of one clusterization to that of another one.

Though in some applications of the semimicroscopic algebraic cluster model the possibility of various clusterizations of special nuclear states have been investigated [12, 13], the quantitative analyses of the experimental data have been carried out so far on the basis of the single channel symmetries [2, 8–10].

There is a heuristic prescription invented by Harvey [14] for describing the fusion and fission processes of light nuclei, which recipe is also based on the harmonic oscillator basis. If we consider the possible fragmentations of a given state [15, 16], then the physical content of Harvey's prescription and the U(3) selection rule of the semimicroscopic algebraic cluster model is very similar (for a detailed discussion, see Refs. [12, 13]). The essential difference is that the multichannel dynamical symmetry, as presented here, involves a spectrum generating algebra too. Therefore, in addition to accounting for the symmetries of the states it also gives the spectra of different clusterizations, which is a completely new feature in comparison with the applications of Harvey's prescription [15, 16].

As an example for the application of the multichannel dynamical symmetry, I consider here the ^{28}Si nucleus in terms of the ^{24}Mg+α and ^{12}C+^{16}O fragmentations. The reasons for choosing this example are the following. (i) SU(3) quantum numbers were associated to several low-lying bands of the ^{28}Si [17], without any reference to the cluster configurations. (ii) The spectrum of these states could be reproduced approximately by supposing a ^{24}Mg+α configuration within the U(3) (single channel) dynamical symmetry scheme [2]. (iii) Both in the ^{24}Mg+α and in the ^{12}C+^{16}O reactions resonances were found in high-lying energy regions [18, 19].

The specific question I address is whether or not the energy spectrum of the high-lying ^{12}C+^{16}O resonances can be described together with the low-lying ^{24}Mg+α bands within the framework of the multichannel dynamical symmetry (2,3,4). The method of this description is very constrained in comparison with those of the phenomenological models (both algebraic and geometric); therefore, this calculation involves much less ambiguity. Nevertheless, the band structure in the highly excited region is not well known, so in order to conclude about the presence of the multichannel dynamical symmetry one has to investigate some global characteristics, like the densities of different cluster states, or number of resonances in the energy and angular-momentum windows defined by the experiments. I make also this kind of comparison.

In the present description the ^{12}C, ^{16}O, ^{24}Mg, and α are considered to have [4,4,0], [4,4,4], [16,8,4], and [0,0,0] $U_C(3)$ quantum numbers, respectively. Each of them is treated as a spin-isospin scalar. The ^{12}C+^{16}O and the ^{24}Mg+α model spaces are defined by requiring the matching between the combined U(3) representation labels of (2) with the corresponding shell model quantum numbers. The sets of basis states obtained this way coincide exactly with the ones determined from the truly microscopic cluster model calculations [20]. The relation of the relative motion quantum numbers is: $n_{\pi_{\rm CO}} = n_{\pi_{\rm Mg\alpha}} + 8$.

From among the low-lying states of ^{28}Si those were considered for which the band structure is known from previous investigations [17] (Fig. 1). The ^{12}C+^{16}O resonances of Fig. 2 are from Ref. [19], and they are arranged into collective bands according to the relative energy differences and the band structure of the model [2, 3]. The model spectra were obtained from the energy formula:

$$E = \epsilon + \gamma n_\pi + \beta L(L+1) + \theta n_\pi L(L+1) + \phi_1 C_2 + \phi_2 C_3 + (\phi_3 C_2 + \phi_4 C_3) L(L+1), \tag{5}$$

where the expectation values of the second- and third-order Casimir operators of the SU(3) group are $C_2 = \lambda^2+\mu^2+\lambda\mu+3(\lambda+\mu), C_3 = (\lambda-\mu)(\lambda+2\mu+3)(2\lambda+2\mu+3)$. The constraints of the multichannel dynamical symmetry (4) were taken into account. A least-squares fitting procedure was carried out, and each state had a weight factor of $1/E_{\rm expt}$ (without dimension) in order to pay more attention to the better-known low-energy part of the spectrum (except for the ground state, which had a unit weight). The parameters (in MeV) corresponding to the model spectra shown in the figures are $\epsilon_{\rm Mg\alpha} = -35.53$, $\epsilon_{\rm CO} = -87.66$, $\gamma_{\rm Mg\alpha} = \gamma_{\rm CO} = 6.52$, $\beta_{\rm Mg\alpha} = 0.131$, $\beta_{\rm CO} = 0.216$, $\theta_{\rm Mg\alpha} = \theta_{\rm CO} = -0.011$, $\phi_{1_{\rm Mg\alpha}} = \phi_{1_{\rm CO}} = -0.072$, $\phi_{2_{\rm Mg\alpha}} = \phi_{2_{\rm CO}} = 0.000\,064$, $\phi_{3_{\rm Mg\alpha}} = \phi_{3_{\rm CO}} = 0.000\,51$, $\phi_{4_{\rm Mg\alpha}} = \phi_{4_{\rm CO}} = -0.000\,008\,9$. In Fig. 2 only the natural parity states are indicated, since the ^{12}C+^{16}O reactions cannot populate others. The model spectrum is shown for each band only in the energy and angular-momentum window similar to that of the experiment. Three bands of the model spectrum in Fig. 1 are also present in the ^{12}C+^{16}O spectrum. They are the ones with the quantum numbers $(12,0)0^+$, $(14,1)1^-$, $(16,2)0^+$. It is appropriate to note here, however, that the other low-lying bands of the ^{28}Si nucleus cannot be described in terms of the ^{12}C+^{16}O clusterization. The situation is different from case of the ^{24}Mg, where the whole low-lying region can be interpreted in terms of the the ^{12}C+^{12}C configuration [9]. This finding is in line with the experimental observation, that no ^{12}C(g.s.)+^{16}O(g.s.) fission of the ^{28}Si takes

TABLE I. The numbers of states of the indicated angular momenta and natural parities in the energy windows defined by the experimental observations. The experimental data are compared with the results of the model calculation for the ^{24}Mg+α and ^{12}C+^{16}O clusterizations.

Reaction	L	$N_{\rm expt}$	$N_{\rm Mg\alpha}$	$N_{\rm CO}$
^{24}Mg+α	0	4	1	0
^{24}Mg+α	1	7	10	1
^{24}Mg+α	2	17	15	2
^{12}C+^{16}O	8	6	42	6
^{12}C+^{16}O	9	6	32	2
^{12}C+^{16}O	10	5	27	3
^{12}C+^{16}O	11	3	30	5
^{12}C+^{16}O	12	4	48	9
^{12}C+^{16}O	13	6	33	8

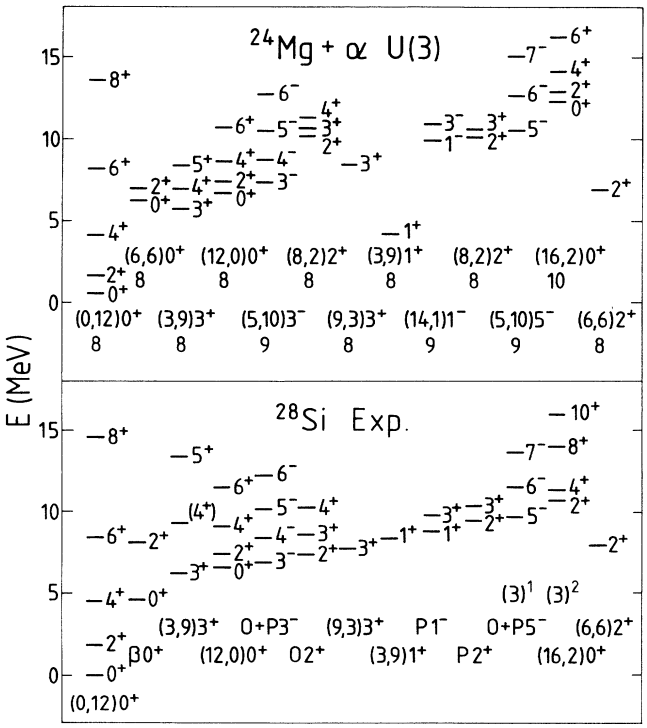

FIG. 1. Band structure of ^{28}Si as established in Ref. [17] in comparison with the model spectrum of the ^{24}Mg+α system. The bands are labeled by the $(\lambda,\mu)K^\pi$ quantum numbers. In the model spectrum the relative motion quantum number $n_{\pi_{\rm Mg\alpha}}$ is given below the SU(3) labels. In the experimental part the excited-shell-model configurations are also indicated, so, e.g., $(3)^1$ means $(0)^4(1)^{12}(2)^{11}(3)^1$. When no such configuration is shown, it should be understood as $(0)^4(1)^{12}(2)^{12}$. Further notations, β, β instable; O, oblate ($\lambda<\mu$); P, prolate ($\lambda>\mu$).

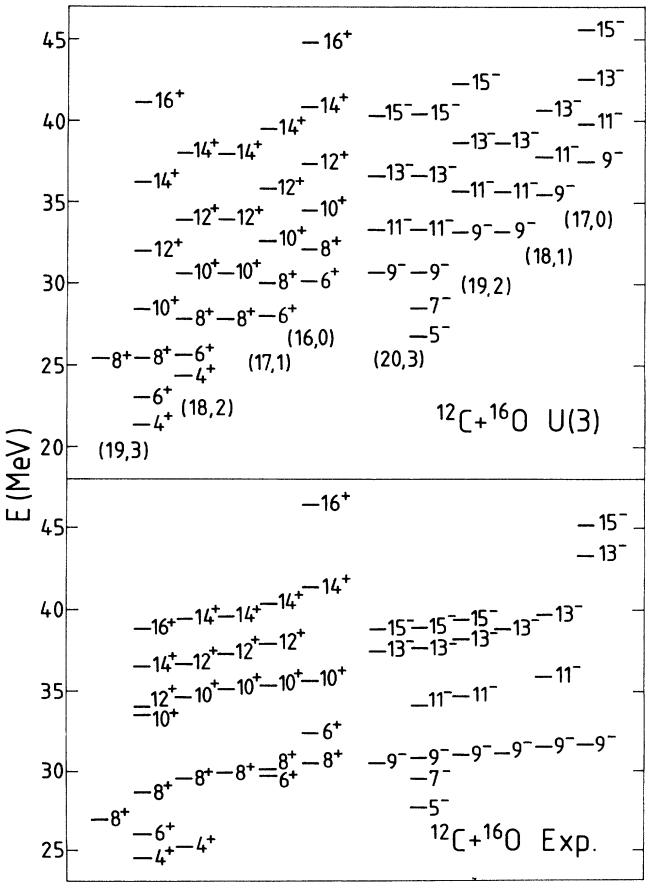

FIG. 2. Experimental and model spectra of the ^{12}C+^{16}O system. When the (λ,μ) numbers are written between two bands, they refer to both. In such a case the corresponding K values are $\min(\lambda,\mu)$, and $\min(\lambda,\mu)-2$; otherwise, $\min(\lambda,\mu)$.

place following the inelastic scattering [21], contrary to the $^{24}Mg \rightarrow {}^{12}C + {}^{12}C$ process. (For further discussion see Refs. [22, 13].)

Table I shows the comparison between the experimental and theoretical level densities. Only partial waves populated well in the $^{24}Mg+\alpha$ and $^{12}C+{}^{16}O$ reactions are given. The data were taken from the [18, 19] compilations, and the energy windows of the model calculation were set between $\min(E_{\rm expt}) - 0.5$ MeV and $\max(E_{\rm expt}) + 0.5$ MeV. Note that in such a comparison the similarity in the numbers of states between the studied reactions and the corresponding cluster configurations is the sign of the validity of the symmetry. In the present case these numbers are comparable, and differ significantly from those of the other configuration. This fact combined with the similarities between the experimental and model spectra of the figures indicate the presence of an approximate multichannel dynamical symmetry. It is proper to note here that the single channel symmetry, serving as a building block of the present multichannel one, is valid also approximately in the ^{28}Si nucleus [2].

In conclusion, I have discussed the possibility of the appearance of a multichannel dynamical symmetry in atomic nuclei. It is related to the presence of different cluster configurations of the same nucleus, and it is a consequence of the single channel dynamical symmetry combined with the antisymmetry of the total wave function. The multichannel symmetry establishes a connection between the cluster-cluster interactions of different configurations. Its signature can be the correlated distribution of different cluster states at low and high energies, the latter ones observed as resonances in heavy ion reactions. The $^{24}Mg+\alpha$ and $^{12}C+{}^{16}O$ data indicate the approximate validity of the multichannel dynamical symmetry in the ^{28}Si nucleus.

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